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Constrained generalized supersymmetries

(with applications to the superparticles with tensorial central charges)

J. LUKIERSKI F.T. PLB 539 (2002) 266 } octonionic M-theory
" " PLB 567 (2003) 125 }
" " PLB 584 (2004) 315 – Euclidean M-theory

H.L. CARRION - M. ROJAS - F.T. JHEP 04 (2003) 040
Quaternionic and octonionic spinors

F.T. JHEP 09 (2004) 016
Constrained holo and hermitian susies

Z. KUZNETSOVA - F.T. hep-th/0502178 To appear in JHEP
Constrained susies and classification of superparticles.

Division algebras & Clifford algebras   Schur lemma

$[S,\Gamma^\mu]=0$ $\Gamma^\mu\Gamma^\nu+\Gamma^\nu\Gamma^\mu = 2\eta^{\mu\nu}$

p-q mod 8

R	C	H
0, 2		4, 6
1	3, 7	5

Also: Γ^μ realized with matrices whose entries are valued in a given division algebra.

Clifford irreps over $\mathbb{R}$.

Basic example $x = x_0 + x_i e_i \in \mathbb{R}, \mathbb{C}, \mathbb{H}$ or $\mathbb{O}$

$x^* = x_0 - x_i e_i$ 0, 1, 3, 7

$e_i \cdot e_j = -\delta_{ij} + C_{ijk}\, e_k$

↖ totally antisymmetric structure constant

Then $e_i \cdot e_j + e_j e_i = -2\delta_{ij}$

Take $i=1,2,3$ (the three imaginary quaternions) $\Rightarrow C(0,3)$

$i=1,2,\dots,7$ (the seven imaginary octonions) $\Rightarrow C(0,7)$

1	2	4	8	16	32	64	128	256

$\mathbb{R}$ $\quad (1,0) \Rightarrow (2,1) \Rightarrow (3,2) \Rightarrow (4,3) \Rightarrow (5,4) \Rightarrow (6,5) \Rightarrow (7,6) \Rightarrow (8,7) \Rightarrow (9,8)$

$\mathbb{C}$ $\quad (0,1)$

$\nearrow (1,2) \to (2,3) \to (3,4) \to (4,5) \to (5,6) \to (6,7) \to (7,8)$

$\searrow (3,0) \to (5,0) \to (6,1) \to (7,2) \to (8,3) \to (9,4) \to (10,5)$

$\mathbb{H}$ $\quad (0,3)$

$\nearrow (1,4) \to (2,5) \to (3,6) \to (4,7) \to (5,8) \to (6,9)$

$\searrow (5,0) \to (6,1) \to (7,2) \to (8,3) \to (9,4) \to (10,5)$

$\mathbb{C}$ $\quad (0,5)$

$\nearrow (1,6) \to (2,7) \to (3,8) \to (4,9) \to (5,10)$

$\searrow (7,0) \to (8,1) \to (9,2) \to (10,3) \to (11,4)$

$\mathbb{R}/\mathbb{D}$ $\quad (0,7)$

$\nearrow (1,8) \to (2,9) \to (3,10) \to (4,11) \to (5,12)$

$\searrow (9,0) \to (10,1) \to (11,2) \to (12,3) \to (13,4)$

$\mathbb{C}$ $\quad (0,9)$

$\nearrow (1,10) \to (2,11) \to (3,12)$

$\searrow (11,0) \to (12,1) \to (13,2)$

$\mathbb{H}$ $\quad (0,11)$

$\nearrow (1,12) \to (2,13)$

$\searrow (13,0) \to (14,1)$

....

I) $\gamma \in C(p,q) \rightsquigarrow \Gamma \equiv \left\{ \begin{pmatrix} & \gamma \\ \gamma & \end{pmatrix} ; \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix} ; \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} \right\} \in C(p+1, q+1)$

II) $\gamma \in C(p,q) \rightsquigarrow \Gamma \equiv \left\{ \begin{pmatrix} & \gamma \\ -\gamma & \end{pmatrix} ; \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} ; \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} \right\} \in C(q+2, p)$

