On the role of quantum corrections in flux compactifications

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hep-th/0508043 (with M. Berg, B. Körs)
work in progress with M. Berg, E. Pajer

Motivation

- Better understanding of the vacuum structure of type IIB compactifications with fluxes
- Soft supersymmetry breaking terms from string theory (direct relevance for LHC!)

Overview

- Review type IIB compactifications with fluxes
- Large volume scenario
- Soft susy breaking terms
- Effects of string loop corrections
- Outlook

Type IIB

- Massless (bosonic) spectrum:

$$g_{MN},\ \phi,\ B_{MN},\ C,\ C_{MN},\ C_{MNPQ}$$

- 10-dimensional action:

$$S_{IIB} \sim \int d^{10}x\sqrt{-g}\left\{e^{-2\phi}\left[R+(\partial\phi)^2\right] - F_1^2 - G_3\cdot\bar{G}_3 - \tilde{F}_5^2\right\}$$

$$+\int e^{\phi} C_4 \wedge G_3 \wedge \bar{G}_3$$

with $G_3 = F_3 - SH_3$, $S = e^{-\phi} + iC_0$

and $\tilde{F}_5 = F_5 - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2 \wedge F_3$

- Type IIB string has symmetry under orientation reversal of the string worldsheet; mod out this symmetry $\Longrightarrow$ orientifold (reduced supersymmetry)

- Calabi-Yau compactification of IIB orientifold $\Longrightarrow\ \mathcal{N}=1,\ d=4$ supergravity

- Moduli problem

Complex structure: U^α (number given by $h^{2,1}$)

Kähler: $T^j = \tau^j + iC^j$ (number given by $h^{1,1}$)

$\mathrm{volume}(\Sigma_4^j)$ $\qquad \int_{\Sigma_4^j} C_4$

Background Fluxes

- IIB string theory contains 2-form fields
- Kinetic term in 10 dimensions:

$$\int d^{10}x \sqrt{-G} F_{IJK} F_{LMN} G^{IL} G^{JM} G^{KN}$$

- Internal components G^{il} of the metric correspond to the moduli fields
- $\langle F_{ijk} \rangle \neq 0$: Potential for the moduli
  [Polchinski, Strominger]
- $\int d^{10}x \sqrt{-g}\, G_3 \cdot \bar{G}_3 \quad \Rightarrow \quad W_{\text{flux}} = \int_{\text{CY}} G_3 \wedge \Omega_3$

[Giddings, Kachru, Polchinski]

$\mathcal{N}=1,\ d=4$ Supergravity

$$\frac{\mathcal{L}_{\text{bos}}}{(-G)^{1/2}}=\frac{1}{2\kappa^2}R-K_{,\bar{\imath}j}D_\mu\bar{\phi}^{\bar{\imath}}D^\mu\phi^j-\frac{1}{4}\mathrm{Re}(f_{ab}(\phi))F^a_{\mu\nu}F^{b\mu\nu}$$

$$-\frac{1}{8}\mathrm{Im}(f_{ab}(\phi))\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^b_{\rho\sigma}-V(\phi,\bar{\phi})$$

- $K_{,\bar{\imath}j}=\dfrac{\partial^2K(\phi,\bar{\phi})}{\partial\bar{\phi}^{\bar{\imath}}\partial\phi^j}$, K Kählerpotential
- f_{ab} gauge kinetic function (holomorphic)
- $V(\phi,\bar{\phi})=e^K(K^{\bar{\imath}j}D_{\bar{\imath}}\overline{W}D_jW-3|W|^2)+\mathrm{Re}(f_{ab})\mathcal{D}^a\mathcal{D}^b$
- $D_jW\equiv\partial_{\phi^j}W+\partial_{\phi^j}KW$, W Superpotential (holom.)

$\equiv F_j$

- $W_{\rm flux}$ leads to potential for dilaton and c.s. moduli
- Compactification manifolds stays Calabi-Yau (in IIB!)
- KKLT: [Kachru, Kallosh, Linde, Trivedi]

  Additional contribution to superpotential from D7-branes wrapped around 4-cycles Σ^j

  Gaugino condensation on D7:

$$W_{np} \sim e^{-af^j}$$

$$f^j_{\rm tree} = T^j = \tau^j + iC^j$$

- Supersymmetric minima:

