From D-Branes to M-Branes:
Up from String Theory

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Plan

• Introduction
• What is String Theory?
• D-branes
• M-Theory
• M-branes
• Conclusions
The World (as seen from CERN)

- Standard Model
- GUT
- LHC
- Quantum Gravity

Graph showing coupling strength vs. energy in GeV, with lines indicating SU(3), SU(2), U(1), and Gravity.
• The Standard Model of particle physics is incredibly successful
  – Describes structure and interactions of all matter* from deep inside nucleons upwards

• General Relativity is also very successful
  – Describes physics on large to cosmologically large scales

• But they are famously hard to reconcile
  – GR is classical
  – Standard Model is an effective low-energy theory

* Well maybe 20% of it
• String Theory seems capable of describing all that we expect in one consistent framework:
  – Quantum Mechanics and General Covariance
  – Standard Model-like gauge theory
  – General Relativity
  – Cosmology (inflation)?
What is String Theory?

String Theory Summarized:

I JUST HAD AN AWESOME IDEA. SUPPOSE ALL MATTER AND ENERGY IS MADE OF TINY, VIBRATING “STRINGS.”

OKAY. WHAT WOULD THAT IMPLY?

I DUNNO.

Well in fact we know an awful lot (although not what string theory really is)
• (perturbative) quantum field theory assumes that the basic states are point-like particles
  – Interactions occur when two particles meet:
• Point particles are replaced by 1-dimensional strings
  – Multitude of particles correspond to the lowest harmonics of an infinite tower of modes
• Feynman diagrams merge and become smooth surfaces

• Only one coupling constant: $g_s$
  - Vacuum expectation value of a scalar field – the dilaton
• A remarkable feature is that gravity comes out of the quantum theory, unified with gauge forces

• The dimension of spacetime is 10
  • Must compactify to 4D
  • There appear to be a plethora of models with Standard Model-like behaviour
    – Estimated $10^{500}$ 4D vacua

Landscape
The World (as seen from the Multiverse)
D-Branes

• In addition to strings, String Theory contains D-branes:
  – p-dimensional surfaces in spacetime
    • 0-brane = point particle
    • 1-brane = string
    • 2-brane = membrane
    • etc.
  – Non-perturbative states: Mass \( \sim 1/g_s \)
  – End point of open strings
• These open strings give dynamics to the D-brane
• At lowest order the dynamics are those of U(n) Super-Yang-Mills
– $g_{YM}$ is determined from $g_s$
– Light modes on the worldvolume arise from the open strings (Higg’s mechanism)
  • Mass = length of a stretched string between the branes
– Vast applications to model building
• At low energy D-branes appear as (extremal) charged black hole solutions
  – Singularity is extended along p-dimensions

• Thus D-branes have both a Yang-Mills description as well as a gravitational one
  – Exact counting of black hole microstates
  – AdS/CFT
What is M-Theory?

• But not all is perfect in String Theory
  – Are there really $10^{500}$ vacua?
  – Can one make any observable predictions?

• What is String Theory really?
  – The construction of vibrating interacting strings is just a perturbative device, not a definition of the theory
    • What are strongly coupled strings?

• Furthermore why 5 perturbative string theories
  – Type I
  – Type II A & B
  – Heterotic $E_8 \times E_8$ & $SO(32)$
Now all 5 are all thought to be related as different aspects of single theory: M-theory.

How?

Duality

Two theories are dual if they describe the same physics but with different variables.

\[ g_s \leftrightarrow \frac{1}{g_s} \]
• The classic example of duality occurs in Maxwell’s equations without sources:

- ‘electric’ variables:

- ‘magnetic’ variables:

Self-dual
• M-theory moduli space:

- 11D Supergravity
- heterotic $E_8 \times E_8$
- heterotic $SO(32)$
- type IIA
- type IIB
- type I
• M-theory moduli space:
  at strong coupling

11D Supergravity

hetero

10D
• M-theory moduli space in 3D:
• An 11D metric tensor becomes a 10D metric tensor plus a vector and a scalar.

\[ g_{\mu \nu} = e^{-2 \frac{\phi}{3}} g_{\mu \nu} e^{4 \frac{\phi}{3}} A_\nu e^{4 \frac{\phi}{3}} A^\mu e^{4 \frac{\phi}{3}} \]
• Thus the String Theory dilaton has a geometric interpretation as the size of the 11th dimension
  – But the vev of is gs
  – String perturbation theory is an expansion about a degenerate 11th dimension
  – As gs→∞ an extra dimension opens up
    • 11D theory in the infinite coupling limit.