Clifford -vs- fundamental spinors

s-t mod 8	Γ	Ψ
0	$\mathbb{R}$	$\mathbb{R}$
1	$\mathbb{R}$	$\mathbb{R}$
2	$\mathbb{R}$	$\mathbb{C}$
3	$\mathbb{C}$	$\mathbb{H}$
4	$\mathbb{H}$	$\mathbb{H}$
5	$\mathbb{H}$	$\mathbb{H}$
6	$\mathbb{H}$	$\mathbb{C}$
7	$\mathbb{C}$	$\mathbb{R}$

$\Leftarrow$ (only associative)

Lorentz generators $\Sigma_{ij} = [\Gamma_i, \Gamma_j]$

If $\Gamma_i \equiv$  $\Rightarrow$ Weyl projection

B.t.w. these Γ-matrices can be "promoted" to be graded matrices, fermionic.

$$\Gamma^\mu \Gamma^\nu + \Gamma^\nu \Gamma^\mu = 2\delta^{\mu\nu} \quad \Big| \quad Q^i Q^j + Q^j Q^i = \delta^{ij} H$$

1.to.1 correspondence with 1D Susy QM short multiplets representations.

Using special features of 1D susies
(long multiplets can be shortened)

A. Pashnev . F.T. JMP 42 5257 (2001)

Weyl Clifford $\Longleftrightarrow$ N-extended SUSY Q.M.

$$D = N$$

$$d = n$$

Series of applications to $d=1$ susy systems.

$N=8$ susy in two inequivalent variants

$N=8$ associative $(\mathbb{R})$

$N=8$ non-associative $(\mathbb{O})$

- Carrion - Rojas - F.T. MPLA 11 (2003) 787
- " " " PLA 291 (2001) 95
  (relation with Englert et al. $N=8$ SCA)
- Carrion - Rojas . F.T. JPA 36 (2003) 3809

  Construction of $N=8$ KdV (Octonionic $N=8$)

Superparticles with tensorial central charges.

$$S = \frac{1}{2} \int d\tau \ \mathrm{tr} \left[Z \cdot \Pi - e (Z)^2 \right]$$

$X^{ab}, \theta^a \longleftrightarrow Z_{ab}, Q_a$

$$\Pi^{ab} = dX^{ab} - \theta^{(a} d\theta^{b)}$$

e: lagrange multiplier

$e^T = \varepsilon e$

$C^T = \varepsilon C$

$$(Z)^2_{ab} = Z_{ac} C^{cd} Z_{db}$$

charge conjugation matrix.

Real formulation (Rudychev - Sezgin)

C symmetric $\rightsquigarrow$ $Z \to Z + m C$ shift

giving the possibility of introducing a mass term.

Complex superparticle models.

$$\mathcal{P} = \begin{pmatrix} P & R \\ R^* & P^* \end{pmatrix} \qquad Q_a \quad Q^*_{\dot{a}}$$

lagrange multiplier $\quad E = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$

$$S = \frac{1}{2}\int d\tau \, \mathrm{tr} \left[\mathcal{P}\cdot\Pi - E(\mathcal{P})^2 \right]$$

$$\mathcal{P}^2 = \mathcal{P}\,\mathcal{C}\,\mathcal{P}$$

i) $\mathcal{C} = \begin{pmatrix} C & 0 \\ 0 & C^* \end{pmatrix}$

ii) $\mathcal{C} = \begin{pmatrix} 0 & A \\ \xi A^+ & 0 \end{pmatrix} \qquad \xi = \pm 1$

iii) $\mathcal{C} = \begin{pmatrix} C & A \\ \varepsilon\delta A^+ & C^* \end{pmatrix} \qquad \begin{matrix} A^+ = \delta A \\ C^T = \varepsilon C \end{matrix}$

dualities		# bos. comp.		
I	a1	$2n^2+n$	$k=3$	$l=1$
II	a2	$\frac{3}{2}(n^2+n)$	$k=3$	$l=0$
III	a3 $\leftrightarrow$ b1	$\frac{1}{2}(3n^2+n)$	$k=2$	$l=1$
IV	a4 $\leftrightarrow$ b2	n^2+n	$k=2$	$l=0$
V	b3 $\leftrightarrow$ c1	n^2	$k=1$	$l=1$
VI	b4 $\leftrightarrow$ c2	$\frac{1}{2}(n^2+n)$	$k=1$	$l=0$
VII	c3	$\frac{1}{2}(n^2-n)$	$k=0$	$l=1$

$$Z = kX + lY$$

$D=3$	M_1	M_0
$D=5$	M_2	M_0+M_1
$D=7$	M_0+M_3	M_1+M_2
$D=9$	$M_0+M_1+M_4$	M_2+M_3
$D=11$	$M_1+M_2+M_5$	$M_0+M_3+M_4$
$D=13$	$M_2+M_3+M_6$	$M_0+M_1+M_4+M_5$