$$D_{T^j}W = D_{U^\alpha}W = D_S W = 0$$

Drawbacks of KKLT

- $W = W_{\rm flux} + W_{\rm np} = W(S,U) + A(S,U)e^{-aT}$

  $K = -\ln(S+\bar{S}) - 3\ln(T+\bar{T}) + K_{\rm cs}(U,\bar{U})$

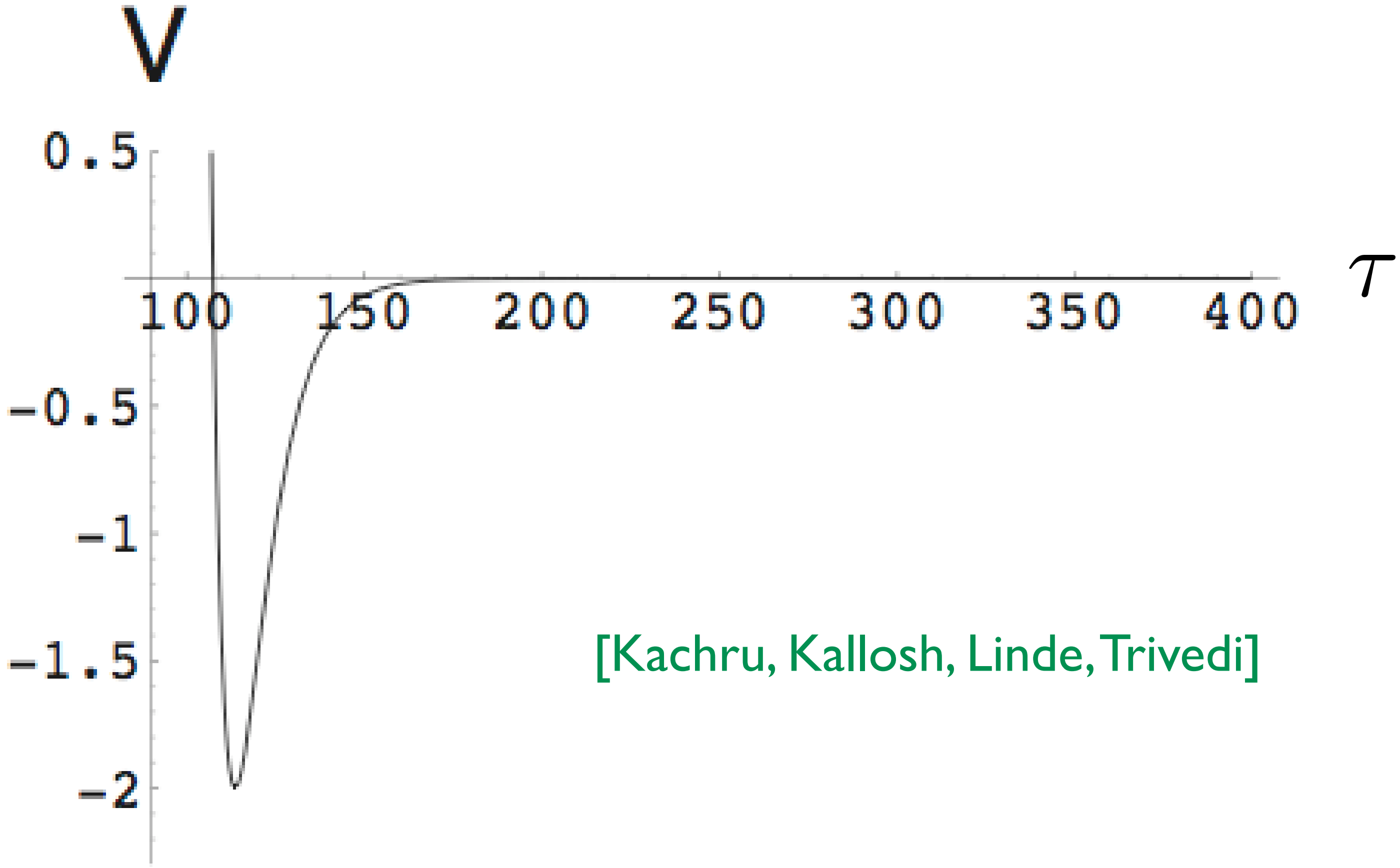
- Stabilization of S and U by $D_U W = 0 = D_S W$

  $W = W_0 + Ae^{-aT} \quad , \quad T = \tau + iC$

  $D_T W = 0 \Longrightarrow W_0 = -Ae^{-a\tau}(1 + \frac{2}{3}a\tau)$

  $\longrightarrow \quad W_0$ very small

- Supersymmetric minimum is AdS:



[Kachru, Kallosh, Linde, Trivedi]

- Supersymmetric minimum is AdS:

  One needs uplift mechanism

$$V = e^K \left(G^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right) + \frac{\epsilon}{\mathcal{V}^\alpha}$$

- Generically the mass matrix has negative eigenvalues!

  [Choi, Falkowski, Nilles, Olechowski, Pokorski]

- Case by case study necessary

  [Lüst, Reffert, Scheidegger, Schulgin, Stieberger]

Large Volume Scenario (LVS)

[Balasubramanian, Berglund, Conlon, Quevedo]

- W_0 of order $\mathcal{O}(1)$ $\longleftarrow$ more generic
- $D_T W \neq 0$ [Balasubramanian, Berglund]
- α'-corrections not negligible [Becker, Becker, Haack, Louis]

$$K = -2\ln(\mathcal{V}) + \ldots \to -2\ln(\mathcal{V} + \tfrac{1}{2}\xi S_1^{3/2}) + \ldots$$

$$\xi = -\zeta(3)\chi/(2(2\pi)^3)$$

- Look at particular direction in Kähler cone

$$\mathcal{V}^{2/3} \sim \tau_b \gg 1 \quad (\mathcal{V} \sim 10^{15} l_s)$$

$$\tau_i \sim \ln \mathcal{V}$$

- Non-supersymmetric AdS minima
- Generically no tachyons after uplift (cf. below)
- Interesting pattern for soft susy breaking terms, e.g. gaugino masses

