Decoherence and transition from quantum to classical in open quantum systems

Aurelian Isar

Department of Theoretical Physics
National Institute of Physics and Nuclear Engineering
Bucharest-Magurele, Romania
isar@theory.nipne.ro

Faculty of Science and Mathematics, Nis, 18 October 2011

Content

- theory of OQS based on q. dyn. semigs (Lindblad)
- master eq. for h.o. interacting with an environment (thermal bath)
- Schrödinger gen-zed uncertainty f.
- q. and thermal fluctuations
- q. decoherence (QD) and degree of QD
- decoherence time
- summary

Introduction

- quantum - classical transition and classicality of q. ss - among the most interesting problems in many fields of physics
- 2 conditions - essential for classicality of a q. s.:

a) quantum decoherence (QD)

b) classical correlations (CC): Wigner f. has a peak which follows the classical eqs. of motion in phase space with a good degree of approx. (q. state becomes peaked along a class. trajectory)

- Classicality: emergent property of OQSs (both main features – QD and CC – strongly depend on the interaction between s. and its external E)
- necessity and sufficiency of both QD and CC as conditions of classicality - subject of debate
- they do not have an universal character (not nec. for all physical models)

Content of the talk

- Theory of OQS (q. dynamical semigroups)
- Partic. case: h.o.
- QD and CC for a *h. o. interacting with an E (thermal bath)* in the framework of the theory of OQS
- *degree of QD and CC* and the possibility of simultaneous realization of QD and CC
- true quantum - classical transition takes place (classicality - temporary phenomenon)
- t_{deco} - of the same scale with time when q. and thermal fluctuations become comparable
- summary and further development (q. fidelity - in the context of CV approach to QIT)

Open systems

- the simplest dynamics for an OS which describes an irreversible process: semigroup of transformations introducing a preferred direction in time (characteristics for dissipative processes)
- in Lindblad axiomatic formalism of introducing dissipation in quantum mechanics, the usual von Neumann-Liouville eq. ruling the time evolution of closed q. ss is replaced by the following Markovian master eq. for the density operator $\rho(t)$ in the Schrödinger rep.:

$$\frac{d\Phi_t(\rho)}{dt} = L(\Phi_t(\rho))$$

Lindblad theory (1)

- Φ_t - the dynamical semigroup describing the irreversible time evolution of the open system and L is the infinitesimal generator of Φ_t
- fundamental properties are fulfilled (positivity, unitarity, Hermiticity)
- the semigroup dynamics of the density operator which must hold for a quantum Markov process is valid only for the weak-coupling regime, with the damping λ typically obeying the inequality $\lambda \ll \omega_0$, where ω_0 is the lowest frequency typical of reversible motions

Lindblad theory of OQS

- Lindblad axiomatic formalism is based on quantum dynamical semigroups (complete positivity property is fulfilled)
- irreversible time evolution of an open system is described by the following general q. Markovian master equation for the density operator $\rho(t)$:

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H, \rho(t)] + \frac{1}{2\hbar}\sum_j([V_j\rho(t), V_j^\dagger] + [V_j, \rho(t)V_j^\dagger])$$

- H - Hamiltonian of the system
- $V_j, V_j^\dagger$ - operators on the Hilbert space of H (they model the environment)

Master equation for damped h.o.

- V_1 and V_2 - linear polynomials in q and p (equations of motion as close as possible to the classical ones) and H - general quadratic form

$$H = H_0 + \frac{\mu}{2}(qp + pq), \quad H_0 = \frac{1}{2m}p^2 + \frac{m\omega^2}{2}q^2$$

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_0, \rho]$$

$$-\frac{i}{2\hbar}(\lambda + \mu)[q, \rho p + p\rho] + \frac{i}{2\hbar}(\lambda - \mu)[p, \rho q + q\rho]$$

$$-\frac{D_{pp}}{\hbar^2}[q, [q, \rho]] - \frac{D_{qq}}{\hbar^2}[p, [p, \rho]] + \frac{D_{pq}}{\hbar^2}([q, [p, \rho]] + [p, [q, \rho]])$$