Constrained complex supersymmetries

Z. Kuznetsova & F.T.

$$\{Q_a, Q_b\} = \mathcal{P}_{ab} \qquad \{Q^*_{\dot a}, Q^*_{\dot b}\} = \mathcal{P}^*_{\dot a \dot b}$$

$$\{Q_a, Q^*_{\dot b}\} = \mathcal{R}_{a\dot b}$$

$\mathcal{P}$ - symmetric

$\mathcal{R}$ hermitian

$\mathcal{P} \backslash \mathcal{R}$	Full	Real	Im.	Abs
Full	$2n^2+n$	$\frac{3}{2}(n^2+n)$	$\frac{1}{2}(3n^2+n)$	n^2+n
Real	$\frac{1}{2}(3n^2+n)$	n^2+n	n^2	$\frac{1}{2}(n^2+n)$
Im.	…	…	…	…
Abs	n^2	$\frac{1}{2}(n^2+n)$	$\frac{1}{2}(n^2-n)$	0

duality formulations.

FREE DYNAMICS FOR OCTONIONIC SPINORS

(ALLOWED TERMS)

t mod8

	0	1	2	3
1		K	K, M	M
2	M_S	K	K, K_S, M	K_S, M, M_S
3	K_J, M_S, M_J	K, M_J	K, K_S, M	K_S, K_J, M, M_{S_i}
4	$K_{J_i}, K_F, M_{S_i}, M_{J_i}$	K, K_F, M_{J_i}, M_F	K, K_{S_i}, M, M_F	$K_{S_i}, K_{J_i}, M, M_{S_i}$

t-s mod8

Weyl projected spinors:

	0	1	2	3
0		$K_{\parallel}$	$K_{\perp}, M_{\parallel}$	$M_{\perp}$
5	$K_{\parallel T_j}$ $K_{\perp J_i}$ $M_{\parallel J_i}$ $M_{\perp F}$	$K_{\parallel}$ $K_{\perp T_j}$ $M_{\parallel T_j}$ $M_{\perp J_i}$	$K_{\perp}$ $K_{\parallel F}$ $M_{\parallel}$ $M_{\perp T_j}$	$K_{\parallel J_i}$ $K_{\perp F}$ $M_{\perp}$ $M_{\parallel F}$
6	$K_{\parallel T_i}$ $M_{\parallel J}$ $K_{\perp J_i}$	$K_{\parallel}$ $K_{\perp T_j}$ $M_{\parallel T_i}$ $M_{\perp J}$	$K_{\perp}$ $M_{\parallel}$ $M_{\perp T_i}$	$K_{\parallel J}$ $M_{\perp}$
7	$K_{\parallel T}$	$K_{\parallel}$ $K_{\perp T}$ $M_{\parallel T}$	$K_{\perp}$ $M_{\parallel}$ $M_{\perp T}$	$M_{\perp}$

T, S, J, F : insertion of extra Γ-matrices

T: time-line

S: space-line

J: product of two Γ's

F: product of three Γ's

$$K = \frac{1}{2} \operatorname{tr}\left[(\Psi^+ A \Gamma^\mu) \partial_\mu \Psi \right] + \frac{1}{2} \operatorname{tr}\left[\Psi^+ (A \Gamma^\mu \partial_\mu \Psi) \right]$$

"tr" ≡ projection over the identity.

Superconformal extension

accomodating 8-octonionic, (64 real components) spinors into the fermionic sector of a super matrix

$$\left(\begin{array}{c|cc} 0 & -\beta^{+} & \alpha^{+} \\ \hline \alpha & 0 & 0 \\ \beta & 0 & 0 \end{array}\right) \Leftarrow \text{fermionic (graded) sector}$$

