Black holes and extra dimensions

D. Stojkovic **PRL** 94: 011603 (2005)
V. Frolov, D. Stojkovic **PRL** 89:151302 (2002)
V. Frolov, D. Fursaev, D. Stojkovic **JHEP** 0406:057 2004

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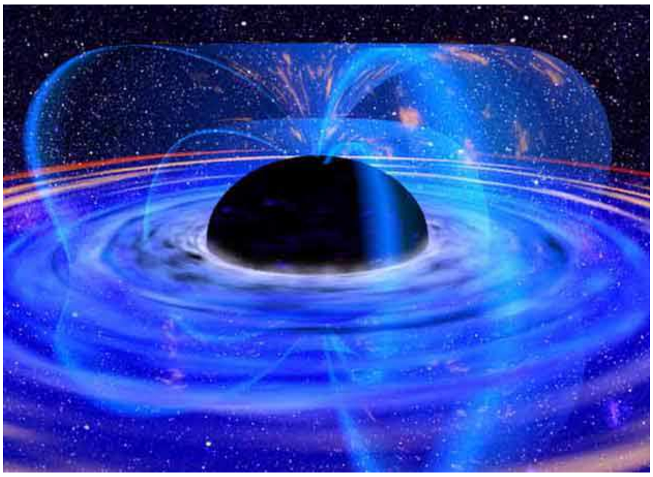

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Motivation

- **Black holes: most interesting and intriguing solutions of Einstein's equations**

- **Extra dimensions seem to be necessary in an ultimate theory of high energy physics**

- **Brane world models ➠ large extra dimensions**

- **Higher dim. black holes as classical solutions**

Black holes in accelerators!

Outline

- Black holes - basics

- Extra dimensions - basics

- Brane world models

- Higher dim. black holes in brane world models

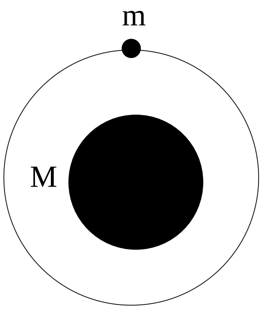
i) black holes on the brane

ii) black holes in the bulk

Black Holes

- Laplace in 18th century

$$E_g = G\frac{mM}{r} = \frac{1}{2}mc^2 \quad \rightarrow \quad r_g = 2GM/c^2$$



- Einstein, 1915, **General Relativity**

- Schwarzschild metric, 1916:

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) cdt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega$$

- $r = 0$ ➠ metric is singular

- $r = r_{horizon} = 2GM/c^2$ ➠ coordinate singularity

- at $r = r_{horizon}$ t and r exchange their roles

Different types of black holes

- **Black holes formed in collapse of stellar matter**

- endpoint of stellar evolution $\&$ mergers and accretion

- mass range: a few M_{Sun} - $10^9 M_{Sun}$

- **Primordial black holes**

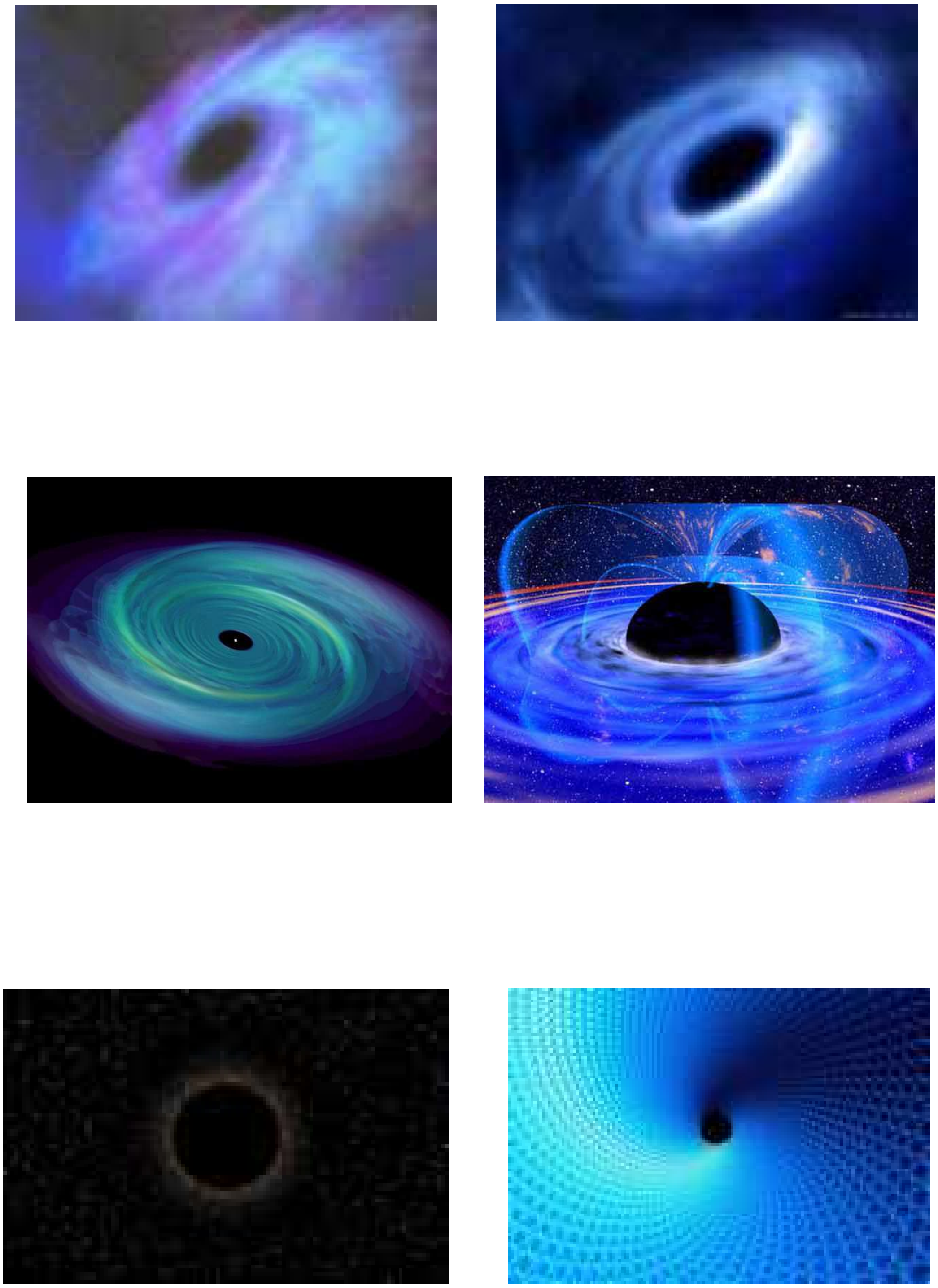
- early universe, large fluctuations in energy density

