From D-Branes to M-Branes:

Up from String Theory

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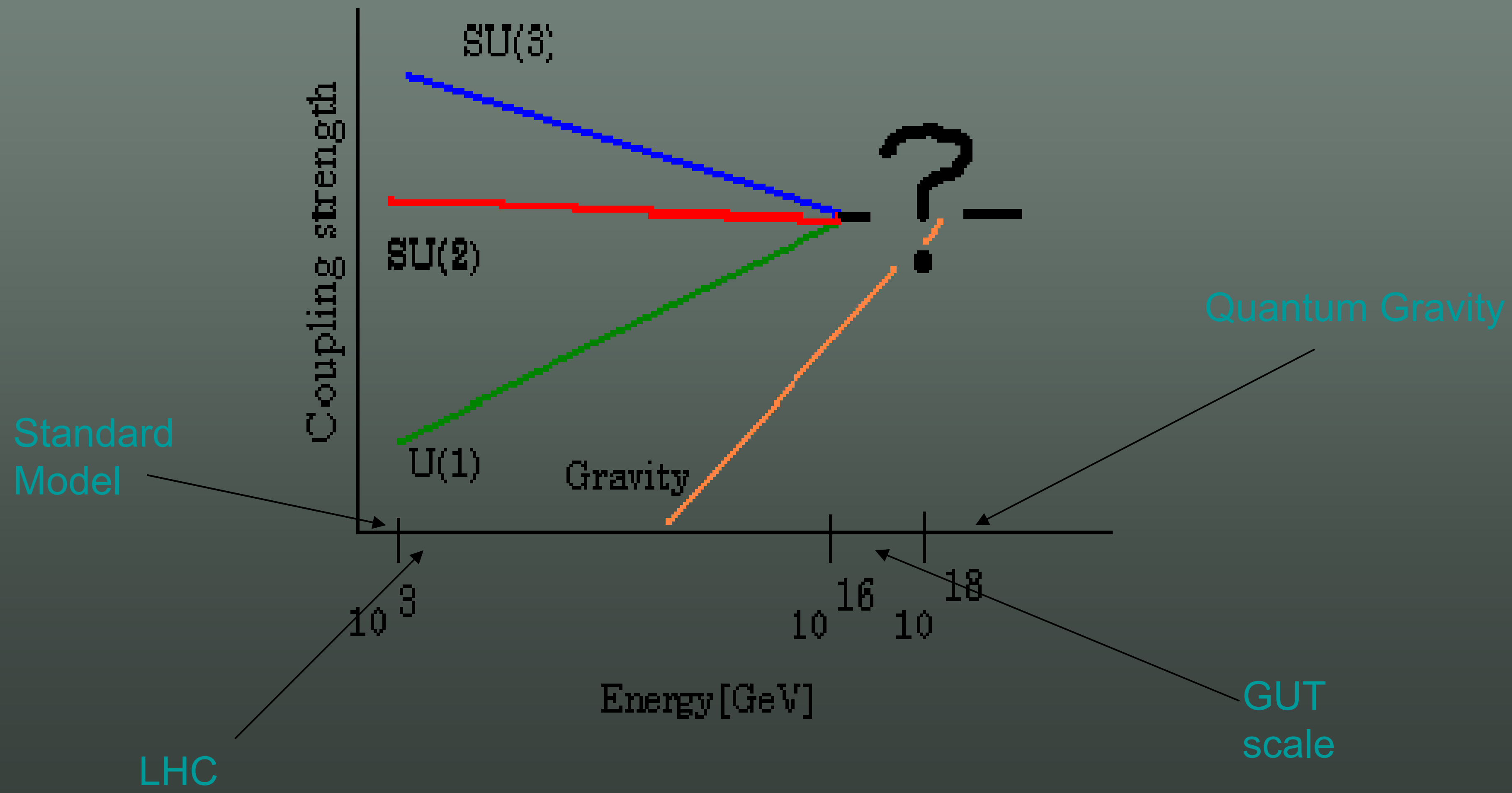
Plan

- Introduction
- What is String Theory?
- D-branes
- M-Theory
- M-branes
- Conclusions

The World (as seen from CERN)

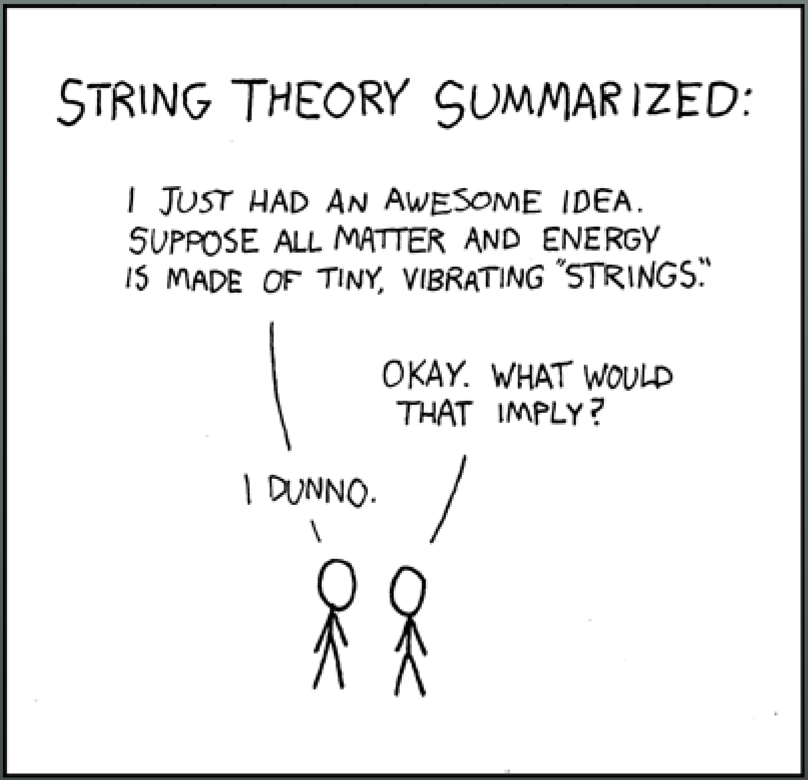


- The Standard Model of particle physics is incredibly successful
  - Describes structure and interactions of all matter* from deep inside nucleons upwards
- General Relativity is also very successful
  - Describes physics on large to cosmologically large scales
- But they are famously hard to reconcile
  - GR is classical
  - Standard Model is an effective low-energy theory