[Conlon, Quevedo; Choi, Falkowski, Nilles, Olechowski]

$$M_a \sim \frac{m_{3/2}}{\ln(M_p/m_{3/2})}\ , \quad m_{3/2} \sim \frac{M_p}{\mathcal{V}}$$

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$$M_a \sim \frac{m_{3/2}}{\ln(M_p/m_{3/2})}\,, \qquad m_{3/2} \sim \frac{M_p}{\mathcal{V}}$$

Question: What effect do string loop corrections have on LVS?

Review LVS (continued)

- Need Calabi-Yau with:
  - ★ Basis of 4-cycles with $\tilde{\chi} = 1$ [Witten]
  - ★ "Swiss cheese" form [Conlon, Quevedo, Suruliz]

Review LVS (continued)

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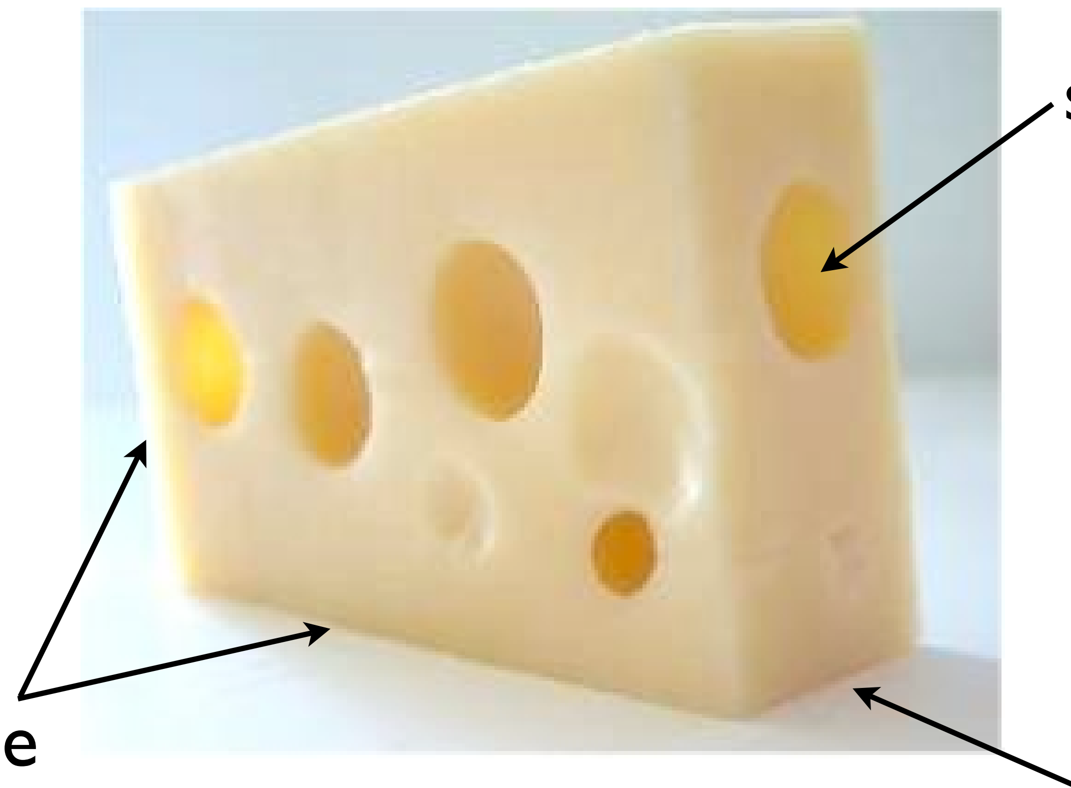


- I.e.

$$\mathcal{V} = \tau_b^{3/2} - \Big(\sum_i a_i \tau_i\Big)^{3/2} - \ldots - \Big(\sum_i b_i \tau_i\Big)^{3/2}$$

- Examples in [Denef, Douglas, Florea]

- Here for simplicity always hypersurface in $\mathbb{P}^4_{[1,1,1,6,9]}$:

  - $h_{1,1} = 2$

    $h_{2,1} = 272$

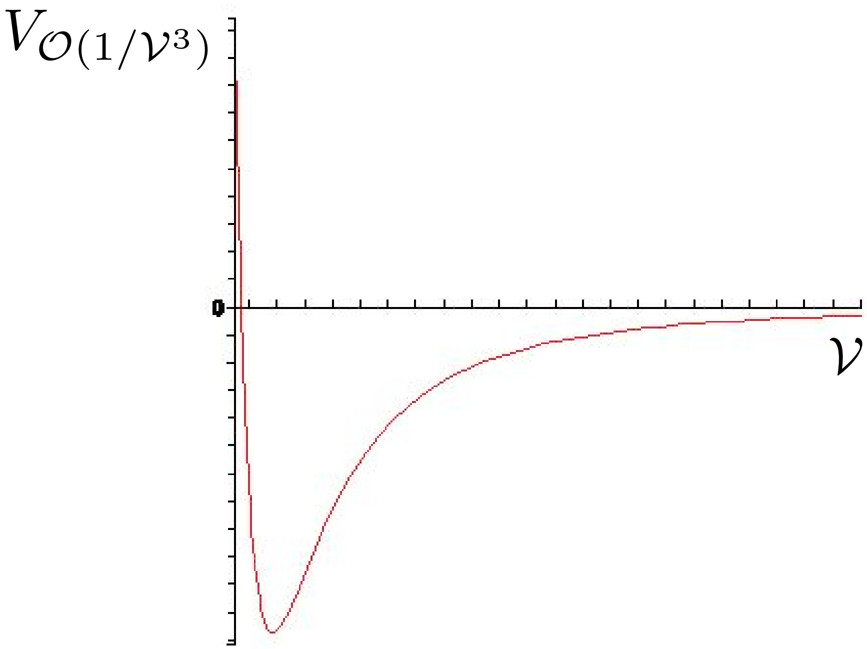
  - $\mathcal{V} = \frac{1}{9\sqrt{2}}\left(\tau_b^{3/2} - \tau_s^{3/2}\right)$

- Look for minimum with

$$\tau_b^{3/2} \sim \mathcal{V} \,, \quad a\tau_s \sim \ln \mathcal{V} \quad (\Longrightarrow \; e^{-a\tau_s} \sim \mathcal{V}^{-1})$$

- Large volume expansion of the potential (already minimized w.r.t. imaginary part in T_s)

$$V_{\mathcal{O}(1/\mathcal{V}^3)} = \frac{12\sqrt{2}|A|^2 a^2 \sqrt{\tau_s} e^{-2a\tau_s}}{\mathcal{V} S_1} - \frac{2a|AW_0|\tau_s e^{-a\tau_s}}{\mathcal{V}^2 S_1} + \xi \frac{3|W_0|^2 \sqrt{S_1}}{8\mathcal{V}^3}$$



$$\Longrightarrow \xi > 0 \,, \quad \text{i.e. } \chi < 0$$

- Minimize w.r.t. $\tau_s, \mathcal{V}$:

$$\tau_s \sim S_1 \xi^{2/3} \quad , \qquad \mathcal{V} \sim \frac{\xi^{1/3}\sqrt{S_1}|W_0|}{a|A|} e^{a\tau_s}$$

- Also minimum in S and U :

$$V = \underbrace{e^K G^{a\bar{b}} D_a W D_{\bar{b}} \bar{W}}_{\sim \mathcal{V}^{-2}} + \underbrace{V_{\mathcal{O}(1/\mathcal{V}^3)} + \ldots}_{\sim \mathcal{V}^{-3}}$$

- Minimum (non-supersymmetric) AdS; needs uplift
  Can be done without changing $\tau_s, \mathcal{V}$ much

[Conlon, Quevedo, Suruliz; Choi, Falkowski, Nilles, Olechowski]

Soft susy breaking terms

- $$\mathcal{L}_{\rm soft} = -\Big(\tfrac{1}{2}M_a\lambda^a\lambda^a + \tfrac{1}{6}a^{IJK}\phi_I\phi_J\phi_K + \tfrac{1}{2}b^{IJ}\phi_I\phi_J\Big) + \text{c.c.}$$
  $$-(m^2)^{IJ}\phi_I^*\phi_J$$

- These are determined by moduli F-terms

- E.g. gaugino masses:

$$M_a = \frac{1}{2}\frac{1}{\mathrm{Re}f_a}\sum_\alpha F^\alpha\partial_\alpha f_a$$

$$F^\alpha = e^{K/2}G^{\alpha i}D_iW$$

- In LVS: SM gauge group arises from D7-branes wrapped around small cycles

$$\mathrm{Re} f_a \sim g_a^{-2} \sim \tau_a + h_a(S,U)$$

- However

$F^U = 0$ (without loop corrections)

$$F^S = \mathcal{V}^{-2}$$

$$F^a = \mathcal{V}^{-1}$$

$$\Longrightarrow \; M_a = \frac{1}{2}\frac{1}{\mathrm{Re} f_a} F^a \overbrace{\partial_a f_a}^{=1} + (\text{suppressed in } \mathcal{V}^{-1})$$

- In $\mathbb{P}^4_{[1,1,1,6,9]}$ model:

$$F^s = 2\tau_s e^{K/2}\bar{W}_0\Big((1-\frac{3}{4a\tau_s}) - 1 + \mathcal{O}((a\tau_s)^{-2})\Big) + \mathcal{O}(\mathcal{V}^{-2})$$

$\longrightarrow$ Cancellation at leading order

- $$M_a \sim \frac{m_{3/2}}{\ln(M_p/m_{3/2})}$$

[Conlon, Quevedo;
Choi, Falkowski, Nilles, Olechowski]

- Similar for other soft susy breaking terms

[Abdussalam, Conlon, Quevedo, Suruliz]

How stable are existence and features of LVS vacua against other quantum corrections?