- mass range: M_{Pl} - M_{Sun}

- **Black holes formed in the Lab**

- need accelerator as big as the whole universe

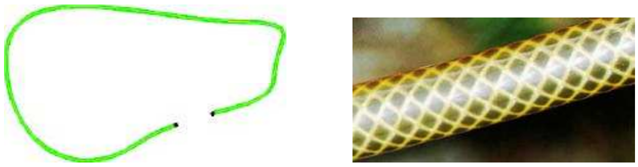
- mass range: a few $M_{Pl} \sim 10^{19} GeV$



Extra Dimensions

- Our world is manifestly $3+1$ dim. on large scales
- Kaluza (1921) and Klein (1926) introduced the fifth dimension to unify gravity with electromagnetism

- 5th dimension rolled on a very small circle



- Size of extra dimension is $L_{Pl} \sim 10^{-33}$cm

Higher dimensional objects?

- To unify all the interactions we need more than one extra dimension
- KK approach ➠ at least 11 dimensions
- String theory ➠ 10 or 11 dimensions

➠ Can have interesting higher dimensional objects

(eg. higher dim. black holes, topological defects etc.)

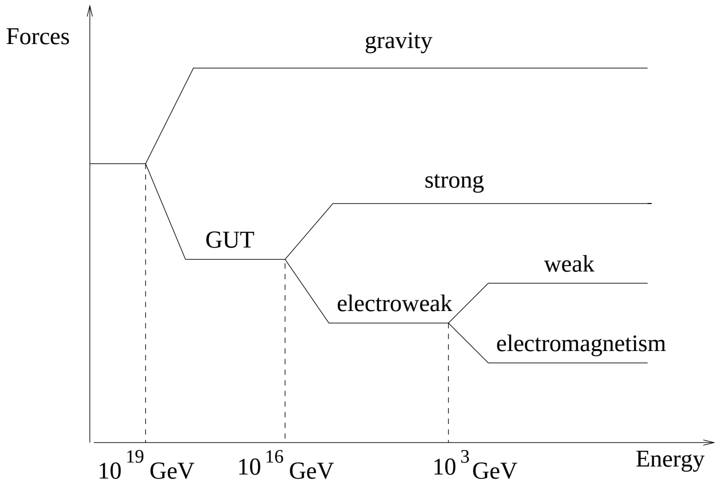
- At distances of $L_{Pl} \sim 10^{-33}$cm quantum gravity effects become very important
- Problem: quantum gravity has not been formulated

➠ Can not describe them properly!

Brane worlds and large extra dimensions

- Brane world models have attracted a lot attention
- Introduced as a solution to the hierarchy problem
- They imply existence of large extra dimensions
- They offer rich higher dimensional phenomenology
- We can study higher dimensional black holes

The hierarchy problem



- Gravity is by far the weakest interaction in nature

- Planck energy scale:

$M_{Pl} \sim 10^{19}\mathrm{GeV} \;\gg M_{EW} \sim 200$GeV

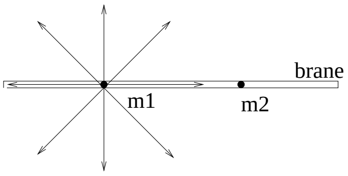
$G_{Newton} = 1/M_{Pl}^2$ ➠ weak gravity