• Predicts a complete quantum theory in eleven dimensions: M-Theory
  – Effective action is 11D supergravity
  – Little else is known
<table>
<thead>
<tr>
<th>Type IIA String Theory</th>
<th>M-Theory</th>
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M-Branes

Type IIA String Theory | M-Theory
---|---
0-Branes | gravitational wave along $X^{11}$
Strings | 2-branes
2-branes | 5-branes
4-branes | 5-branes
5-branes | Kaluza-Klein monopoles
M-Branes

Type IIA String Theory  M-Theory

0-Branes  gravitational wave along $X^{11}$
Strings
2-branes
4-branes
5-branes  purely gravitational excitations
5-branes
6-Branes  Kaluza-Klein monopoles

The branes of M-theory
• So there are no strings in M-theory
  – 2-branes and 5-branes
• In particular no open strings and no $g_s$
  – No perturbative expansion
  – No microscopic understanding
• The dynamics of a single M-branes act to minimize their worldvolumes
  – With other fields related by supersymmetry
    • M2 [Bergshoeff, Sezgin, Townsend]
    • M5 [Howe, Sezgin, West]
• What about multiple M-branes?
• In string theory you can derive the dynamics of multiple D-branes from symmetries:
  – Effective theory has 16 supersymmetries and breaks $\text{SO}(1,9) \rightarrow \text{SO}(1,p) \times \text{SO}(9-p)$
  – This is in agreement with maximally supersymmetric Yang-Mills gauge theory

\[
L = - \frac{1}{4} \text{tr} (F^2) - \frac{1}{2} \text{tr} (D^\mu \Psi \Gamma^\mu \Psi) + \text{tr} (\bar{\Psi} \Gamma [X, \Psi]) + \frac{1}{4} \text{tr} ([X_i, X_j]^2)
\]
Can we derive the dynamics of M2-branes from symmetries?

- Conformal field theory
  - Strong coupling (IR) fixed point of 3D SYM
- No perturbation expansion
- The only maximally supersymmetric Lagrangians are Yang-Mills theories
  - Wrong symmetries for M-Theory
  - need $SO(1,2) \times SO(8)$ not $SO(1,2) \times SO(7)$
• Can we derive the dynamics of M2-branes from symmetries?
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• Well that turns out not to be true
• The Yang-Mills theories living on D-branes are determined by the susy variation

\[ \delta \Psi = \Gamma_\mu \Gamma_i D_\mu X^i + [X^i, X^j] \Gamma_{ij} \Gamma_{10} + ... \]

\[ A \otimes A \rightarrow A \]

\[ \delta X^i = [\Lambda, X^i] + [X, [\Lambda, Y]] = [[\Lambda, X], Y] + [X, [\Lambda, Y]] \]

• Here we find a Lie-algebra with a bi-linear anti-symmetric product:

• Closure of the susy algebra leads to gauge symmetry:

• Consistency of this implies the Jacobi identity:
• What is required for M2-branes?
  – Now \(0\) and \(1\) so we require

  \[
  \Gamma_{012}^2 = \Gamma_{012}^2 \Psi = -\Psi [\cdot, \cdot, \cdot] = A \otimes A \otimes A \rightarrow A
  \]

  \[
  \delta \Psi = \Gamma_{\mu}^{\mu} \Gamma_{i}^{i} D_{\mu} X_{i}^{2} + [X_{I}, X_{J}, X_{K}] \Gamma_{IJK}^{2}
  \]

  \[
  \delta X = [X, A, B] + [A, [Y, A, B], Z] + [X, Y, [Z, A, B]]
  \]

  – Thus we need a triple product: 3-algebra

  – Closure implies a gauge symmetry:

  – Consistency requires a generalization of the Jacobi identity (fundamental identity)
• The fundamental identity implies the gauge symmetry \[ \delta X = [X, A, B] \] acts as a (non simple) Lie algebra \( g \) acting on \( \mathfrak{g} \).

