Supersymmetric cascade decays

based on: B. K. Gjelsten, D. J. Miller, P. Osland

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Per Osland, University of Bergen

Outline

- Introduction
- Graphic overview of parameter space
- Kinematical endpoints (squark chain)
- Inverting endpoint formulas
- Fitting MC data: precision
- 10,000 LHC experiments: precision
- LC input: high precision
- gluino chain (b-tagging)

Supersymmetry may be realized at the LHC

Unstable particles produced copiously, cascade decays, e.g.

$$\tilde{q} \to \tilde{\chi}_2^0 q \to \tilde{l} l q \to \tilde{\chi}_1^0 l l q$$

Challenge: determine masses with high precision

Refs: Baer et al, hep-ph/9512383; Hinchliffe et al, hep-ph/9610544; Bachacou et al, hep-ph/9907518; Polesello, ATLAS Int Note 1997; Allanach et al, hep-ph/0007009; Gjelsten et al, ATLAS Note 2004; Chiorboli, Tricomi, CMS Note 2004

mSUGRA (CMSSM)

- Unification at high energies, fewer parameters

$$m_0 \quad m_{1/2} \quad A_0 \quad \tan\beta \quad \mathrm{sign}\,\mu$$

- Snowmass Points and Slopes: Allanach et al, hep-ph/0202233: SPS 1a, SPS 1b, SPS 3, SPS 5.
  Other set: Battaglia et al, hep-ph/0306219
- WMAP constraints: Bennett et al, astro-ph/0302207; Spergel et al, astro-ph/0302209

footnote: Benchmark Points

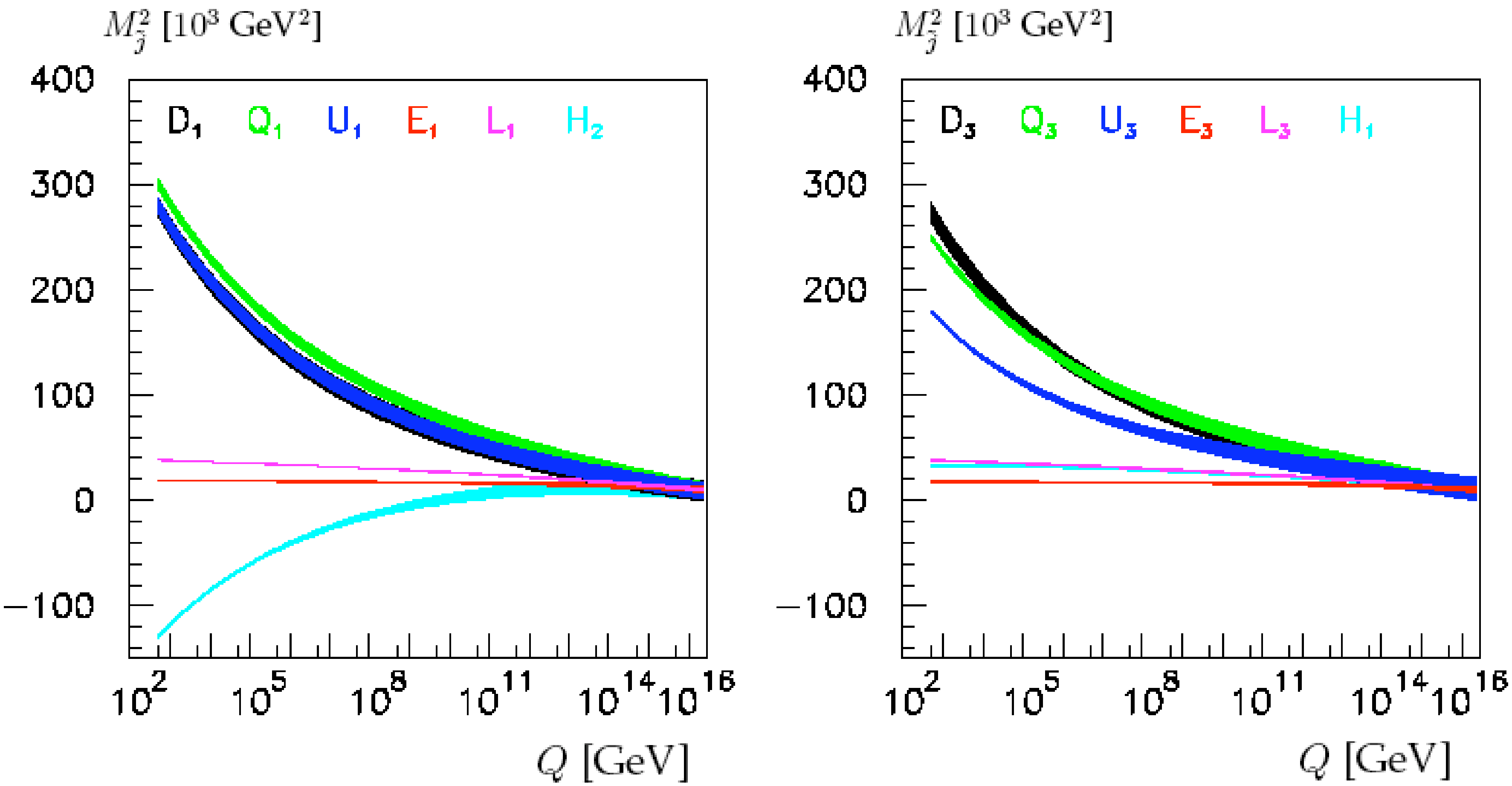
- LHC Points ('SUGRA'): Hinchliffe et al, hep-ph/9610544: Point 1, Point 2, Point 3, Point 4, Point 5
- Post-LEP Benchmarks ('CMSSM'): Battaglia et al, hep-ph/0106204: A, B, C, ..., M
- Snowmass Points and Slopes ('mSUGRA'): Allanach et al, hep-ph/0202233: SPS 1a, SPS 1b, SPS 2, SPS 3, SPS 4, SPS 5, SPS 6, .., SPS 9
- Post-WMAP Benchmarks ('CMSSM'): Ellis et al, hep-ph/0303043, Battaglia et al hep-ph/0306219: A', B', C', ..., M'