$F_{EM} = \frac{q_1 q_2}{r^2} \qquad F_G = G\frac{m_1 m_2}{r^2}$

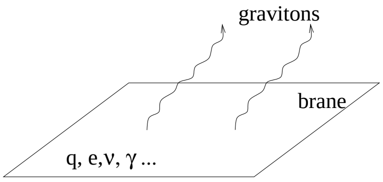
- For protons, gravity is 10^{36} times weaker

Idea

- Gravity is as strong as the other interactions
- Gravitational force is diluted due to the presence of extra dimensions



- Assume that the standard model particles are confined on a 3-dim subspace (i.e. brane) while gravitons can propagate everywhere



Flat compact extra dimensions

- $V_{extra} = R^d$
- $G_4 \equiv \frac{1}{M_{Pl}^2} = \frac{G_{4+d}}{V_{extra}}$
- Fundamental scale $M_* \sim 1$TeV
- Compactification radius: $R \sim 10^{\frac{32}{d}} \cdot 10^{-19} m$

$d = 1 \quad => \quad R \sim 10^{13} m$ (excluded)

$d = 2 \quad => \quad R \sim 1 mm$ (current lab limit)

$d = 3 \quad => \quad R \sim 10^{-5} mm$

$d = 6 \quad => \quad R \sim 10^{-11} mm$

$R >>$TeV$^{-1} \sim 10^{-16}$mm

➠ a lot of room for higher dim. classical objects

Higher dimensional black hole metric

- A static black hole in $(N+1)$-dim space-time:

$$dS^2 = -F dT^2 + \frac{dR^2}{F} + R^2\, d\Omega^2_{N-1}$$

$$F = 1 - \left(\frac{R_0}{R}\right)^{N-2}$$

- N ➠ number of spatial dimensions
- R_0 ➠ gravitational radius

- Schwarzschild radius of a $(N+1)$-dim black hole

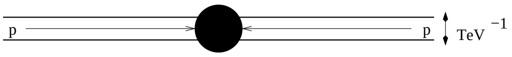
$$R_S = \tfrac{1}{M_*}\left[\tfrac{M}{M_*}\right]^{\frac{1}{N-2}}$$

Black Holes in accelerators

- **Particle accelerator (e.q. Large Hadron Collider):**
- Collision of two particles with COM energy $\sqrt{\hat{s}}$

- If an impact parameter is $< R_S$ for a given $\sqrt{\hat{s}}$

Black hole with a mass $M = \sqrt{\hat{s}}$ forms



- Large Hadron Collider ➠ CERN (2007)
- LHC: $\sqrt{\hat{s}} = 14$TeV
- Geometrical cross section for black hole production:

$$\sigma(M) \approx \pi R_S^2$$

➠ Numerical estimates:

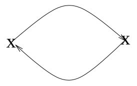
$$\boxed{10^7 \textbf{ black holes per year if } M_* = 1\textbf{TeV !}}$$

- LHC - black hole factory

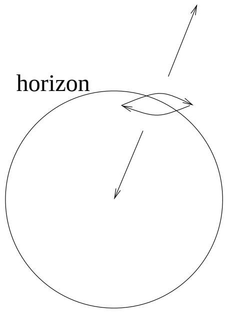
Hawking radiation

- Hawking (1973): **black holes radiate**

- Virtual particle-antiparticle pairs are being created all the time in vacuum



- Usually they disappear almost instantaneously



- If a virtual pair is created near the event horizon

➠ one of them could escape and become real

➠ the black hole thereby loses mass

- Black hole Hawking radiation is thermal
- Black hole decays into all degrees of freedom available at a given temperature democratically

- Black hole Hawking radiation temperature

$$T_H = 1/R_S$$

- The number of particles emitted $S \sim \frac{R_S^2}{M_*^2}$
- If $M_* = 1$TeV and $N + 1 = 10$

➠ $M = 5$TeV black hole emits of order 30 quanta

- BH event quite different from any other SM event

Black holes radiate mostly on the brane ?

R. Emparan, G. Horowitz, R. Myers, PRL **85** 499 (2000) ➠

- $\lambda_T > R_S$ ➠ point radiator ➠ s-mode dominant
- $\#$ of deg. of freedom much larger on the brane ? (~ 60)

- $\#$ of degrees of freedom of gravitons in the $N+1$-dimensional space-time is $\mathcal{N} = (N+1)(N-2)/2$

➠ For $N+1 = 10$ we have $\mathcal{N} = 35$!

- LHC ➠ non-zero impact parameter ➠ rotating bh

V. Frolov, D. Stojkovic **PRD** 67:084004 (2003) ; D. Stojkovic **PRL** 94: 011603 (2005)

- rotating black holes ➠ superradiance ➠ graviton emission dominant

Black holes radiate mostly OFF the brane !