Diffusion and dissipation coeffs

- fundamental constraints $D_{pp} > 0, D_{qq} > 0$,

$$D_{pp}D_{qq} - D_{pq}^2 \geq \frac{\lambda^2\hbar^2}{4}$$

- when the asymptotic state is a Gibbs state
$\rho_G(\infty) = e^{-\frac{H_0}{kT}}/\mathrm{Tr}e^{-\frac{H_0}{kT}}$,

$$D_{pp} = \frac{\lambda+\mu}{2}\hbar m\omega \coth\frac{\hbar\omega}{2kT}, D_{qq} = \frac{\lambda-\mu}{2}\frac{\hbar}{m\omega}\coth\frac{\hbar\omega}{2kT},$$

$$D_{pq} = 0, \ \ (\lambda^2-\mu^2)\coth^2\frac{\hbar\omega}{2kT} \geq \lambda^2, \ \ \lambda > \mu$$

- fundamental constraint is a *necessary* condition for the generalized uncertainty relation

$$\sigma_{qq}(t)\sigma_{pp}(t) - \sigma_{pq}^2(t) \geq \frac{\hbar^2}{4}$$

Evolution Eq. in coordinate rep.

$$\frac{\partial\rho}{\partial t}=\frac{i\hbar}{2m}\left(\frac{\partial^2}{\partial q^2}-\frac{\partial^2}{\partial q'^2}\right)\rho-\frac{im\omega^2}{2\hbar}(q^2-q'^2)\rho$$

$$-\frac{1}{2}(\lambda+\mu)(q-q')\left(\frac{\partial}{\partial q}-\frac{\partial}{\partial q'}\right)\rho$$

$$+\frac{1}{2}(\lambda-\mu)\left[(q+q')\left(\frac{\partial}{\partial q}+\frac{\partial}{\partial q'}\right)+2\right]\rho$$

$$-\frac{D_{pp}}{\hbar^2}(q-q')^2\rho+D_{qq}\left(\frac{\partial}{\partial q}+\frac{\partial}{\partial q'}\right)^2\rho$$

$$-2iD_{pq}\hbar(q-q')\left(\frac{\partial}{\partial q}+\frac{\partial}{\partial q'}\right)\rho$$

Fokker-Planck Eq. for Wigner f.

$$\frac{\partial W}{\partial t} = -\frac{p}{m}\frac{\partial W}{\partial q} + m\omega^2 q\frac{\partial W}{\partial p}$$

$$+(\lambda+\mu)\frac{\partial}{\partial p}(pW) + (\lambda-\mu)\frac{\partial}{\partial q}(qW)$$

$$+D_{pp}\frac{\partial^2 W}{\partial p^2} + D_{qq}\frac{\partial^2 W}{\partial q^2} + 2D_{pq}\frac{\partial^2 W}{\partial p\partial q}$$

Physical signification

- first two terms generate a purely *unitary* evolution (usual Liouvillian evolution)
- third and forth terms - *dissipative* (damping effect: exchange of energy with environment)
- last three terms: *noise (diffusive)* (*fluctuation* effects)
- D_{pp} : diffusion in p + generates *decoherence* in q: it reduces the off-diagonal terms, responsible for correlations between spatially separated pieces of the wave packet
- D_{qq} : diffusion in q + generates *decoherence* in p
- D_{pq} : "anomalous diffusion" term - does *not* generate decoherence)

Initial Gaussian wave function

- correlated coherent state (CCS) or squeezed CS (special class of pure states, which realizes equality in generalized uncertainty relation)

$$\Psi(q) = (\frac{1}{2\pi\sigma_{qq}(0)})^{\frac{1}{4}}$$

$$\times \exp[-\frac{1}{4\sigma_{qq}(0)}(1 - \frac{2i}{\hbar}\sigma_{pq}(0))(q - \sigma_q(0))^2 + \frac{i}{\hbar}\sigma_p(0)q],$$

$$\sigma_{qq}(0) = \frac{\hbar\delta}{2m\omega}, \sigma_{pp}(0) = \frac{\hbar m\omega}{2\delta(1 - r^2)}, \sigma_{pq}(0) = \frac{\hbar r}{2\sqrt{1 - r^2}}$$