Mutations: Point 5 → B → SPS 1a → B'

Precision in masses allows extrapolation to Unification scale

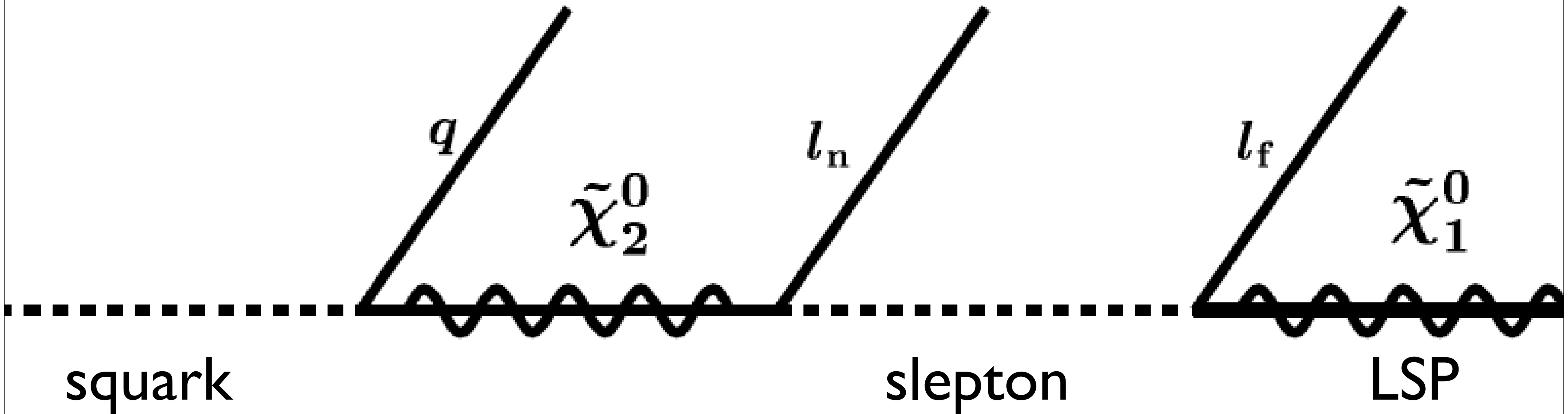
Example: Sfermion mass parameters

Allanach et al, hep-ph/0403133