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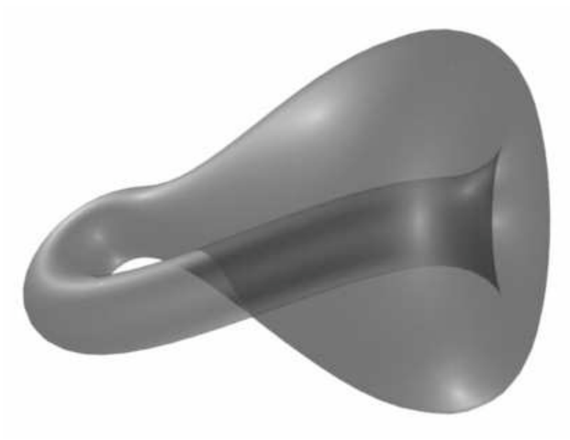
- α'-corrections [Conlon, Quevedo, Surulitz]

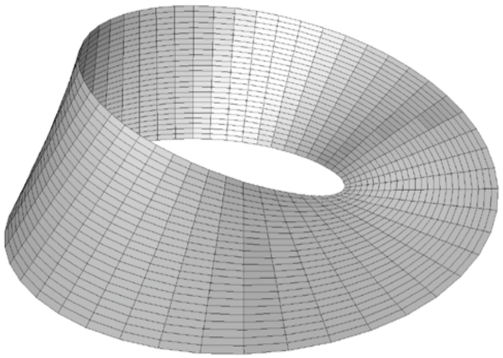
How stable are existence and features of LVS vacua against other quantum corrections?

- α'-corrections [Conlon, Quevedo, Surulitz]
- No discussion of additional loop-corrections in presence of D-branes/O-planes:

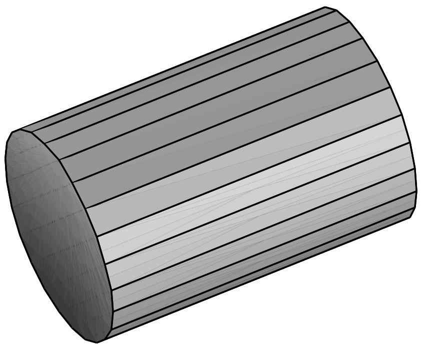
Kleinbottle:



Möbius strip:



Cylinder:



1-loop Kähler potential

- Not known for the $\mathbb{P}^4_{[1,1,1,6,9]}$ model

- Result for $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$:

$$\boxed{K^{(1)} = c\sum_{I=1}^{3}\left[\frac{E_2(U^I)}{(S+\bar{S})(T^I+\bar{T}^I)} + \frac{E_2(U^I)}{(T^J+\bar{T}^J)(T^K+\bar{T}^K)}\bigg|_{K\neq I\neq J}\right]}$$

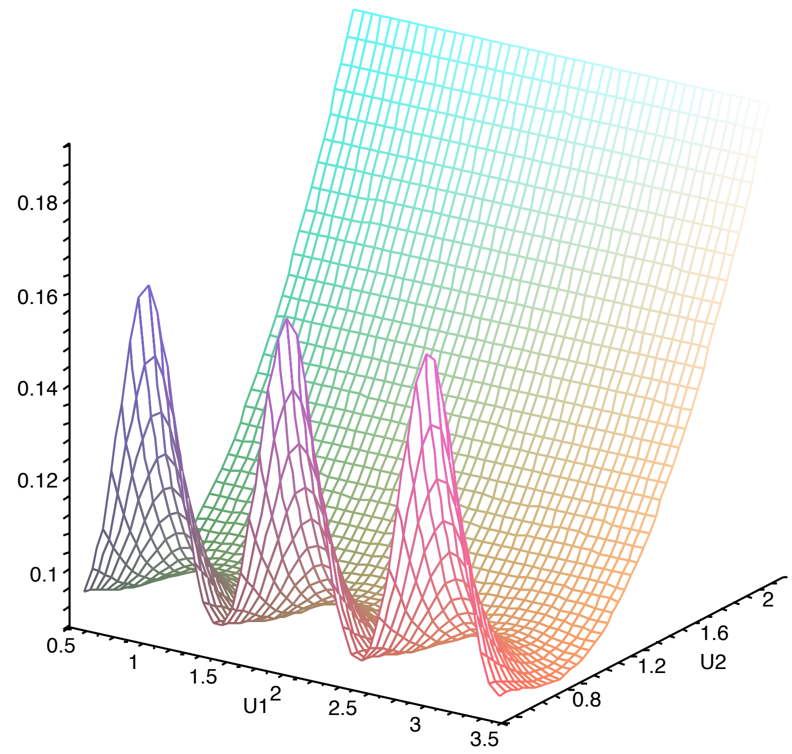
[Berg,Haack,Körs]

where $c = 15/(2\pi^6)$ and

$$E_2(U) = \sum_{(n,m)\neq(0,0)} \frac{U_2^2}{|n+mU|^4}$$

(Eisenstein series)

- The function $cE_2(U)$:



- Gets large for large U_2 : proportional to U_2^2

  $\longrightarrow$ Corrections can get large for degenerate tori

- Compared to α'-correction, 1-loop correction is suppressed in S - but leading in T-expansion

- Generalization to $\mathbb{P}^4_{[1,1,1,6,9]}$ model?