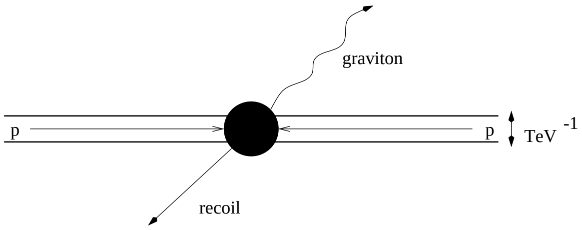
Recoil Effect

V. Frolov, D. Stojkovic **PRL** 89:151302 (2002) ➠

- Any particle emitted in the bulk can cause a recoil of the black hole from the brane

➠ Recoil due to Hawking radiation can be very significant for small black holes

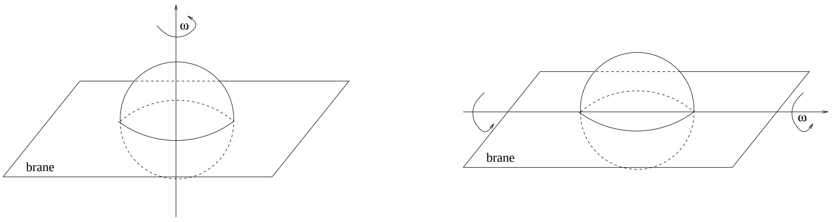
➠ Black hole radiation would be terminated and an observer located on the brane would register the virtual energy non-conservation



Rotating black hole on the brane

V. Frolov, D. Fursaev, D. Stojkovic **JHEP** 0406:057 2004 ; **CQG** 21:3483 (2004)

D. Stojkovic **JHEP** 0409:061 (2004)



- Kerr metric: small rotational parameter a

$$ds^2 = ds^2_{Sch} - 2a\sin^2\theta\, d\varphi \left(\frac{r_0}{r}\, dv + dr\right)$$

- Brane deformed by the black hole rotation is described by $\varphi = \bar{\varphi} + \psi$

$${}^{(3)}\bar{\Delta}\psi + \frac{2}{r^2}\cot\theta\psi_{,\theta} + 2\frac{B}{r}\psi_{,r} = \frac{a}{r^3}$$

- Solution ➠

$$\boxed{\psi = -a/r}$$

Angular momentum loss

- The stress-energy tensor of the brane is

$$\sqrt{-g}T^{\mu\nu} = \sigma \int d^{n+1}\zeta\delta^{(N+1)}(X - X(\zeta))\sqrt{-\gamma}\gamma^{ab}X^{\mu}_{,a}X^{\nu}_{,b}$$

- $\dot{J}$ ➠ rate of loss of the angular momentum

$$\dot{J} = \int_{r=const} \sqrt{-g}T^{r\nu}\xi_{\nu}d^{N-1}\Omega$$

$$\boxed{\dot{J} = -\pi\sigma a r_0 \cos^2\alpha}$$

$\alpha = \pi/2$ ➠ $\dot{J} = 0$ ➠ final stationary equilibrium configuration of the rotating black hole

- The relaxation time ➠ $T \sim (\pi G_4 \sigma \cos^2\alpha)^{-1}$

Generalization to higher dimensions

- Theorem:

V. Frolov, D. Fursaev, D. Stojkovic

Black hole in its final stationary state can have only those components of angular momenta which are connected with Killing vectors generating transformations preserving a position of the brane

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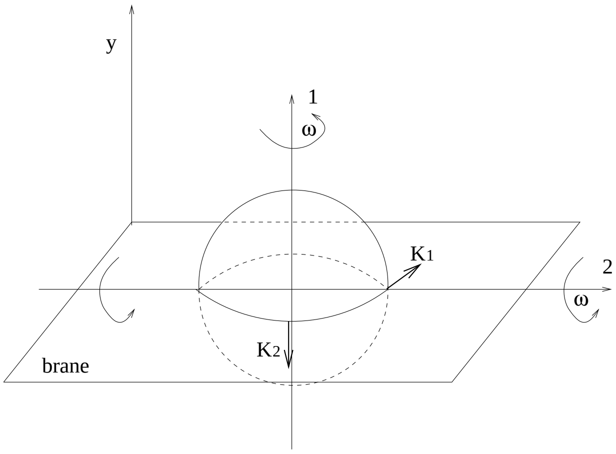
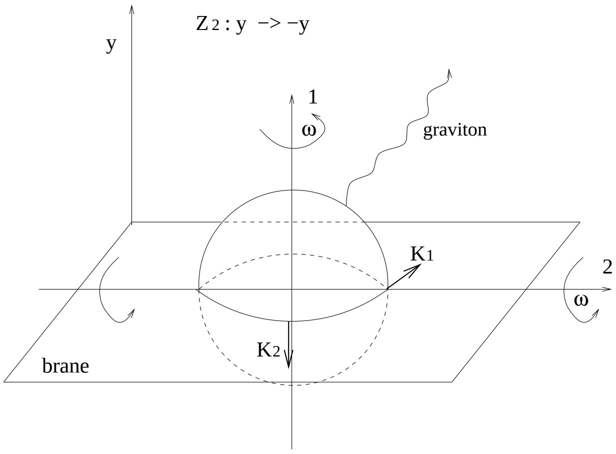