* Well maybe 20% of it

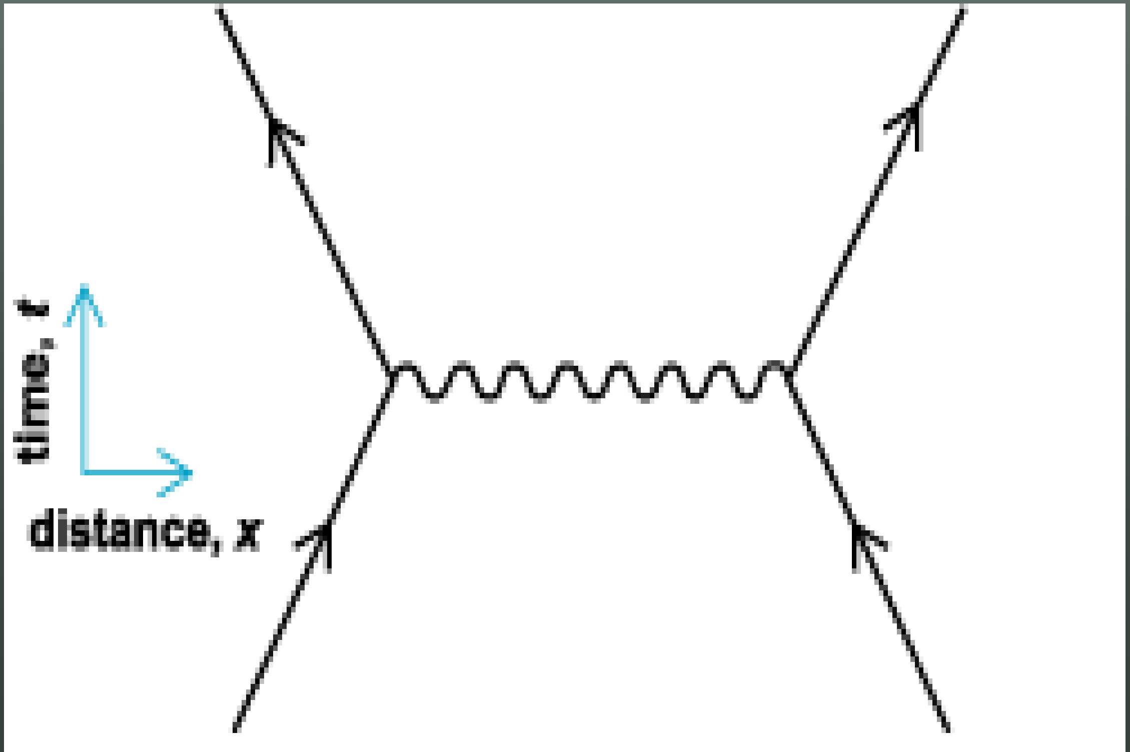
- String Theory seems capable of describing all that we expect in one consistent framework:
  - Quantum Mechanics and General Covariance
  - Standard Model-like gauge theory
  - General Relativity
  - Cosmology (inflation)?

What is String Theory?

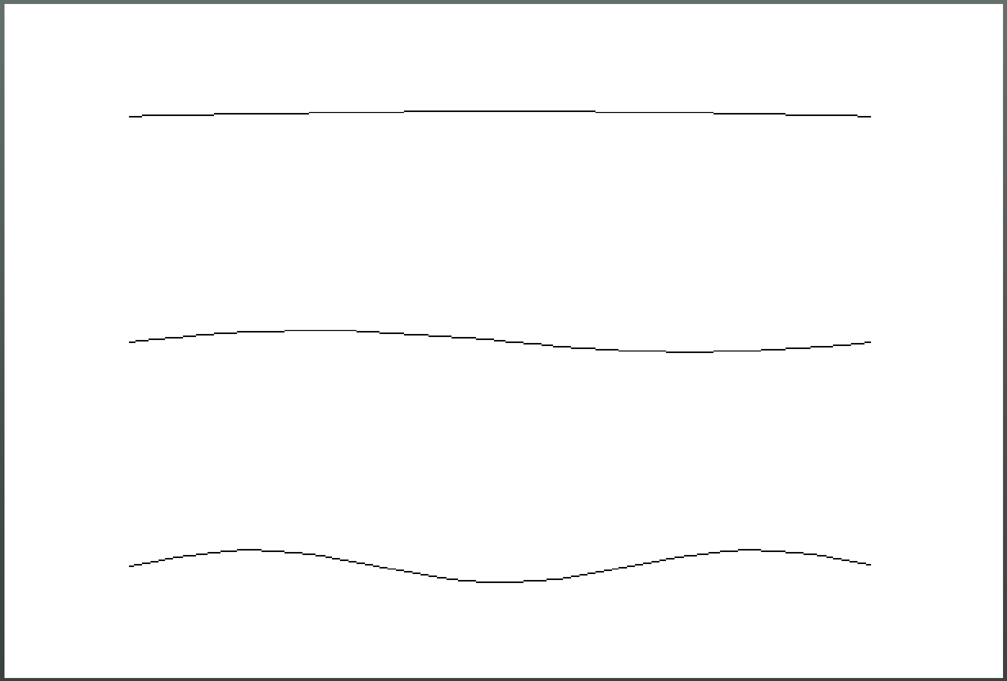


Well in fact we know an awful lot (although not what string theory really is)

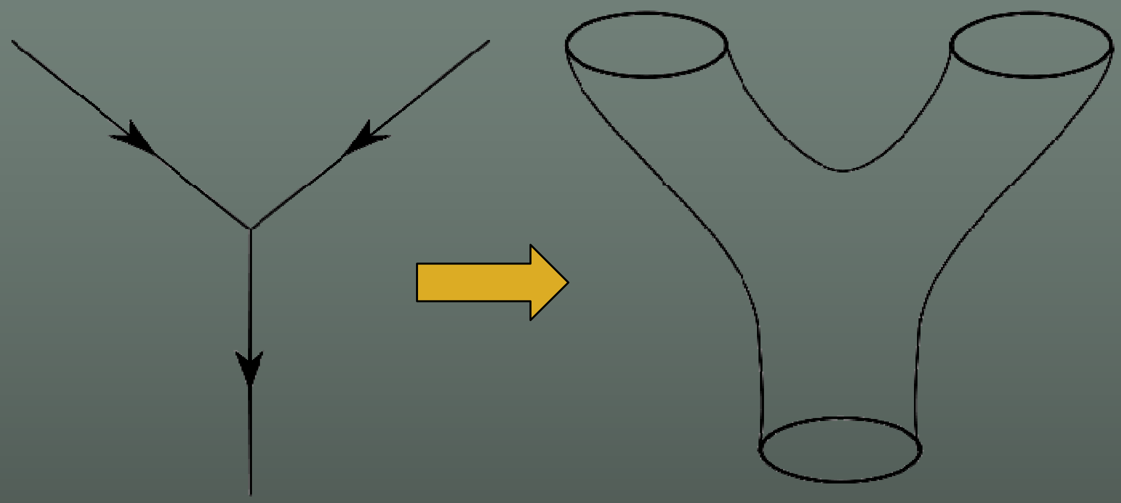
- (perturbative) quantum field theory assumes that the basic states are point-like particles
  - Interactions occur when two particles meet:



- Point particles are replaced by 1-dimensional strings
  - Multitude of particles correspond to the lowest harmonics of an infinite tower of modes



- Feynman diagrams merge and become smooth surfaces



- Only one coupling constant: g_s
  - Vacuum expectation value of a scalar field – the dilaton

- A remarkable feature is that gravity comes out of the quantum theory, unified with gauge forces

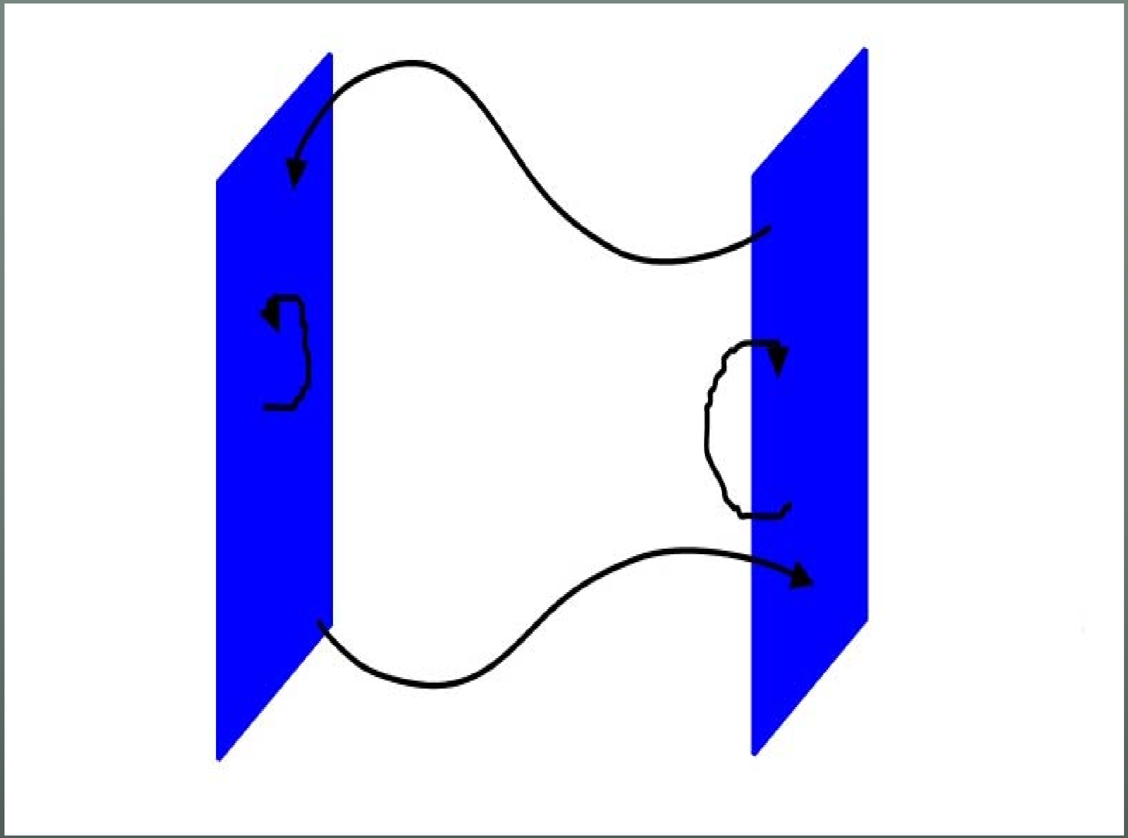
- The dimension of spacetime is 10
  - Must compactify to 4D
  - There appear to be a plethora of models with Standard Model-like behaviour
    - Estimated 10^{500} 4D vacua

Landscape

The World (as seen from the Multiverse)

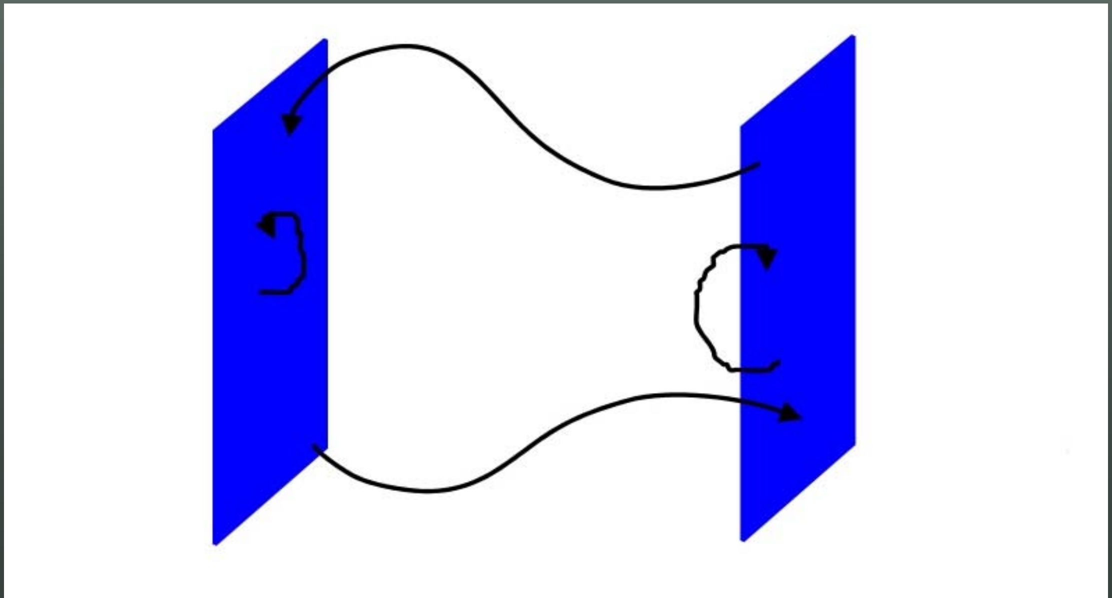
D-Branes

- In addition to strings, String Theory contains D-branes:
  - p-dimensional surfaces in spacetime
    - 0-brane = point particle
    - 1-brane = string
    - 2-brane = membrane
    - *etc*….
  - Non-perturbative states: Mass ~ $1/g_s$
  - End point of open strings