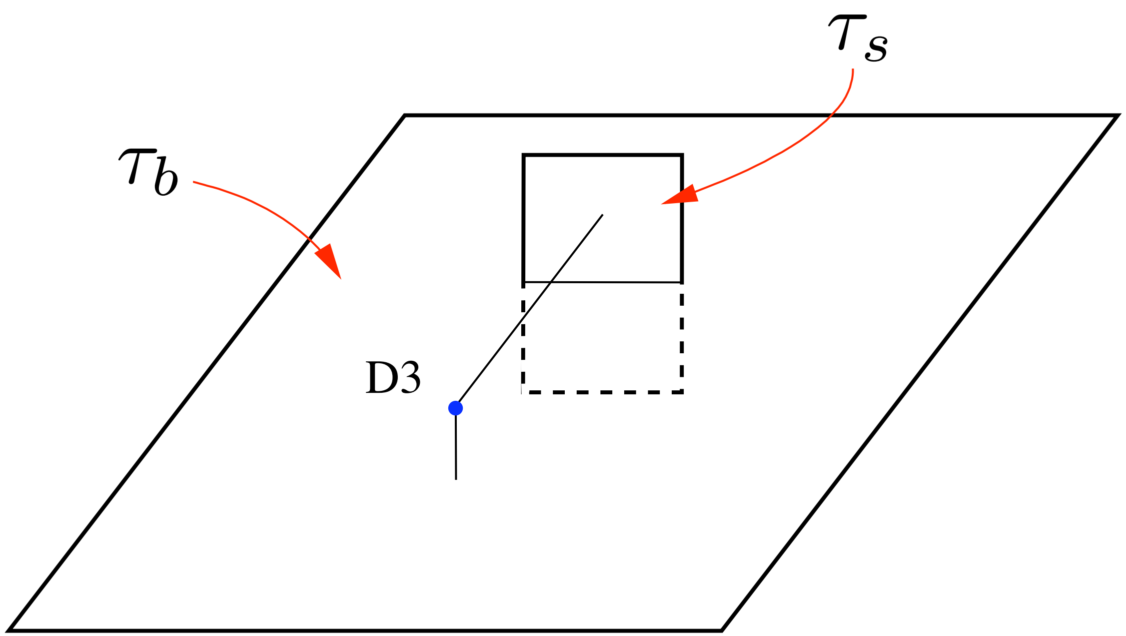
  Does one expect corrections $\delta K \sim \frac{E(U)}{S_1 \tau_s}$?

- Could lead to very strong constraints for LVS

Origin of loop corrections

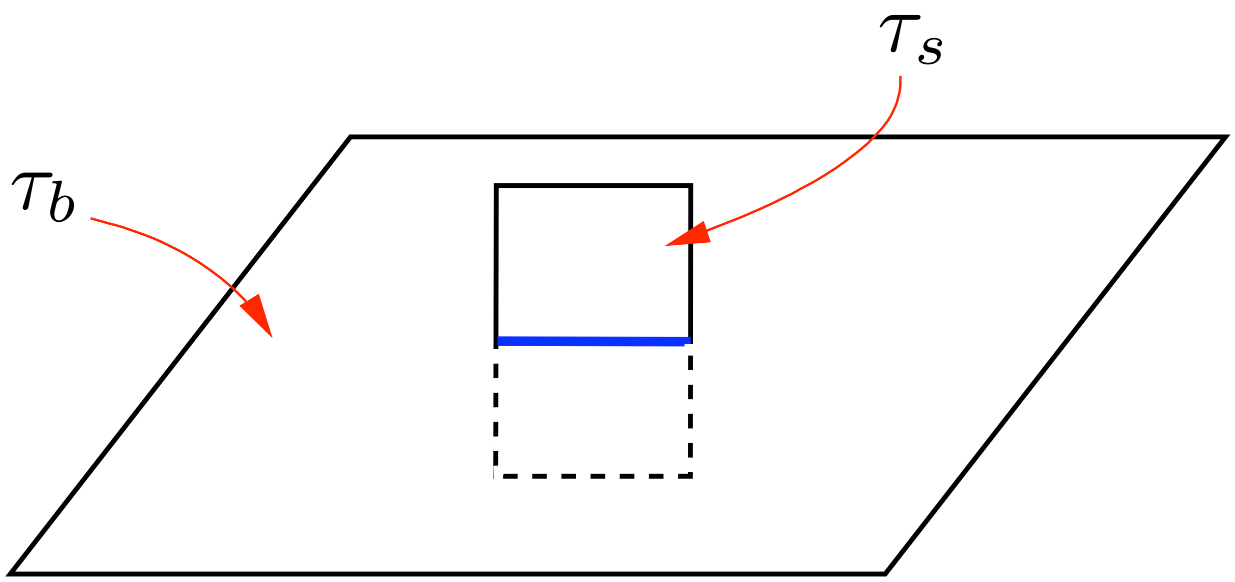
- Exchange of Kaluza-Klein modes $m_{KK}^2 \sim t^{-1}$