Parameters and variances

- δ - squeezing parameter (measures the spread in the initial Gaussian packet), r, $|r| < 1$ - correlation coefficient at time $t = 0$
- for $\delta = 1$, $r = 0$ CCS - red Glauber coherent state - σ_{qq} and σ_{pp} denote the dispersion (variance) of the coordinate and momentum, respectively, and σ_{pq} denotes the correlation (covariance) of the coordinate and momentum
- in the case of a thermal bath

$$\sigma_{qq}(\infty) = \frac{\hbar}{2m\omega} \coth \frac{\hbar\omega}{2kT}, \quad \sigma_{pp}(\infty) = \frac{\hbar m\omega}{2} \coth \frac{\hbar\omega}{2kT},$$

$$\sigma_{pq}(\infty) = 0$$

Density matrix

$$< q|\rho(t)|q' >= (\frac{1}{2\pi\sigma_{qq}(t)})^{\frac{1}{2}} \exp[-\frac{1}{2\sigma_{qq}(t)}(\frac{q+q'}{2} - \sigma_q(t))^2$$

$$-\frac{\sigma(t)}{2\hbar^2\sigma_{qq}(t)}(q-q')^2 + \frac{i\sigma_{pq}(t)}{\hbar\sigma_{qq}(t)}(\frac{q+q'}{2} - \sigma_q(t))(q-q')$$

$$+\frac{i}{\hbar}\sigma_p(t)(q-q')] \ - \ \text{general Gaussian form}$$

- thermal bath, $t \to \infty$ (stationary solution)

$$< q|\rho(\infty)|q' >= (\frac{m\omega}{\pi\hbar \coth\epsilon})^{\frac{1}{2}} \exp\{-\frac{m\omega}{4\hbar}[\frac{(q+q')^2}{\coth\epsilon}$$

$$+(q-q')^2 \coth\epsilon]\}, \quad \epsilon \equiv \hbar\omega/2kT$$

Quantum decoherence (QD)

- irreversible, uncontrollable and persistent formation of q. correlations (entanglement) of the s. with its environment (interference between different states are negligible - decay (damping) of off-diagonal elements representing coherences between q. states below a certain level, so that density matrix becomes approximately diagonal)
- strongly depends on the interaction between s. and environment (an isolated s. has unitary evolution and coherences of states are not lost – pure states evolve in time only to pure states)

Role of QD

- an isolated system has an unitary evolution and the coherence of the state is not lost – pure states evolve in time only to pure states
- loss of coherence can be achieved by introducing an interaction between the system and environment: an initial pure state with a density matrix (containing nonzero off-diagonal terms) can non-unitarily evolve into a final mixed state with an approx. diagonal density matrix
- in QI processing and computation we are interested in understanding the specific causes of QD: to prevent decoherence from damaging q. states and to protect the information stored in these states from the degrading effect of the interaction with the environment

Degree of quantum decoherence

$$\Sigma = (q+q')/2, \Delta = q - q',$$
$$\alpha = \frac{1}{2\sigma_{qq}(t)}, \gamma = \frac{\sigma(t)}{2\hbar^2\sigma_{qq}(t)}, \beta = \frac{\sigma_{pq}(t)}{\hbar\sigma_{qq}(t)}$$

$$\rho(\Sigma, \Delta, t) = \sqrt{\frac{\alpha}{\pi}} \exp[-\alpha\Sigma^2 - \gamma\Delta^2 + i\beta\Sigma\Delta]$$

(for zero initial mean values of q and p)

- representation-independent measure of the degree of QD : ratio of the dispersion $1/\sqrt{2\gamma}$ of the off-diagonal element to the dispersion $\sqrt{2/\alpha}$ of the diagonal element

$$\delta_{QD}(t) = (1/2)\sqrt{\alpha/\gamma} = \hbar/2\sqrt{\sigma(t)}$$

Schrödinger gen-zed uncert. f.

$$\sigma(t) \equiv \sigma_{qq}(t)\sigma_{pp}(t) - \sigma_{pq}^2(t)$$

$$\sigma(t) = \frac{\hbar^2}{4}\{e^{-4\lambda t}[1 - (\delta + \frac{1}{\delta(1-r^2)})\coth\epsilon + \coth^2\epsilon]$$

$$+e^{-2\lambda t}\coth\epsilon[(\delta + \frac{1}{\delta(1-r^2)} - 2\coth\epsilon)\frac{\omega^2 - \mu^2\cos(2\Omega t)}{\Omega^2}$$

$$+(\delta - \frac{1}{\delta(1-r^2)})\frac{\mu\sin(2\Omega t)}{\Omega} + \frac{2r\mu\omega(1-\cos(2\Omega t))}{\Omega^2\sqrt{1-r^2}}]$$

$$+\coth^2\epsilon\}$$

- underdamped case ($\omega > \mu,\ \Omega^2 \equiv \omega^2 - \mu^2$)