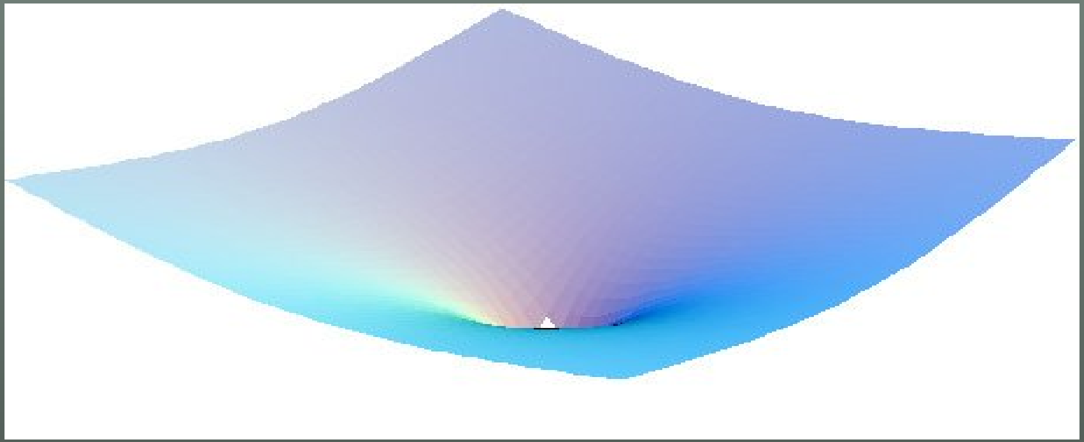


- These open strings give dynamics to the D-brane
- At lowest order the dynamics are those of U(n) Super-Yang-Mills

- g_{YM} is determined from g_s
- Light modes on the worldvolume arise from the open strings (Higg's mechanism)
  - Mass = length of a stretched string between the branes
- Vast applications to model building



- At low energy D-branes appear as (extremal) charged black hole solutions
  - Singularity is extended along p-dimensions



- Thus D-branes have both a Yang-Mills description as well as a gravitational one
  - Exact counting of black hole microstates
  - AdS/CFT

What is M-Theory?

- But not all is perfect in String Theory
  - Are there really 10^{500} vacua?
  - Can one make any observable predictions?
- What is String Theory really?
  - The construction of vibrating interacting strings is just a perturbative device, not a definition of the theory
    - What are strongly coupled strings?
- Furthermore why 5 perturbative string theories
  - Type I
  - Type II A & B
  - Heterotic $E_8 x E_8$ & SO(32)

- Now all 5 are all thought to be related as different aspects of single theory:

M-theory

- How?

Duality

- Two theories are dual if they describe the same physics but with different variables.

e.g. S-duality $g_s \leftrightarrow 1/g_s$

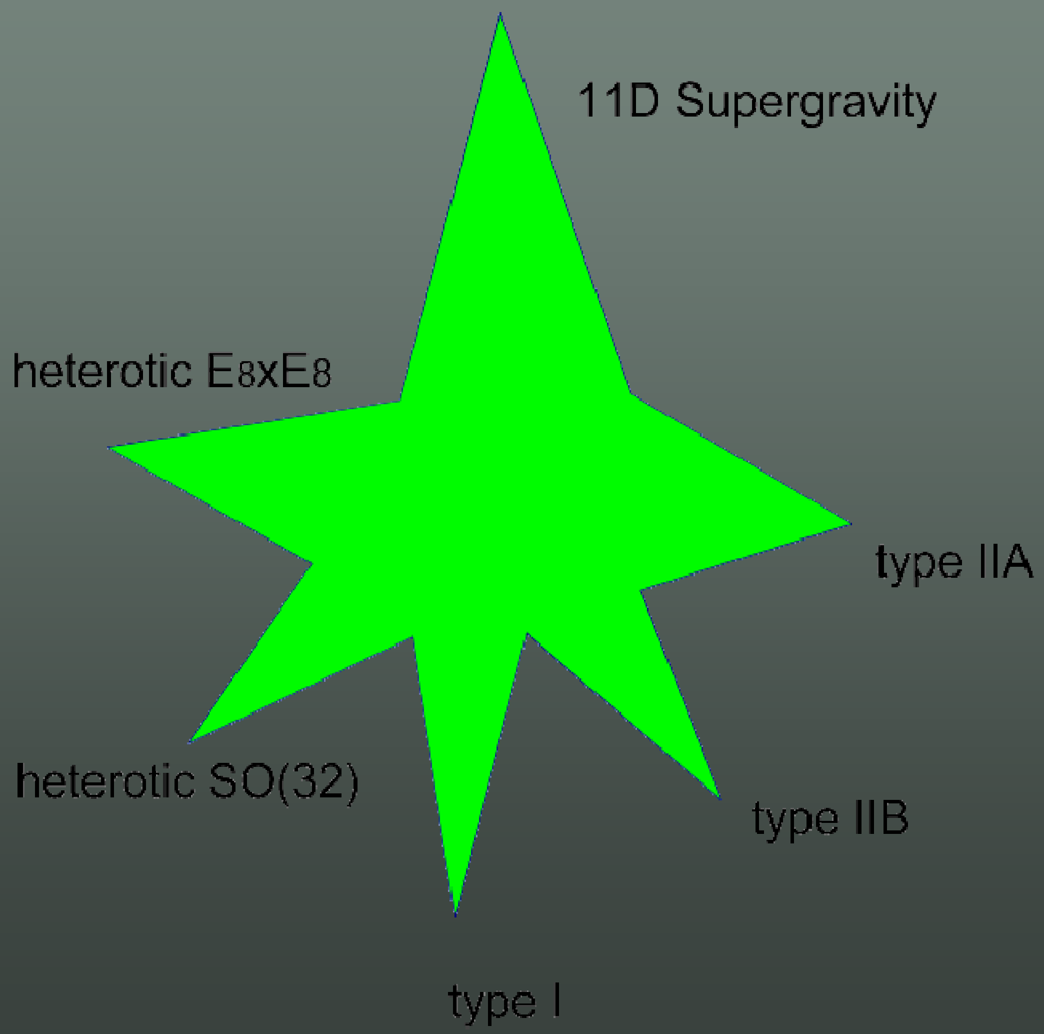
- The classic example of duality occurs in Maxwell’s equations without sources:

  – ‘electric’ variables:

  – ‘magnetic’ variables:

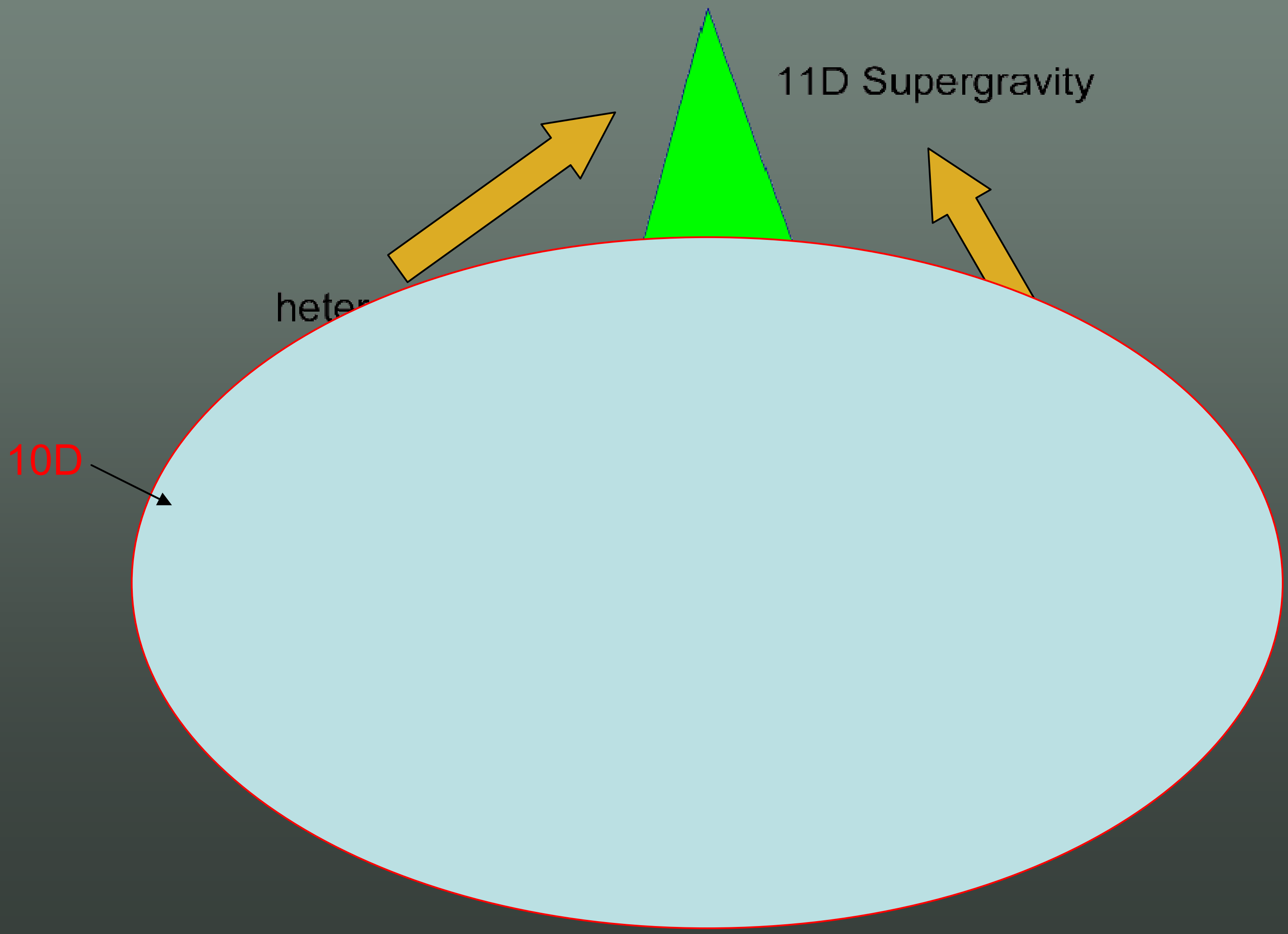
Self-dual

- M-theory moduli space:

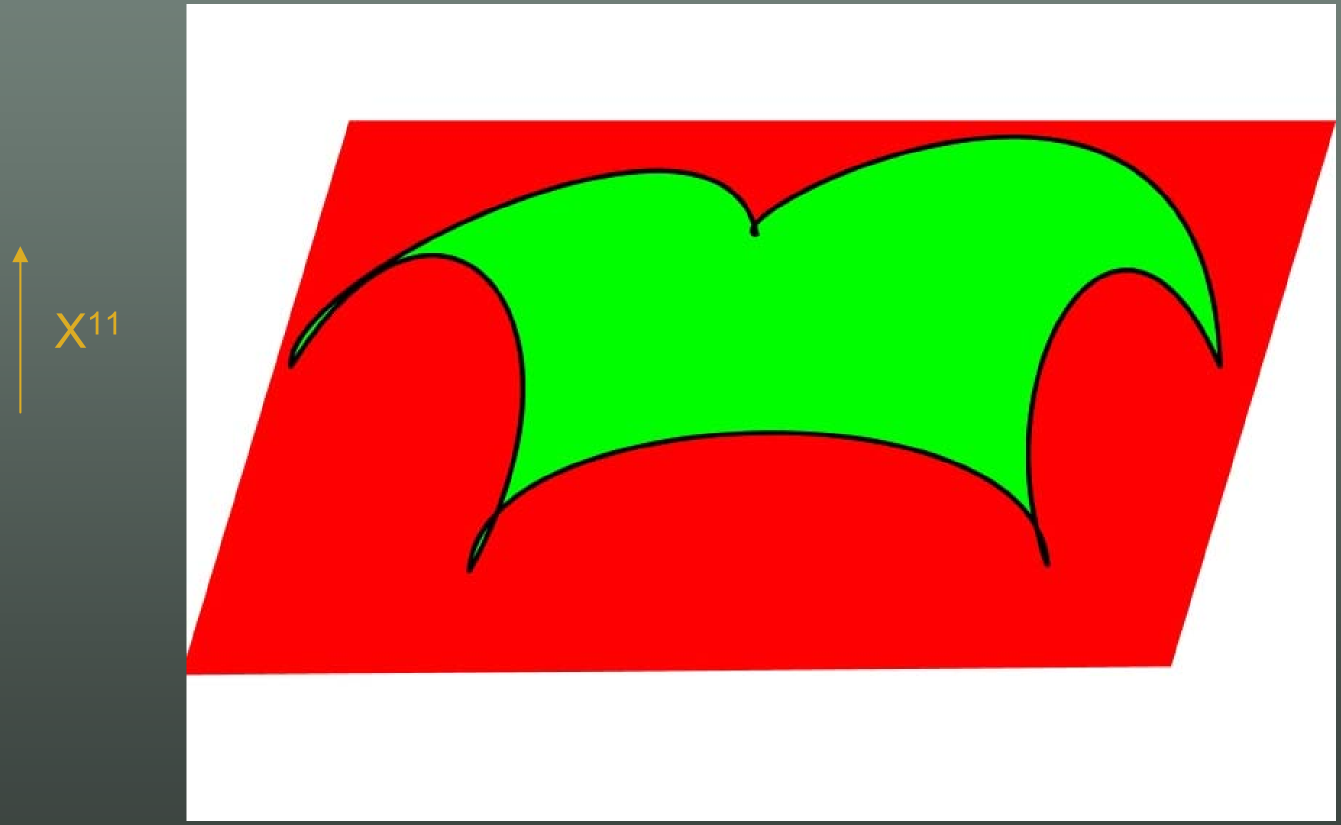


- M-theory moduli space:

at strong coupling



- M-theory moduli space in 3D:



- An 11D metric tensor becomes a 10D metric tensor plus a vector and a scalar

10D metric

U(1) gauge field

Scalar that controls the size of the 11th dimension

- Thus the String Theory dilaton has a geometric interpretation as the size of the 11th dimension
  - But the vev of is g_s
  - String perturbation theory is an expansion about a degenerate 11th dimension
  - As $g_s \rightarrow \infty$ an extra dimension opens up
    - 11D theory in the infinite coupling limit.
- Predicts a complete quantum theory in eleven dimensions: M-Theory
  - Effective action is 11D supergravity
  - Little else is known

M-Branes

Type IIA String Theory	M-Theory
0-Branes	gravitational wave along X^{11}
Strings	
2-branes	2-branes
4-branes	
5-branes	5-branes
6-Branes	Kaluza-Klein monopoles

M-Branes

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M-Branes

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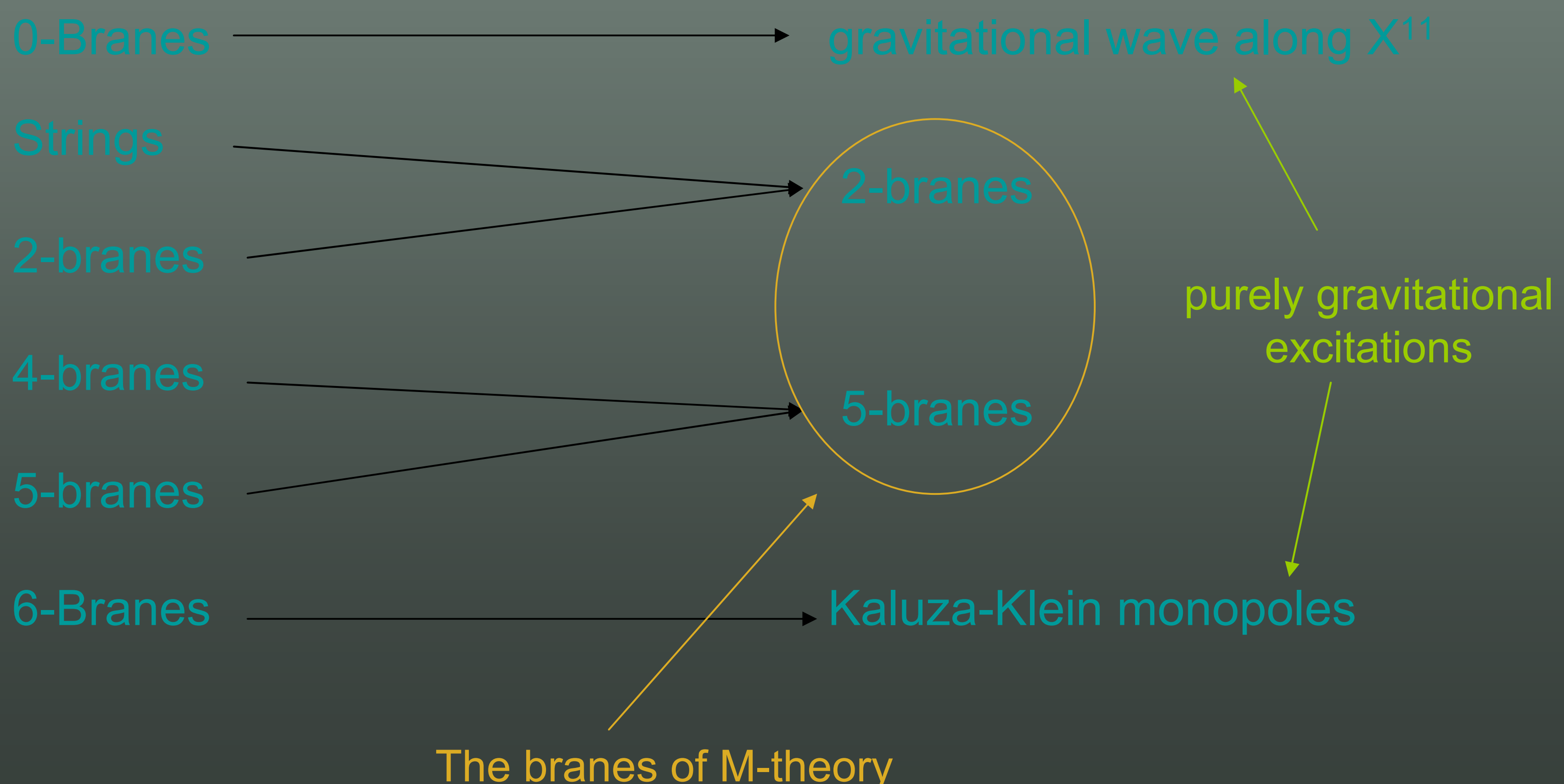

- So there are no strings in M-theory
  - 2-branes and 5-branes
- In particular no open strings and no g_s
  - No perturbative expansion
  - No microscopic understanding
- The dynamics of a single M-branes act to minimize their worldvolumes
  - With other fields related by supersymmetry
    - M2 [Bergshoeff, Sezgin, Townsend]
    - M5 [Howe, Sezgin, West]
- What about multiple M-branes?

- In string theory you can derive the dynamics of multiple D-branes from symmetries:
  - Effective theory has 16 supersymmetries and breaks SO(1,9) $\rightarrow$ SO(1,p) x SO(9-p)
  - This is in agreement with maximally supersymmetric Yang-Mills gauge theory

- Can we derive the dynamics of M2-branes from symmetries?
  - Conformal field theory
    - Strong coupling (IR) fixed point of 3D SYM
  - No perturbation expansion
  - The only maximally supersymmetric Lagrangians are Yang-Mills theories
    - Wrong symmetries for M-Theory
    - need SO(1,2) x SO(8) not SO(1,2) x SO(7)

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- Well that turns out not to be true

- The Yang-Mills theories living on D-branes are determined by the susy variation

- Here we find a Lie-algebra with a bi-linear anti-symmetric product:

- Closure of the susy algebra leads to gauge symmetry:

- Consistency of this implies the Jacobi identity:

- What is required for M2-branes?
  - Now and so we require
  - Thus we need a triple product: 3-algebra
  - Closure implies a gauge symmetry:
  - Consistency requires a generalization of the Jacobi identity (fundamental identity)

- The fundamental identity implies the gauge symmetry acts as a (non simple) Lie algebra acting on

- 3-algebra data is equivalent to specifying a Lie-algebra with a (split) metric and a representation acting on vector space space (with an invariant metric).