Then the bosonic sector is given by

$$\left(\begin{array}{c|cc} A & 0 & 0 \\ \hline 0 & D & B \\ 0 & C & -D^{+} \end{array}\right) \Leftarrow \text{bosonic sector}$$

with $-A = A^{+}$ $\qquad A \approx S^{7}$

bosonic components $7 + 232 = 239$

$Osp(1,8|\mathbb{O})$

OCTONIONIC CONFORMAL AND SUPERCONFORMAL M-ALGEBRA

- mimicking Sudbery & Chung '87

M versus F-theory description

(10,1) Majorana
(10,2) Majorana-Weyl

$$\{Q_a, Q_b^*\} = C\Gamma_{[\mu_1\mu_2]} Z^{[\mu_1\mu_2]}$$

$\uparrow$ 4 octonionic components

$\nwarrow$ $66 - 14 = 52$

$\llcorner$ G_2 coset

Doubling the spinors : 8-components of (11,2)

$$C\Gamma_{[\mu_1\mu_2]} Z^{[\mu_1\mu_2]} + C\Gamma_{[\mu_1\mu_2\mu_3]} Z^{[\mu_1\mu_2\mu_3]} \equiv C\Gamma_{[\mu_1\ldots\mu_6]} Z^{[\mu_1\ldots\mu_6]}$$

$$64 + 168 = \underline{232}$$

Conformal algebra as algebra of transformations leaving invariant the inner product of Dirac's spinors $\psi^+ C \eta$

octonionic-valued matrices M s.t.

$$M^+ C + C M = 0$$

since $C \equiv \begin{pmatrix} 0 & \mathbb{1}_4 \\ -\mathbb{1}_4 & 0 \end{pmatrix}$ symplectic it defines the quasi-group of symplectic transformations $M = \begin{pmatrix} D & B \\ C & -D^+ \end{pmatrix}$; $B = B^+$, $C = C^+$

$Sp(8|\mathbb{O})$

$\uparrow$ $\underline{232}$ independent components

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Comment about spin-algebras (Ferrara et al.)

For $D=3,4,6$ the spinorial covering of the conformal algebra $O(D,2)$ is described by $U_\alpha(4|\mathbb{F})$ (bosonic), i.e. a classical Lie group.

The situation is different for $D=5$, $D>6$. (this is why one needs to introduce extra-generators in super Poincaré)

Spin-algebra: Fundamental spinor representation of $O(n,m)$
$\mathbb{F}_{n,m}$ ($\mathbb{F}$-valued)

$Spin(n,m)$: group of $\mathbb{F}$-valued $N\times N$ endomorphisms of $\mathbb{F}_{n,m}$ containing the spinorial covering $\overline{O(n,m)}$

Minimal spin group: $Spin_{min}(n,m)$ containing the minimal number of generators.

Spinors $D=4$ $\mathbb{C}$ $\longrightarrow$ $\widetilde{Spin}_{min} = U_\alpha U(4;1|\mathbb{C})$

$D=5$ $\mathbb{H}$ $\longrightarrow$ $\widetilde{Spin}_{min} = U_\alpha U(4;1|\mathbb{H})$

$D=7$ $\mathbb{H}$ $\longrightarrow$ $\widetilde{Spin}_{min} = U_\alpha U(8;1|\mathbb{H})$

minimal conformal superalgebras.

For $D=7$ the two constructions coincide.

For $D=4,5$ they differ

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$D=4 \quad \{Q_a, Q_b\} = C\Gamma_\mu P^\mu + C\Gamma_{[\mu\nu]} Z^{[\mu\nu]}$

$D=5 \quad \{Q_a, Q_b^+\} = C\Gamma_\mu P^\mu + C\Gamma_{[\mu\nu]} Z^{[\mu\nu]} + CZ$

$D=7 \quad \{Q_a, Q_b^+\} = C\Gamma_\mu P^\mu + C\Gamma_{[\mu\nu]} Z^{[\mu\nu]}$

To go to conformal superalgebra: construct a replica of the superalgebra (generators S_a, $\tilde{Z}_{ab}$) and introduce L_{ab} from the anticommutators $\{Q_a, S_b\} = L_{ab}$

$L_{ab} \in GL(4|\mathbb{F}_i)$

$$\begin{array}{ccccc} I_{-2} & I_{-1} & I_0 & I_1 & I_2 \\ \Psi & \Psi & \Psi & \Psi & \Psi \\ \tilde{Z}_{ab} & S_a & L_{ab} & Q_a & Z_{ab} \end{array}$$

The bosonic sector is given by the conformal algebra $U_\alpha(8|\mathbb{F}_i)$

The full conformal super. algebra is $UU_\alpha(8;1|\mathbb{F}_i)$

For $D=4$ we get $Osp(1|8)$, ...