Limit of long times

$$\sigma(\infty) = \frac{\hbar^2}{4} \coth^2 \epsilon,$$

$$\delta_{QD}(\infty) = \tanh \frac{\hbar\omega}{2kT},$$

- high T :

$$\delta_{QD}(\infty) = \frac{\hbar\omega}{2kT}$$

Discussion of QD (1)

- QD increases with t and T, i.e. the density matrix becomes more and more diagonal and the contributions of the off-diagonal elements get smaller and smaller
- the degree of purity decreases and the degree of mixedness increases with t and T
- for $T = 0$ the asymptotic (final) state is pure and δ_{QD} reaches its initial maximum value 1
- a pure state undergoing unitary evolution is highly coherent: it does not lose its coherence, i.e. off-diagonal coherences never vanish and there is no QD

Discussion of QD (2)

- the considered system interacting with the thermal bath manifests QD - dissipation promotes quantum coherences, whereas fluctuation (diffusion) reduces coherences and promotes QD; the balance of dissipation and fluctuation determines the final equilibrium value of δ_{QD}
- the quantum system starts as a pure state (Gaussian form) and this state remains Gaussian, but becomes a quantum mixed state during the irreversible process of QD

Decoherence time scale

- in the case of a thermal bath

$$t_{deco} = \frac{2\hbar}{(\lambda + \mu) m\omega\sigma_{qq}(0) \coth \epsilon}, \quad \epsilon \equiv \frac{\hbar\omega}{2kT}$$

where we have taken $(q - q')^2$ of the order of the initial dispersion in coordinate $\sigma_{qq}(0)$
- the decoherence time scale t_{deco} is very much shorter than the relaxation time $\rightarrow$ in the macroscopic domain QD occurs very much faster than relaxation
- t_{deco} is of the same order as the time when thermal fluctuations overcome q. fluctuations

Q. and thermal fluctuations

- when $t \gg t_{rel} \approx \lambda^{-1}$ (relaxation time, which governs the rate of energy dissipation), the particle reaches equilibrium with the environment
- $\sigma(t)$ is insensitive to λ, μ, δ and r and approaches $\sigma^{BE} = \frac{\hbar^2}{4} \coth^2 \epsilon$ (Bose-Einstein relation for a system of bosons in equilibrium at temperature T)
- in the case of $T = 0$, $\sigma_0 = \frac{\hbar^2}{4}$ - q. Heisenberg relation (limit of pure q. fluctuations)
- at high T $(T \gg \hbar\omega/k)$, $\sigma^{MB} = (\frac{kT}{\omega})^2$ - Maxwell - Boltzmann distribution for a s. approaching a classical limit (limit of pure thermal fluctuations)

Figures (1)

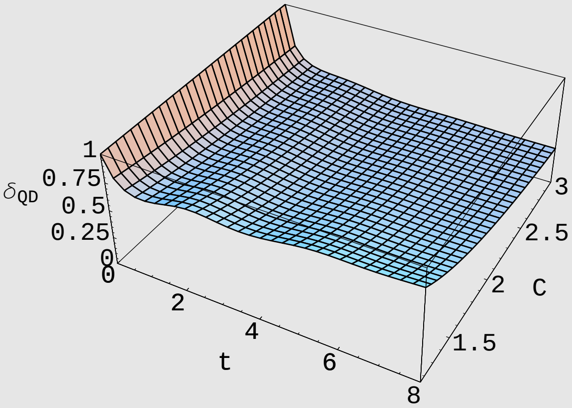

Figure: δ_{QD} on T ($C \equiv \coth \hbar\omega/2kT$) and t
($\lambda = 0.2, \mu = 0.1, \delta = 4, r = 0$).