Precision of order per cent achievable with LHC plus LC

“Easy” SPS 1a squark cascade



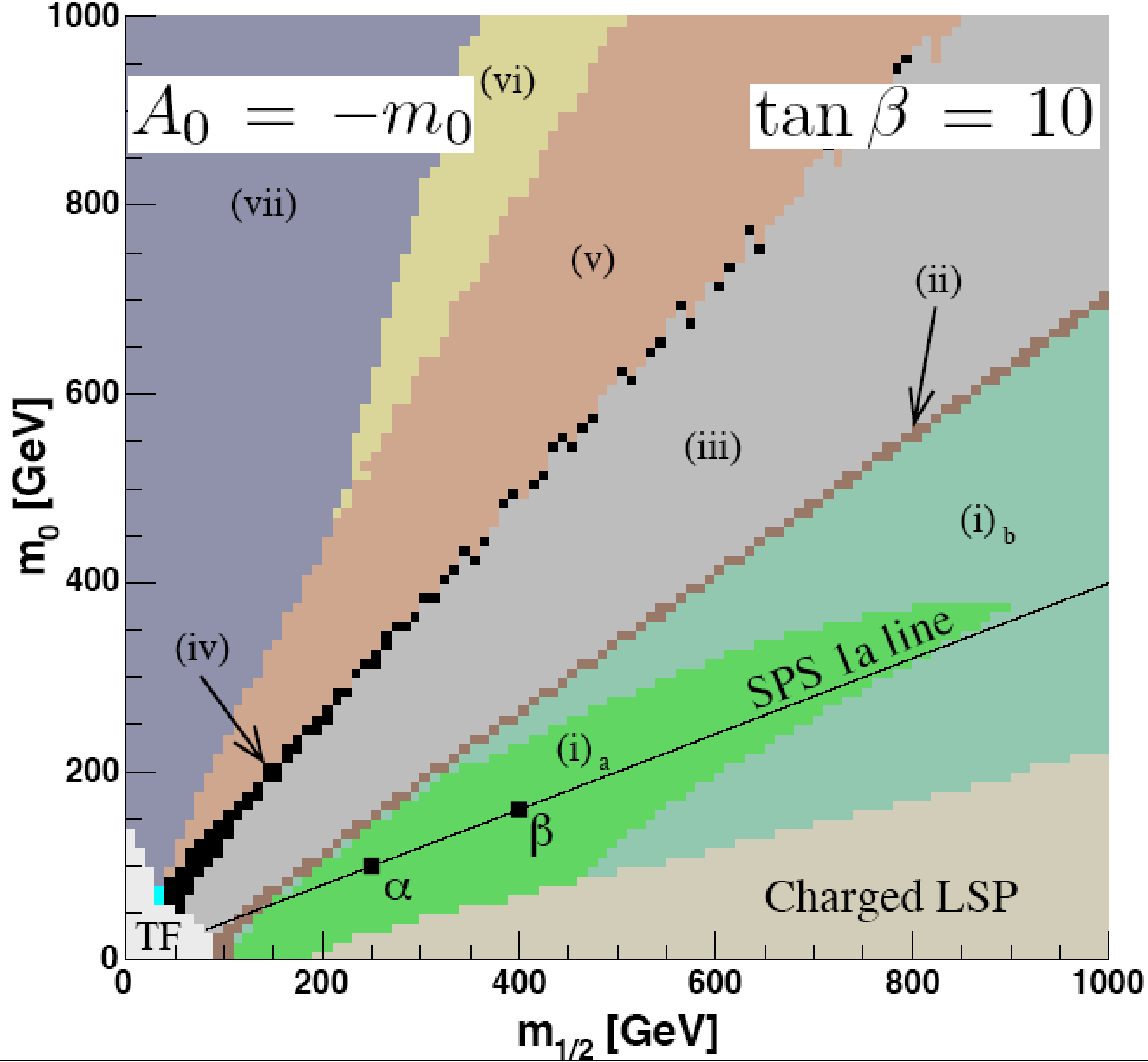
Detect: quark jet and two leptons

Aim: determine squark, slepton and neutralino masses

Question: Is this mass hierarchy “typical”?