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The 1024 generators L_{RS} form $GL(32;\mathbb{R})$

The algebra admits a 5-grading.

I_{-2}	I_{-1}	I_0	I_1	I_2
$\tilde{Z}_{rs}$	S_R	L_{RS}	Q_R	Z_{rs}

F

B

Osp(1164) as conformal M-superalgebra.

Next: Extension of this construction to generalized Poincaré superalgebras in $D<11$.

Starting from $D=4$ we get $\mathcal{F}$-series for $D = 4, 5, 7$

$\uparrow \ \uparrow \ \uparrow$

$\mathbb{R} \ \mathbb{C} \ \mathbb{H}$

$(3,1)$ $\{Q_R, Q_S\} = Z_{rs}$ $\mathbb{R}^4 \times \mathbb{R}^4$

$(4,1)$ .... $\mathbb{C}^4 \times \mathbb{C}^4$

$(6,1)$ ... $\mathbb{H}^4 \times \mathbb{H}^4$

$D=4$	$4+6=10$
$D=5$	$5+11=16$
$D=7$	$7+21=28$

Extended spacetime.

STANDARD CONSTRUCTION:

From ordinary super Poincaré algebra (defined in any dimension) the extension to conformal superalgebras can be realized only in $D=3,4,6$.

Superalgebras $U_\alpha U(4;n|\mathcal{F})$ for $\mathcal{F} \equiv \mathbb{R}, \mathbb{C}, \mathbb{H}$.

($U_\alpha U(n;m|\mathcal{F})$ is the algebra of the $\mathcal{F}$ valued graded transformations which preserve the bilinear form

$$\underline{q_i^+ A_{ij}} q_j \underline{+ \theta_k^+ \theta_k},$$

θ_k $k=1,\dots,m$ are $\mathcal{F}$-valued Grassmann variables

$A_{ij} = -A_{ji}^+$ is antihermitian

q_i^+ is the main conjugation in $\mathcal{F}$

For $D=3$ $U_\alpha U(4;n|\mathbb{R}) \equiv Osp(n;4|\mathbb{R})$ / $D=4,\dots$

In $D=11$ to introduce conformal superalgebras one needs to start from the M-algebra first.

$$\{Q_r, Q_s\} = Z_{rs} = (C\Gamma_\mu)_{rs} P^\mu + (C\Gamma_{[\mu\nu]})_{rs} Z^{[\mu\nu]} + (C\Gamma_{[\mu_1\dots\mu_5]})_{rs} Z^{[\mu_1\dots\mu_5]}$$

Introduce the conformal accelerations sectors $\tilde{Z}_{rs}$ by adding a second copy of the superalgebra $\{S_r, S_s\} = \tilde{Z}_{rs}$

$\tilde{Z}_{rs}$ symmetric 32×32 matrices $\quad [Z_{rs}, Z_{tu}] = [\tilde{Z}_{rs}, \tilde{Z}_{tu}] = 0$

The crossed anticommutator $\{Q_r, S_s\} = L_{rs}$ is closed with the help of the Jacobi identities.

$D=11$ $\qquad$ $11 = 4 + 7$

real $\qquad$ octonionic indices

$a, b, \dots$ $\qquad$ $i, j, \dots$

0	1	2	3	4	5	6	7	← p-form
1	7	7	1	1	7	7	1	← # of components

$M1_a$	4	$M_{5\,abcdi} \equiv M_{5\,i}$	7
$M1_i$	7	$M_{5\,abcij} \equiv M_{5\,ai}$	4×7
$M_{2\,[a,b]}$	6	$M_{5\,abijk} \equiv M_{5\,ab}$	6
$M_{2\,[a,i]}$	4×7	$M_{5\,aijkl} \equiv M_{5\,a}$	4
$M_{2\,[i,j]} \equiv M_{2\,i}$	7	$M_{5\,ijklm} \equiv \hat{M}_{5\,i}$	7

$52 = 2\times 7 + 28 + 6 + 4$ (in both cases)

Bosonic sector:

$$Z_{ab} \equiv (C\Gamma^{\mu})_{\mathbb{O}} P_{\mu} + (C\Gamma^{[\mu\nu]})_{\mathbb{O}} Z_{[\mu\nu]} + (C\Gamma^{[\mu_1\ldots\mu_5]})_{\mathbb{O}} Z_{\mu_1\ldots\mu_5}$$

11 components

41 components

$41 = 55 - 14$ ← G_2 automorphism

$[e_i, e_j] \sim 2C_{ijk}\, e_k$

$\Sigma_{\mu\nu} = [\Gamma_\mu, \Gamma_\nu]$ generators of $so(10,1)/G_2$

$D=7$ $S^7 \equiv Spin(7)/G_2$

$M_1 + M_2$ saturates the bosonic degrees of freedom

What about M_5?

$$M_5 \equiv M_1 + M_2$$

(based on octonionic p-forms identities)

What about octonions?