- This gives a maximally supersymmetric Lagrangian with SO(8) R-symmetry [Bagger,NL]

- ‘twisted’ Chern-Simons gauge theory

- Conformal, parity invariant

- But it turns out to only have one example:

integer

  - a,b,c,d = 1,2,3,4

- SU(2)xSU(2) Chern-Simons at level (k,-k) and matter in the bi-fundamental
- Vacuum moduli space:

- Two M2-branes in $\mathbf{R}^8/\mathbf{Z}_2$
  - agrees with M-theory when k=2

- Need to generalize:
  - Weak coupling arises from orbifold
  - Consider $\mathbf{C}^4/\mathbf{Z}_k$
  - 12 susys and breaks SO(8) $\rightarrow$SU(4)xU(1)
- Look for theories with SU(4)xU(1) R-symmetry and N=6 supersymmetry

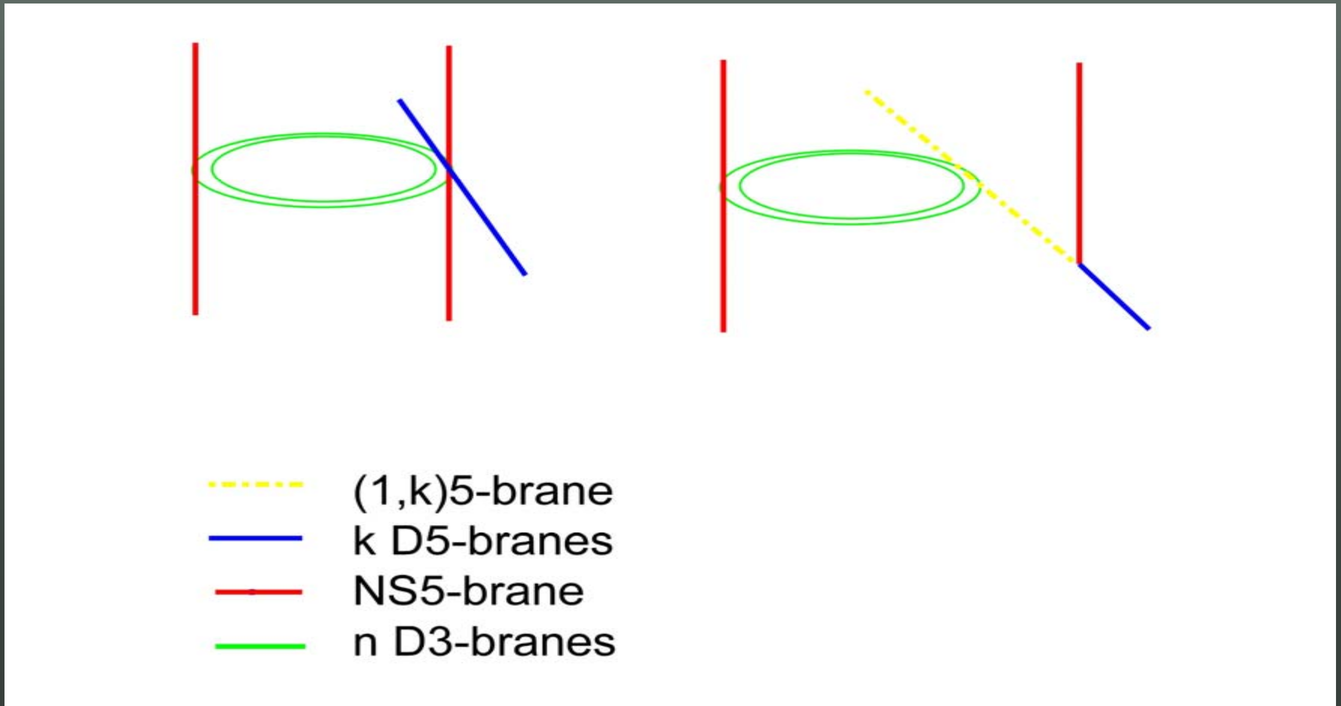
- From the 3-algebra this is achieved if the triple product is no longer totally anti-symmetric:

X,Y,Z are Complex Scalar Fields

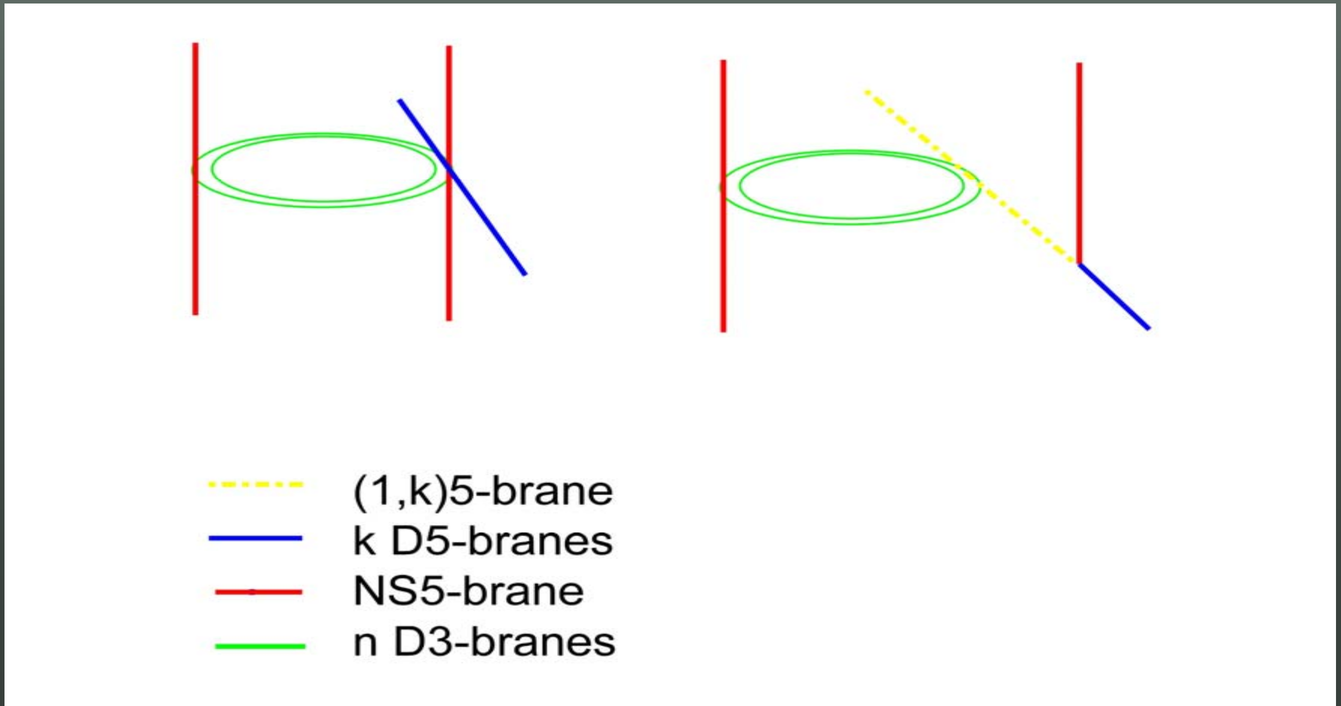
- Consistency requires a related fundamental identity
- For example we can take (for nxm matrices):