Randall-Sundrum black hole

D. Stojkovic **PRL** 94: 011603 (2005)

- Models with Z_2 symmetry (e.g. RS) ➠

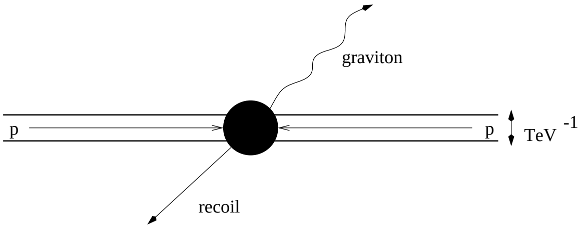
bulk angular momentum of the black hole is strictly zero



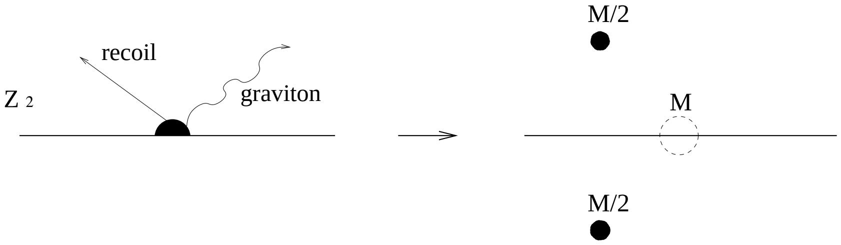
➠ Brane behaves as if it were an infinite tension brane (absorbs all the incoming angular momentum)

Recoil

ADD black hole



RS black hole



- ➠ Brane behaves as if it were an infinite tension brane (absorbs all the incoming linear momentum)

Consequences for Hawking radiation

• Black holes produced in collisions of particles (e.g. LHC) ➠ highly rotating

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• Small ADD black hole on the brane

- Spin dependent super-radiance effect ➠ Hawking radiation dominated by higher spin particles i.e. bulk gravitons

- Black hole quickly loses bulk angular momentum ➠ second phase of radiation is mainly on the brane

- Existence of relaxation time

- Can recoil and leave the brane

• Small RS black hole on the brane

- Bulk emission strongly suppressed

- No bulk rotation (absence of relaxation time)

- Can not recoil and leave the brane

Black hole in the bulk

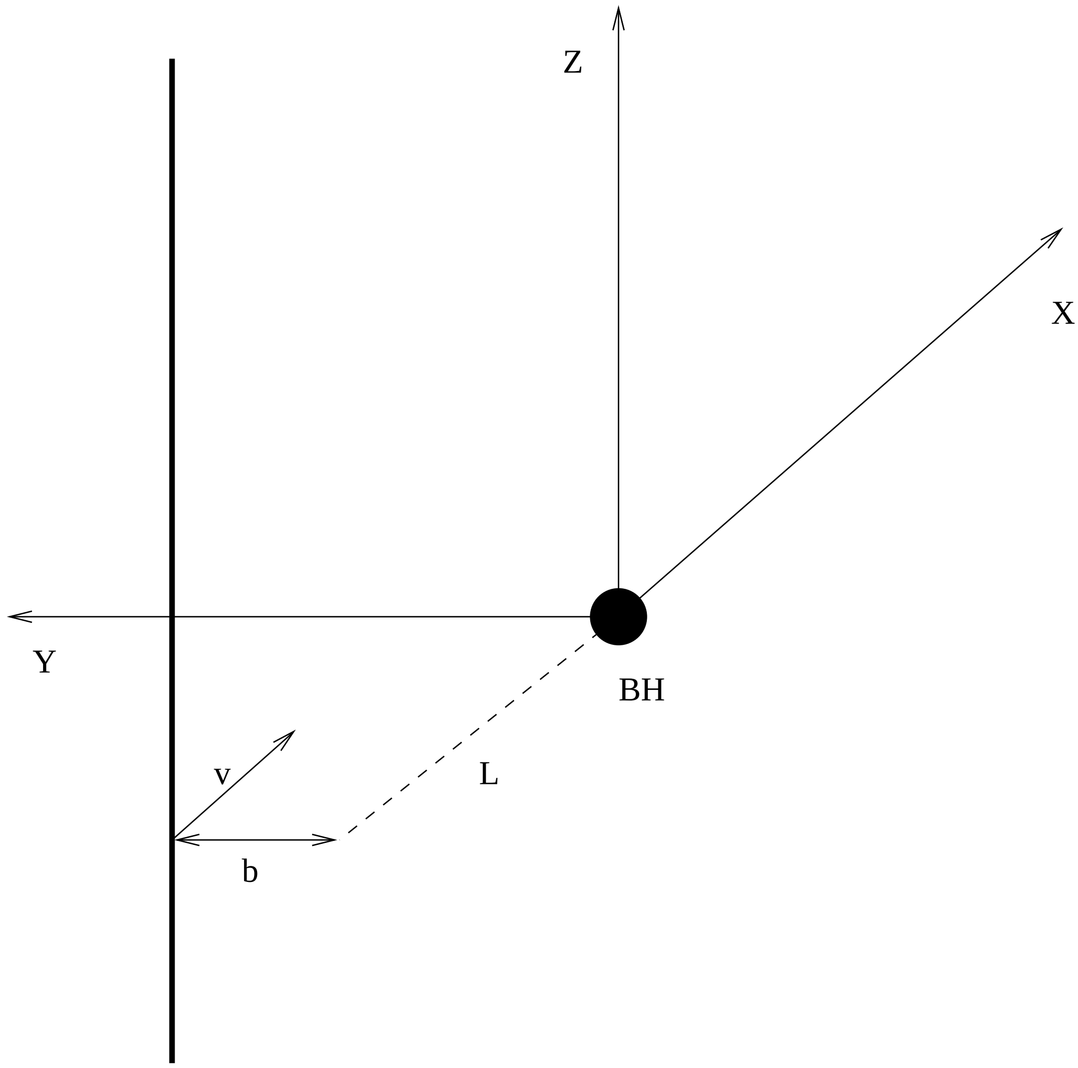
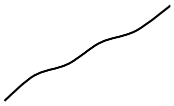