2-cycle volume



This effect should generalize to $\mathbb{P}^4_{[1,1,1,6,9]}$ model

- Exchange of strings, winding around 1-cycles within the intersection of two D7-brane stacks



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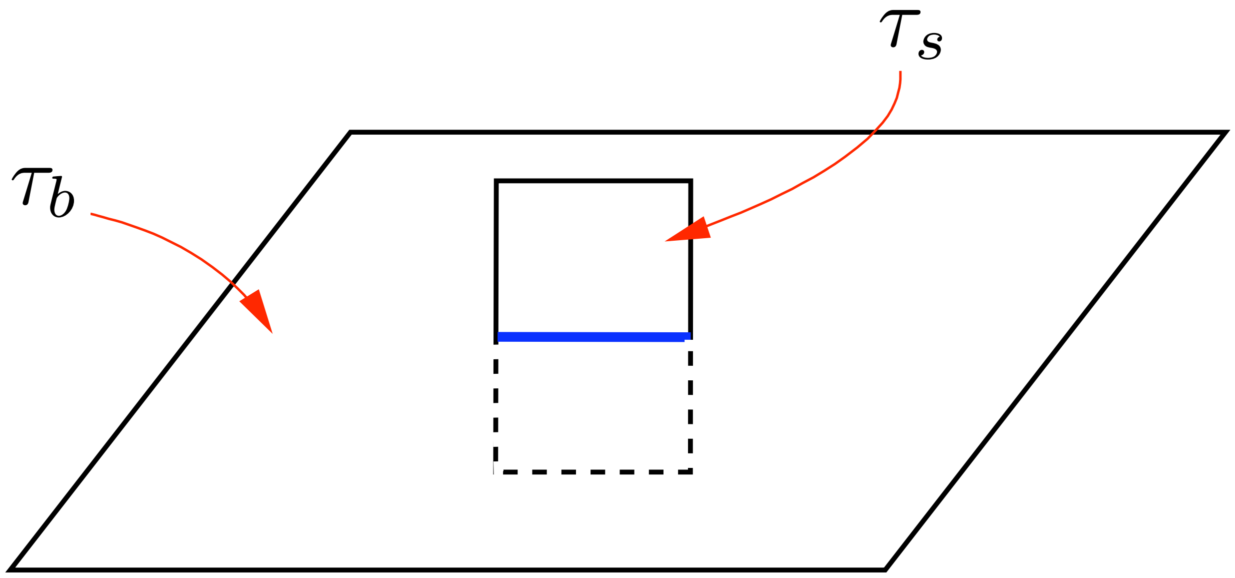


- However: In $\mathbb{P}^4_{[1,1,1,6,9]}$ model the D7-branes do not intersect $\Longrightarrow$ does not generalize (other models?)

- Closer look at 1-loop calculation; it actually gives

$$\int d^4x\sqrt{-g}\Big[\tilde{\mathcal{V}}e^{-2\phi}R+\tilde{t}^I\partial_{U^I}\partial_{\bar{U}^I}E_2(U^I)\partial_\mu U^I\partial^\mu\bar{U}^I+\ldots\Big]$$

$$\xrightarrow[\text{rescaling}]{\text{Weyl}}\int d^4x\sqrt{-g}\Big[R+\frac{\tilde{t}^I\partial_{U^I}\partial_{\bar{U}^I}E_2(U^I)}{\tilde{\mathcal{V}}e^{-2\phi}}\partial_\mu U^I\partial^\mu\bar{U}^I+\ldots\Big]$$

- $\delta K=\sum_I\frac{t^IE_2(U^I)}{S_1\mathcal{V}}+\sum_I\frac{E_2(U^I)}{t^I\mathcal{V}}\qquad(\tau=\tilde{\tau}e^{-\phi})$

- Get old result by using:

$$\mathcal{V}=t^I\tau^I$$

$$\tau^1=t^2t^3$$ (& permutations)

- More plausible form of 1-loop corrections:

$$K^{(1)} = \frac{\sqrt{\tau_s} E_s^{(K)}(U)}{S_1 \mathcal{V}} + \frac{\sqrt{\tau_b} E_b^{(K)}(U)}{S_1 \mathcal{V}} \left(+ \frac{E_s^{(W)}(U)}{\sqrt{\tau_s}\mathcal{V}} + \frac{E_b^{(W)}(U)}{\sqrt{\tau_b}\mathcal{V}} \right)$$

with unknown functions $E_s^{(K)}(U), E_b^{(K)}(U)$

- Note:

$$\mathcal{V} \sim \tau_b^{3/2} \quad \Longrightarrow \frac{\sqrt{\tau_b} E_b^{(K)}}{S_1 \mathcal{V}} \sim \frac{E_b^{(K)}}{S_1 \mathcal{V}^{2/3}}$$

more leading than α'-correction

- $V = V_{\rm np1} + V_{\rm np2} + V_3$

- $V_{\rm np1} = e^{K_{cs}} \dfrac{24\sqrt{2}a^2|A|^2\tau_s^{3/2}e^{-2a\tau_s}}{\Delta\mathcal{V}}$