• 3-algebra data is equivalent to specifying a Lie-algebra \( \mathfrak{g} \) with a (split) metric and a representation acting on vector space \( \mathbb{R}^n \) (with an invariant metric).
• This gives a maximally supersymmetric Lagrangian with SO(8) R-symmetry [Bagger,NL]

\[ L_{\text{CS}} = \frac{1}{4\pi} \text{tr}(\tilde{A} \wedge d\tilde{A}) + \frac{2}{3} \text{tr}(\tilde{A} \wedge \tilde{A} \wedge \tilde{A}) \]

• ‘twisted’ Chern-Simons gauge theory

• Conformal, parity invariant
• But it turns out to only have one example:
  \[ a, b, c, d = 1, 2, 3, 4 \]

• SU(2) x SU(2) Chern-Simons at level (k, -k) and matter in the bi-fundamental

• Vacuum moduli space:

• Two M2-branes in \( \mathbb{R}^8/\mathbb{Z}_2 \)
  – agrees with M-theory when \( k = 2 \)
• Need to generalize:
  – Weak coupling arises from orbifold
  – Consider $\mathbb{C}^4/\mathbb{Z}_k$

– 12 susys and breaks $\text{SO}(8) \rightarrow \text{SU}(4) \times \text{U}(1)$

• Look for theories with $\text{SU}(4) \times \text{U}(1)$ R-symmetry and $N=6$ supersymmetry
• From the 3-algebra this is achieved if the triple product is no longer totally anti-symmetric:

\[
\]

• Consistency requires a related fundamental identity

• For example we can take (for nxm matrices):

\[
M_{k, n} = \text{Sym}_n \left( \frac{R_8}{Z_k} \right)
\]

• Resulting action is similar to the N=8 case but:
  – U(n)xU(m) Chern-Simons theory at level (k,-k) with matter in the bifundamental

X, Y, Z are Complex Scalar Fields

M_{k, n} = S \text{ym}_n \left( \frac{R_8}{Z_k} \right)
• These theories were first proposed by [Aharony, Bergman, Jafferis and Maldacena]

• They gave a brane diagram derivation
  – Consider the following Hannay-Witten picture
• In terms of the D3-brane SYM worldvolume theory:
  – Integrating out D5/D3-strings and flowing to IR gives a $U(n) \times U(n)$ CS theory with level $(k, -k)$ coupled to bi-fundamental matter
  – $N=3$ is enhanced to $N=6$
• The final configuration is just n M2s in a curved background preserving 3/16 susys.
  – Metric can be written explicitly
  – smooth except where the centre's intersect
  – near horizon limit gives n M2's in $\mathbb{R}^8/\mathbb{Z}_k$.
  – Preserved susy's are enhanced to 6/16.

• Note that this works for all n and all k
  – even k=1,2 where we expect N=8 susy
    • Two supersymmetries are not realized in the Lagrangian (carry U(1) charge)
    • For k=1 even the centre of mass mode is obscured
• One success of these models is an understanding of the mysterious \( n^{3/2} \) growth of the degrees of freedom

- Free energy = \( f(\lambda)n^2 \)
  - \( \lambda = n/k \)
  - \( f(\lambda) = \begin{cases} 
    1 & \lambda << 1 \\
    \lambda^{-1/2} & \lambda >> 1 
  \end{cases} \)

• This has recently been confirmed in Chern-Simons Theory for all \( \lambda \) [Drukker, Marino, Putrov]
• How does one recover D2-branes from this [Mukhi, Papageorgakis]

  – Give a vev to a scalar field
    • breaks $U(n) \times U(n) \to U(n)$ and $SO(8) \to SO(7)$

  – becomes a dynamical $U(n)$ gauge field
    • Similar to a Higgs’s effect where a non-dynamical vector eats a scalar to become dynamical
• What can we learn about M-theory?
  – Hints at microscopic dynamics of M-branes

• e.g. in the N=8 theory one finds mass = area of a triangle with vertices on an M2
• Mass deformations give fuzzy vacua:
  – M2-branes blow up into fuzzy M5-branes
  – Can we learn about M5-branes
    • Also M2s can end on M5’s: Chern-Simons gauge fields become dynamical
• There are also infinite dimensional totally antisymmetric 3-algebras: Nambu bracket

\[
X, Y, Z = (dX \wedge dY \wedge dZ)
\]

– Related to M5-branes?

• Infinitely many totally anti-symmetric 3-algebras with a Lorentzian metric
  – Seem to be equivalent to 3D N=8 SYM but with manifest SO(8) and conformal symmetry
Conclusions

• M-Theory and M-branes are poorly understood but there has been much recent progress:
  – Complete proposal for the effective Lagrangian of $n$ M2’s in $\mathbb{R}^8/\mathbb{Z}_k$
  – Novel highly supersymmetric Chern-Simons gauge theories based on a 3-algebra.
  – Gives a Lagrangian description of strongly coupled 3D super Yang-Mills

• M5-branes remain very challenging as does M-Theory itself but hopefully progress will be made
  – M2-brane CFT’s ‘define’ M-theory in AdS$_4 \times X_7$