(10,1) can be realized either associatively $(\mathbb{R})$ or non-associatively $(\mathbb{O})$

spinors: 32-component real

4-octonionic components $4 \times 8 = 32$

octonionic M-algebra J. Lukierski - F.T. PLB 200[illegible] PLB 200[illegible]

octonionic identities for p-forms Carrion-Rojas-F.T. JHEP 2003

octonionic M-algebra as octonionic hermitian supersymmetry.

$$\{Q_a, Q_b^*\} = Z_{ab}$$

4×4 octonionic hermitian matrix

$$\begin{pmatrix} \cdot & \times & \times & \times \\ \cdot & \cdot & \times & \times \\ \cdot & \cdot & \cdot & \times \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix} \qquad 4 + 6 \cdot 8 = 52 \text{ components}$$

HH_{II} (quaternionic holomorphic susy)

It does not admit tensorial central charges rank ≥ 2.

It only exists in given space-time dimensions

	bosonic sector	d.o.f.
$D=3$	M_0+M_1	$1+3=4$
$D=4$	M_1	4
$D=5$	M_1	5
$D=6$	—	—
$D=7$	—	—
$D=8$	—	—
$D=9$	M_0	1
$D=10$	M_0+M_1	$1+10=11$
$D=11$	M_0+M_1	$1+11=12$
$D=12$	M_1	12
$D=13$	M_1	13

Through mod 8

—	$D=0,6,7$	mod 8
M_0	$D=1$	mod 8
M_1	$D=4,5$	mod 8
M_0+M_1	$D=2,3$	mod 8

	Bosonic sector	d.o.f.
$D=3$	M_1	3
$D=4$	$\widetilde{M}_2$	3
$D=5$	M_2	10
$D=6$	$\widetilde{M}_3$	10
$D=7$	M_0+M_3	$1+35=36$
$D=8$	$M_0+\widetilde{M}_4$	$1+35=36$
$D=9$	$M_0+M_1+M_4$	$1+9+126=136$
$D=10$	$M_1+\widetilde{M}_5$	$10+126=136$
$D=11$	$M_1+M_2+M_5$	$11+55+462=528$
$D=12$	$M_2+\widetilde{M}_6$	$66+462=528$
$D=13$	$M_2+M_3+M_6$	$78+286+1716=2080$

N.B. in $D=11,12$ quaternionic spacetime with complex structure for spinors singled out

$\Rightarrow$ Euclidean equivalent of the M & F theory

(holomorphic supersymmetry)

Quaternionic space-times

D=3	(0,3)			
D=4	(0,4)	(4,0)		
D=5	(1,4)	(5,0)		
D=6	(1,5)	(5,1)		
D=7	(2,5)	(6,1)		
D=8	(2,6)	(6,2)		
D=9	(3,6)	(7,2)		
D=10	(3,7)	(7,3)		
D=11	(4,7)	(8,3)	(0,11)	
D=12	(4,8)	(8,4)	(0,12)	(12,0)
D=13	(5,8)	(9,4)	(1,12)	(13,0)

HH_I

	Bosonic sector	d.o.f.
D=3	M_0	1
D=4	M_0	1
D=5	M_0+M_1	$1+5=6$
D=6	M_1	6
D=7	M_1+M_2	$7+21=28$
D=8	M_2	28
D=9	M_2+M_3	$36+84=120$
D=10	M_3	120
D=11	$M_0+M_3+M_4$	$1+165+330=496$
D=12	M_0+M_4	$1+495=496$
D=13	$M_0+M_1+M_4+M_5$	$1+13+715+1287=2016$

CLASSIFICATION (series of tables)

SUSY	Hodge	d.o.f. (bos.)
RR	yes	S
RC	no	S
RH	yes	S
CC_I	no	S
CH_I	yes	S
HH_I	yes	S
CC_{II}	no	S^b
CH_{II}	yes	$S^\#$
HH_{II}	yes	<

S = saturation

S^b = consistent reality constraint $\frac{1}{2}$ d.o.f.