Figure 1: Brane moving in X-direction with an impact parameter b and initial position -L

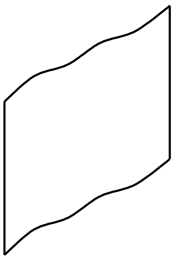
n - branes

n=0 point particle

n=1 string



n=2 domain wall



n=3 brane world ...

Dynamics of an n- brane

- Action for a relativistic point particle: $S = -m \int ds$
- Nambu-Goto action for an extended object:

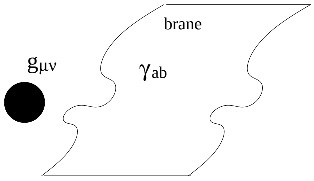
$$S = -\sigma \int \sqrt{-\gamma} d^n \zeta$$

➠ σ is the brane tension

➠ γ is the determinant of an induced metric γ_{ab}

$$\gamma_{ab} = g_{\mu\nu} \partial_a X^\mu \partial_b X^\nu$$

➠ $g_{\mu\nu}$ is the background metric



Brane perturbation equation of motion

- Consider a brane $\mathcal{X}^A(x^\mu)$ moving in the space-time with a metric $G_{AB}(X^C)$
- Nambu-Goto action ➠ the brane equation:

$$\left(\sqrt{-\gamma}\,\gamma^{\mu\nu}\;\mathcal{X}^A_{,\mu}\right)_{,\nu}+\sqrt{-\gamma}\,\gamma^{\mu\nu}\,\Gamma^A_{BC}\,\mathcal{X}^B_{,\mu}\,\mathcal{X}^C_{,\nu}=0$$

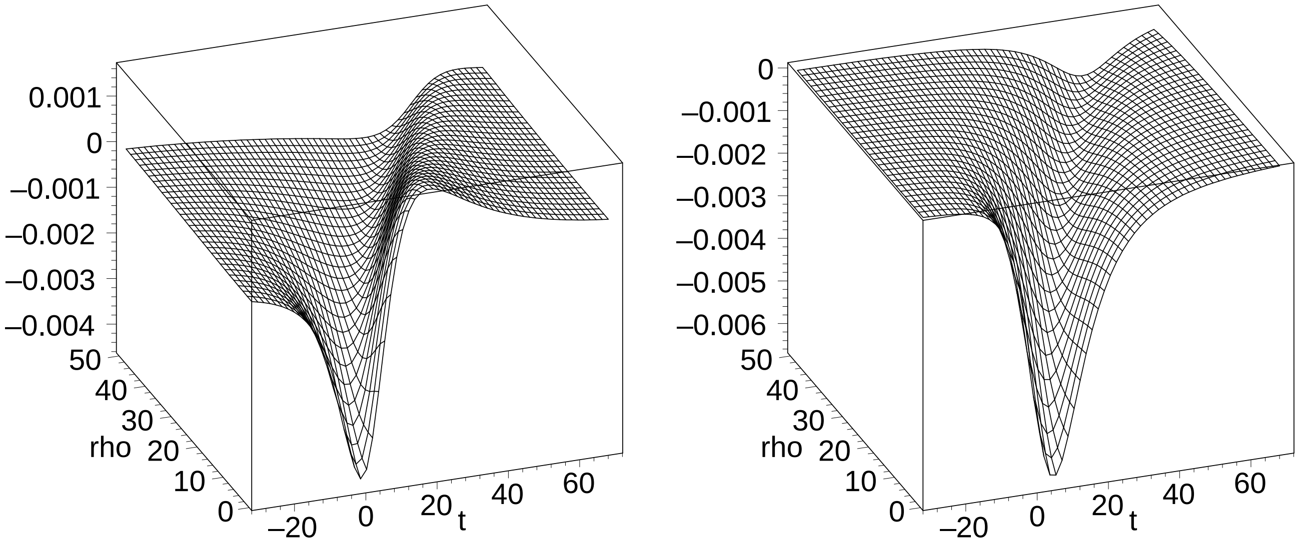
- Linearized problem:

$$\mathcal{X}^A=\mathcal{X}^A_0+\chi^A(x)$$

- χ^A are the physical degrees of freedom of the brane
- The linearized Nambu-Goto brane equation is:

$$\boxed{\Box^{(n+1)}\,\chi_{\hat{m}}=f_{\hat{m}}}$$

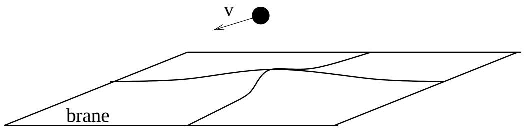
Solutions for 2 extra dimesnions



a) χ_4 **b)** χ_5

Figure 2: Plots of χ_4 and χ_5 for $\beta = 1$, $R_0 = 1$, $b = 10$, $k = 2$. Disturbance of the brane developed around $t = 0$ which travels at a speed of light outward from the point of the brane closest to the black hole

Black hole energy loss



- Energy loss calculated at the future null infinity:

$$\Delta E = \frac{8\,\pi^2\,\sigma\,\sinh^5\beta}{9\cosh^3\beta}\,\frac{R_0^6}{b^3}$$

- If extra dimensions are compact, the black hole will keep passing near the brane. Because of the friction the black hole will be slowing down and finally stop

If $R_0 \sim b \sim$TeV^{-1}, $\sigma \sim$TeV^4 and $v \sim c$

➠ only one turn before the black hole loses all of its kinetic energy

Induced geometry on the brane

- The metric induced on the brane by a bulk black hole:

$$ds^2 = -\left[1-(k+1)\,\Psi\right]dt^2 + (1+\Psi)\left[d\rho^2 + \rho^2\, d\Omega_2^2\right]$$

- The Einstein tensor is:

$$G_{\hat{0}\hat{0}} \sim \left[-\rho^2 k + 3b^2\right]$$

$$G_{\hat{1}\hat{1}} \sim (\rho^2 + b^2)$$

$$G_{\hat{2}\hat{2}} \sim \left[-\rho^2(k+1) + 2b^2\right]$$

- For an observer located on a brane:

$$T_{\hat{\mu}\hat{\nu}} = \frac{1}{8\pi G^{(4)}}\, G_{\hat{\mu}\hat{\nu}}$$

- The energy density ε becomes negative at $\rho \geq \rho_c \approx b$

- The transverse pressures $p_{\theta\perp}$ and $p_{\phi\perp}$ become negative at $\rho \geq \rho_c \approx b$

Energy conditions

- "Normal" matter satisfies certain energy conditions:

- Strong energy condition requires $\varepsilon \geq 0$ and $p \geq 0$

- Weak energy condition allows $\varepsilon \geq 0$ and $p < 0$

- $\varepsilon < 0$ **all the energy conditions are violated**

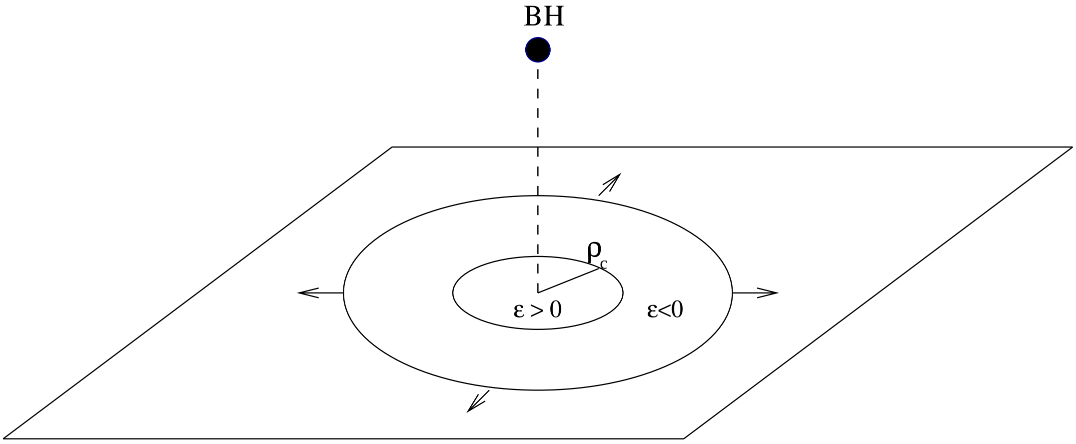


Figure 3: The energy density in the central region is positive. For $\rho > \rho_c$ the energy density is negative

➠ For $\rho \geq \rho_c \approx b$:

all the energy conditions are (apparently) violated

- At large distances $\gg 1$mm ➠ 4-dim Newtonian gravity must be recovered

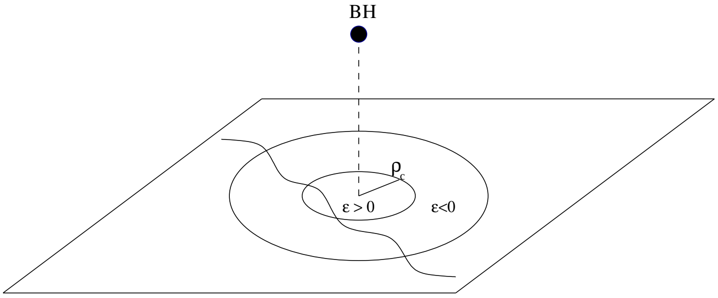
- The total induced positive mass is:

$$m_+ \sim \frac{R_0}{G^{(4)}} \left(\frac{R_0}{b} \right)^k$$

- for $R_0 \sim b \sim$TeV^{-1} we have $m_+ \sim 10^{11}$g

➠ electron size “particle" with $m = 10^{11}$g !