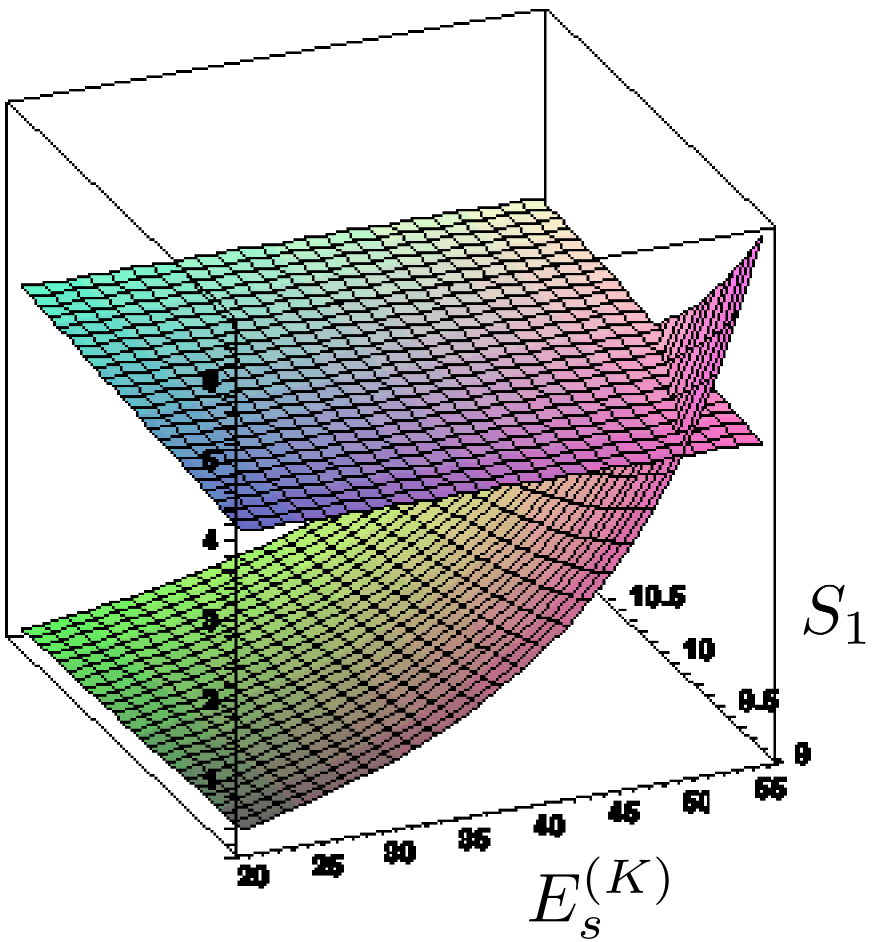
$$V_{\rm np2} = -e^{K_{\rm cs}} \frac{2a|AW_0|\tau_s e^{-a\tau_s}}{S_1\mathcal{V}^2}\left[1 + \frac{6E_s^{(K)}}{\Delta}\right]$$

$$V_3 = e^{K_{cs}} \frac{3|W_0|^2}{8\mathcal{V}^3}\left[\sqrt{S_1}\xi\left(1 + \frac{\pi^2}{3\zeta(3)S_1^2}\right) + \frac{4\sqrt{\tau_s}(E_s^{(K)})^2}{S_1^2\Delta}\right]$$

- $\Delta \equiv \sqrt{2}S_1\tau_s - 3E_s^{(K)}$ $\quad$ $E_b^{(K)}$ appears at $\mathcal{O}(\mathcal{V}^{-10/3})$

- The two terms in V_3 :

$(A=1, W_0=1,$
$a=2\pi/8, \xi=1.31)$



- Mainly quantitative changes

- $\log_{10}\mathcal{V} \sim -0.129E_s^{(K)}+13.99$, $\tau_s \sim -0.379E_s^{(K)}+41.98$

- $\chi=0$ possible?

- What about gaugino masses?

  Before there was a cancellation in F^s

- $F^U = \mathcal{V}^{-2}$

  $F^S = \mathcal{V}^{-2}$

  $$F^s = 2\tau_s e^{K/2}\bar{W}_0\Big(-\frac{3}{4a\tau_s} - \frac{9\bar{W}_0}{16a^2\tau_s} + \frac{9\bar{W}_0(12aE_s^{(K)} - S_1)}{64S_1a^3\tau_s^3} + \ldots\Big)$$

- Loop corrections only appear sub-sub-leading!

Conclusion

- LVS seems surprisingly stable against 1-loop corrections
- Is our conjecture right?
- Further corrections? (Higher loops? α'?)
- Perturbative volume stabilization?
- Departure from "swiss cheese" form?

Winding strings in a CY?