$S^\#$ = automatically implemented reality constraint

< = less than the expected d.o.f.
(saturation only reached for D = 3, 4)
due to consistency conditions imposed by
Lorentz

Example: (4,1) C-type.

$(4,1) \subset (4,3)$ RR - susy. 8-real component spinors

36 components (bosonic)

$$M_0^{D=7} + M_3^{D=7} \equiv M_0^{D=5} + M_3^{D=5} + 2M_2^{D=5} + M_1^{D=5}$$

$$1 + 35 = 1 + 10 + 2\cdot 10 + 5$$

4-complex spinors $(0,1) \to (3,0) \to (4,1)$

CC_I : 16 components (bosonic)

$$M_1 + M_3 + M_5$$

$$16 = 5 + 10 + 1$$

CC_{II} : 20 components

$$M_2 + M_3$$ no Hodge duality

$$10 + 10$$

Extra reality constraint kills $\frac{1}{2}$ the bosonic components

Let us denote susies with "XY"

type of rep used ↗ ↖ division-algebra type

Cases: RR

RC CC_{I} CC_{II}

RH CH_{I} CH_{II} HH_{I} HH_{II}

$$\begin{cases} RC \equiv CC_{I} \\ RH \equiv CH_{I} \equiv HH_{I} \end{cases}$$

$$\left[\, RR \approx CC_{I} + CC_{II} \right.$$

	SYM	HER
R(4)	10	
C(2)	6	4
H(1)	4	1
R(8)	36	
C(4)	20	16
H(2)	12	6
R(16)	136	
C(8)	72	56
H(4)	40	28
R(32)	528	
C(16)	272	256
H(8)	144	120
R(64)	2080	
C(32)	1056*	1024
H(16)	544	496
⋮		

← M-algebra (saturated)

* $1056 = 2 \cdot 528$

What about if spinors are $\mathbb{C}$, $\mathbb{H}$ or even $\mathbb{O}$-valued? (27)

$$\{Q_a, Q_b\} = Z_{ab} \leftarrow \boxed{\{Q^*_a, Q^*_b\} = Z^*_{ab}}$$

symmetric

conjugate

$$\{Q_a, Q^*_b\} = W_{ab}$$

hermitian

N.B. $*$ is the principal conjugation.

Count the maximal number of components (not necessarily saturated)

Notice: we can also impose constraints

I) $Z \equiv 0$ (hermitian supersymmetry)

II) $W \equiv 0$ (holomorphic supersymmetry)

M-algebra $D=11$ $(10,1)$ 32-real component spinors

$$\{Q_a, Q_b\} = Z_{ab}$$

symmetric

528 saturated components (32×32 symmetric)

$$Z_{ab} = C\Gamma^{\mu} P_{\mu} + C\Gamma^{[\mu\nu]} Z_{[\mu\nu]} + C\Gamma^{[\mu_1 \dots \mu_5]} Z_{[\mu_1 \dots \mu_5]}$$

$$528 = 11 + \underset{55}{\binom{11}{2}} + \underset{462}{\binom{11}{5}}$$

F-algebra viewpoint $(10,2)$ Majorana-Weyl spinors

$$\{\tilde{Q}_{\tilde{a}}, \tilde{Q}_{\tilde{b}}\} = \tilde{Z}_{\tilde{a}\tilde{b}}$$

$$\tilde{Z}_{\tilde{a}\tilde{b}} = P(\tilde{C}\,\tilde{\Gamma}^{[\tilde{\mu}\tilde{\nu}]})\tilde{Z}_{[\tilde{\mu}\tilde{\nu}]} + P(\tilde{C}\,\Gamma^{[\tilde{\mu}_1 \dots \tilde{\mu}_6]})\tilde{Z}_{[\tilde{\mu}_1 \dots \tilde{\mu}_6]}$$

self dual

$$528 = \underset{66}{\binom{12}{2}} + \underset{462}{\frac{1}{2}\binom{12}{6}}$$

Z_{ab} abelian: superconformal algebra recipe: take two copies and close the algebra via Jacobi identities ⇒ $Osp(1,64)$