- Visible only for test particles with $\lambda \sim$ TeV^{-1}



Gravitational lensing

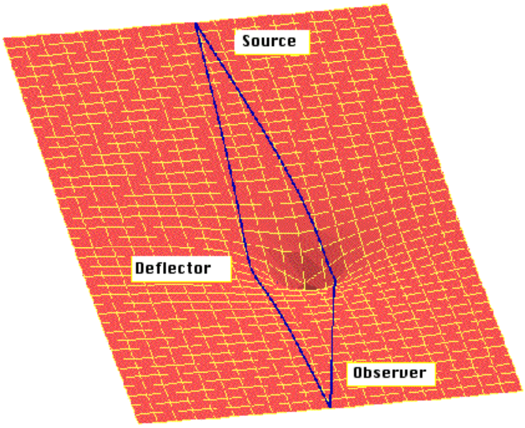


Figure 4: Gravitational lensing

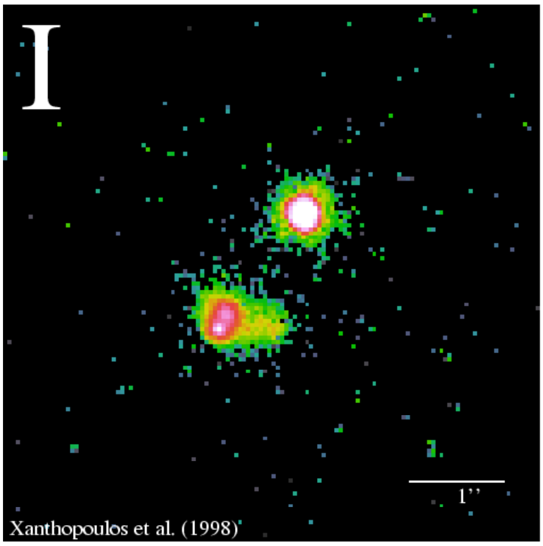


Figure 5: Two images of one object

Deflection of light

- Derive the deflection angle of a light ray passing in the region of influence of the ‘shadow-matter’
- For comparison, the standard $(3+1)$-dim result is:

$$\Delta\phi \sim \frac{1}{b_0}$$

b_0 ➠ the impact parameter of light

- k extra dimensions

$$\Delta\phi \sim \frac{1}{b_0^{1+k}}$$

➠ Can distinguish between the real matter on the brane and the bulk ‘shadow matter’ as a cause of deflection

Lensing by shadow matter

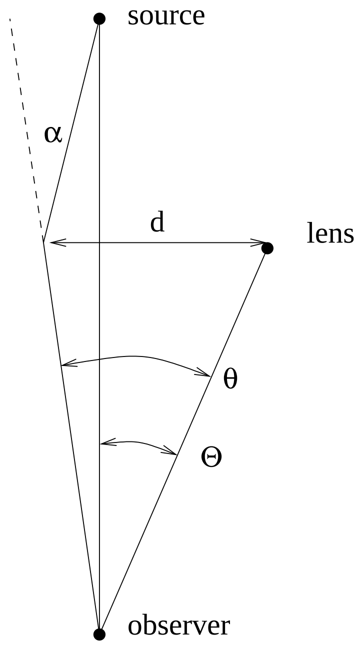


Figure 6: Lens diagram

- $\alpha \propto \frac{1}{d}$ ➠ $\theta^2 - \Theta\theta - \frac{2R_0}{R} = 0$
- $\alpha \propto \frac{1}{d^k}$ ➠ $\theta^{2+k} - \Theta\theta^{1+k} - \left(\frac{R_0}{R}\right)^{1+k} = 0$

➠ shadow matter produces more images and different amplification

➠ quite different light curves

Conclusions

We studied black holes in brane world models

1. Black holes on the brane

- Strong TeV gravity ➠ black holes in the lab

2. Black holes in the bulk

- Strong TeV gravity ➠ weak field approximation does not mean weak effects on the brane

Respectfully Quoted: A Dictionary of Quotations. 1989.

Dialog between Lord Michael Faraday (1791 - 1867) and the Chancellor of the Exchequer

QUOTATION:

Mr. Gladstone, then Chancellor of the Exchequer, had interrupted Lord Faraday in a description of his work on electricity to put the impatient inquiry:

“Very well Lord Faraday, but after all, what use is it?"

Like a flash of lightning came the response:

“Well sir, there is every probability that you will soon be able to tax it!"