- Resulting action is similar to the N=8 case but:
  - U(n)xU(m) Chern-Simons theory at level (k,-k) with matter in the bifundamental

- These theories were was first proposed by [Aharony, Bergman, Jafferis and Maldacena]
- They gave a brane diagram derivation
  - Consider the following Hannay-Witten picture



- In terms of the D3-brane SYM worldvolume theory:
  - Integrating out D5/D3-strings and flowing to IR gives a U(n)xU(n) CS theory with level (k,-k) coupled to bi-fundamental matter
  - N=3 is enhanced to N=6



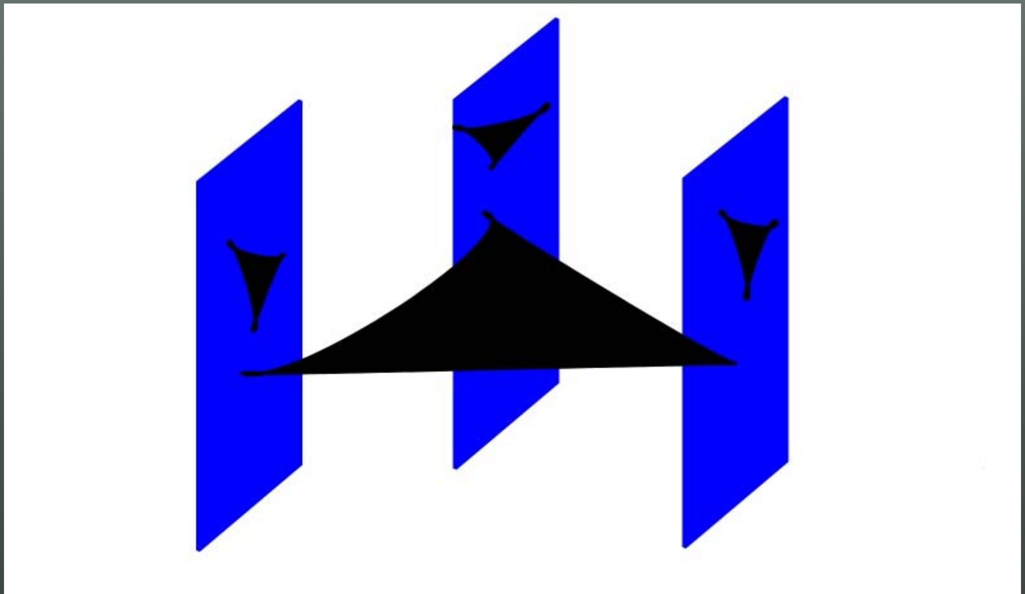
- The final configuration is just n M2s in a curved background preserving 3/16 susys.
  - Metric can be written explicitly
  - smooth except where the centre's intersect
  - near horizon limit gives n M2's in $\mathbf{R^8/Z_k}$.
  - Preserved susy's are enhanced to 6/16.

- Note that this works for all n and all k
  - even k=1,2 where we expect N=8 susy
    - Two supersymmetries are not realized in the Lagrangian (carry U(1) charge)
    - For k=1 even the centre of mass mode is obscured

- One success of these models is an understanding of the mysterious $n^{3/2}$ growth of the degrees of freedom
  - Free energy = $f(\lambda)n^2$
    - $\lambda = n/k$
    - $f(\lambda) = \begin{cases} 1 & \lambda \ll 1 \\ \lambda^{-1/2} & \lambda \gg 1 \end{cases}$
- This has recently been confirmed in Chern-Simons Theory for all λ [Drukker,Marino,Putrov]

- How does one recover D2-branes from this [Mukhi, Papageorgakis]
  - Give a vev to a scalar field
    - breaks U(n)xU(n) → U(n) and SO(8) → SO(7)
  - becomes a dynamical U(n) gauge field
    - Similar to a Higg's effect where a non-dynamical vector eats a scalar to become dynamical
  -

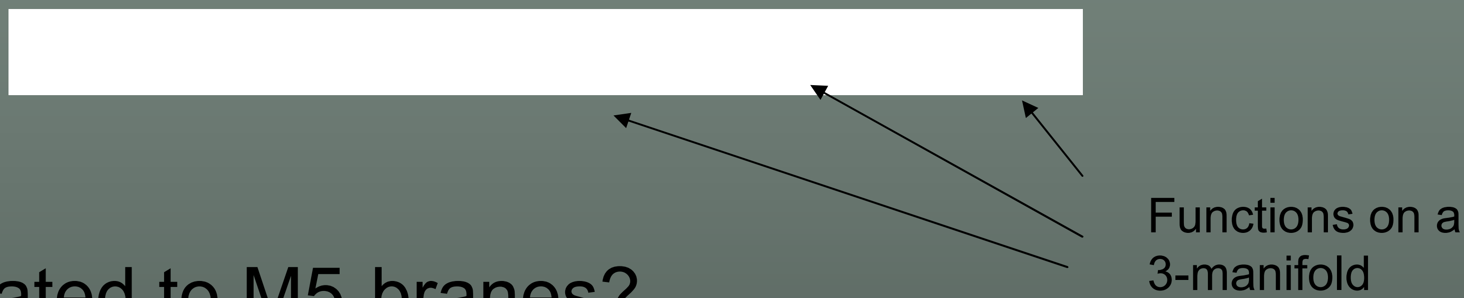
- What can we learn about M-theory?
  - Hints at microscopic dynamics of M-branes
    - e.g. in the N=8 theory one finds mass = area of a triangle with vertices on an M2



- Mass deformations give fuzzy vacua:

  - M2-branes blow up into fuzzy M5-branes
  - Can we learn about M5-branes
    - Also M2s can end on M5's: Chern-Simons gauge fields become dynamical

- There are also infinite dimensional totally antisymmetric 3-algebras: Nambu bracket



  - Related to M5-branes?

- Infinitely many totally anti-symmetric 3-algebras with a Lorentzian metric
  - Seem to be equivalent to 3D N=8 SYM but with manifest SO(8) and conformal symmetry

Conclusions

- M-Theory and M-branes are poorly understood but there has been much recent progress:
  - Complete proposal for the effective Lagrangian of n M2's in $\mathbf{R}^8/\mathbf{Z}_k$
  - Novel highly supersymmetric Chern-Simons gauge theories based on a 3-algebra.
  - Gives a Lagrangian description of strongly coupled 3D super Yang-Mills
- M5-branes remain very challenging as does M-Theory itself but hopefully progress will be made
  - M2-brane CFT's 'define' M-theory in $AdS_4 x X_7$