Figures (2)

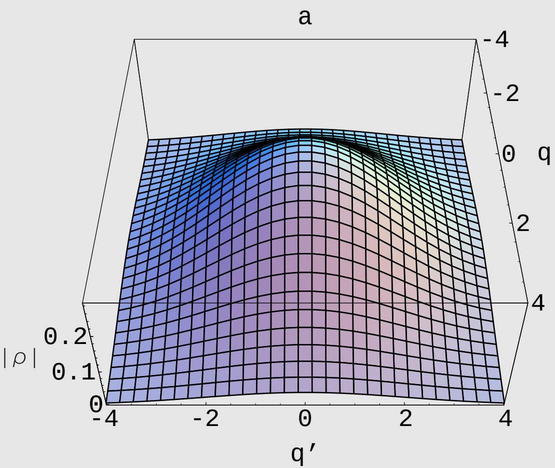

Figure: ρ in $q-$ representation $(\lambda = 0.2, \mu = 0.1, \delta = 4, r = 0)$ at $t = 0$.

Figures (3)

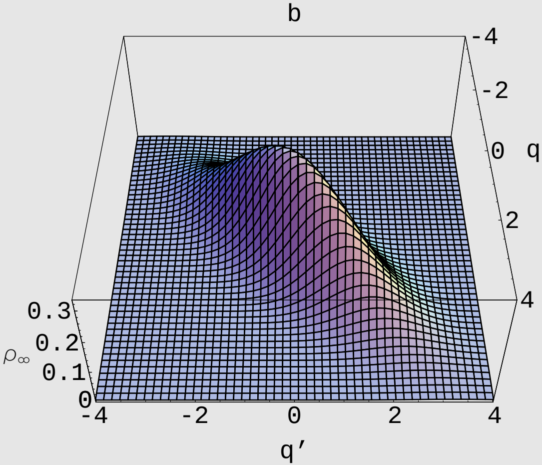

Figure: ρ in $q-$ representation $(\lambda = 0.2, \mu = 0.1, \delta = 4, r = 0)$ at $t \to \infty$ and $C = 3$.

Figures (4)

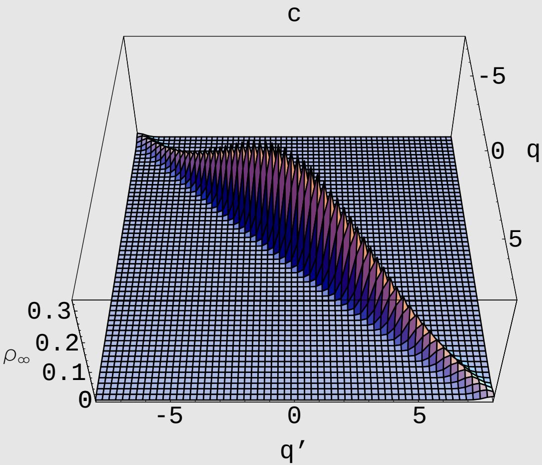

Figure: ρ in $q-$ representation ($\lambda = 0.2, \mu = 0.1, \delta = 4, r = 0$) at $t \to \infty$ and $C = 20$.

Classical correlations

classical correlations - the s. should have, with a good approx., an evolution according to classical laws: this implies that the Wigner f. has a peak along a classical trajectory (there exist CC between the canonical variables of coordinate and momentum) - of course, the correlation between the canonical variables, necessary to obtain a classical limit, should not violate Heisenberg uncertainty principle, i.e. the position and momentum should take reasonably sharp values, to a degree in concordance with the uncertainty principle.

General Gaussian Wigner f.

- most gen. mixed squeezed states described by Wigner f. of Gauss. form with 5 real parameters

$$W(p,q) = \frac{1}{2\pi\sqrt{\sigma}} \exp\{-\frac{1}{2\sigma}[\sigma_{pp}(q-\sigma_q)^2 + \sigma_{qq}(p-\sigma_p)^2$$

$$-2\sigma_{pq}(q-\sigma_q)(p-\sigma_p)]\}$$

- for $\sigma > \hbar^2/4 \rightarrow$ mixed quantum states
- for $\sigma = \hbar^2/4 \rightarrow$ pure correlated coherent states

Summary

- we have studied QD with the Markovian Lindblad Eq. for an one-dimensional h. o. in interaction with a thermal bath in the framework of the theory of OQS based on q. dyn. semigs
- the s. manifests a QD which increases with t and T, i.e. the density matrix becomes more and more diagonal at higher T (loss of q. coherence); at the same time the degree of purity decreases and the degree of mixedness increases with T
- q. and thermal fluctuations in gen-zed Schrödinger uncertainty f.