Want “heavy” gluino and “heavy” neutralino $\tilde{\chi}^0_2$

Hierarchies:

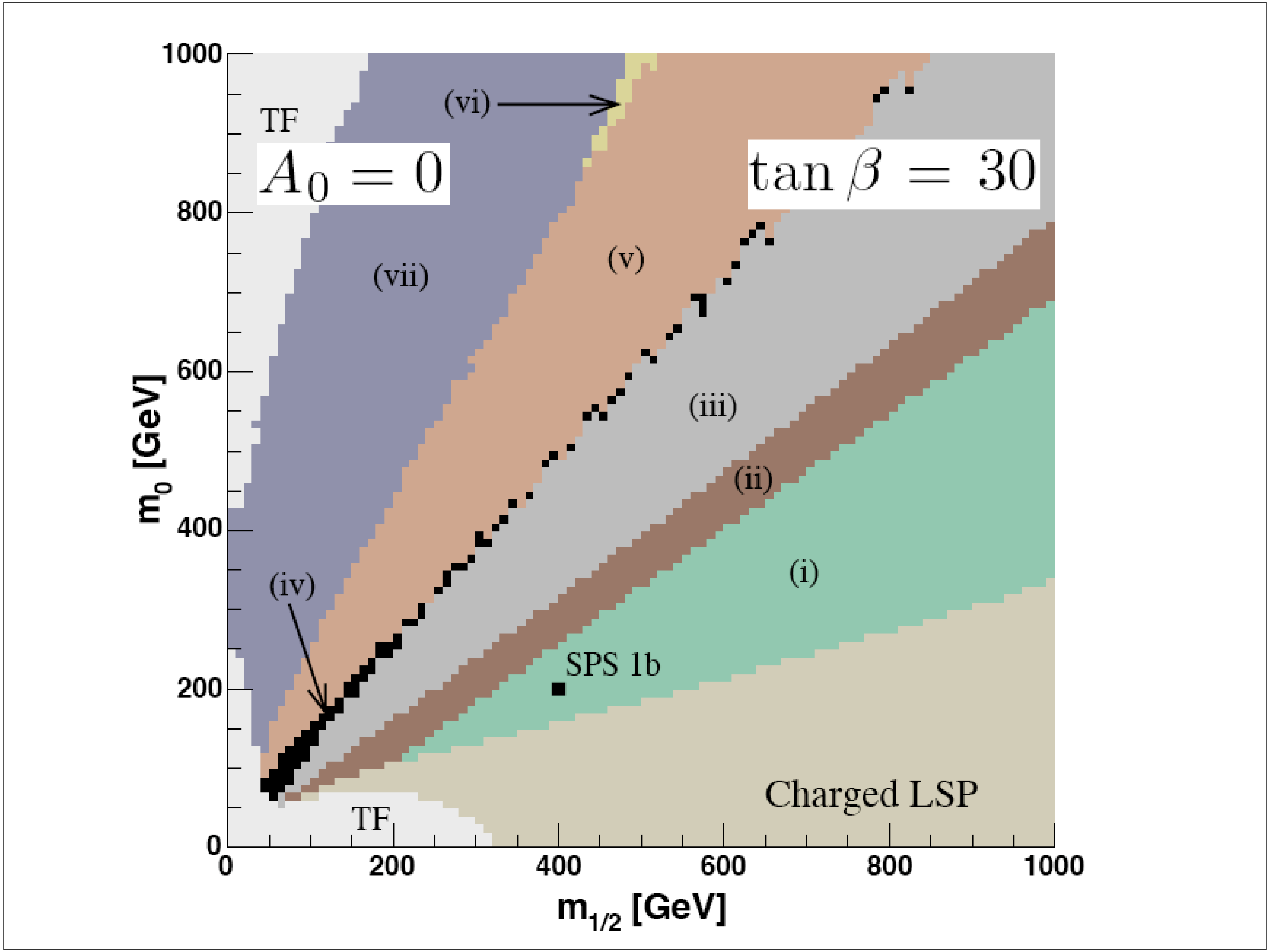


heavy gluino

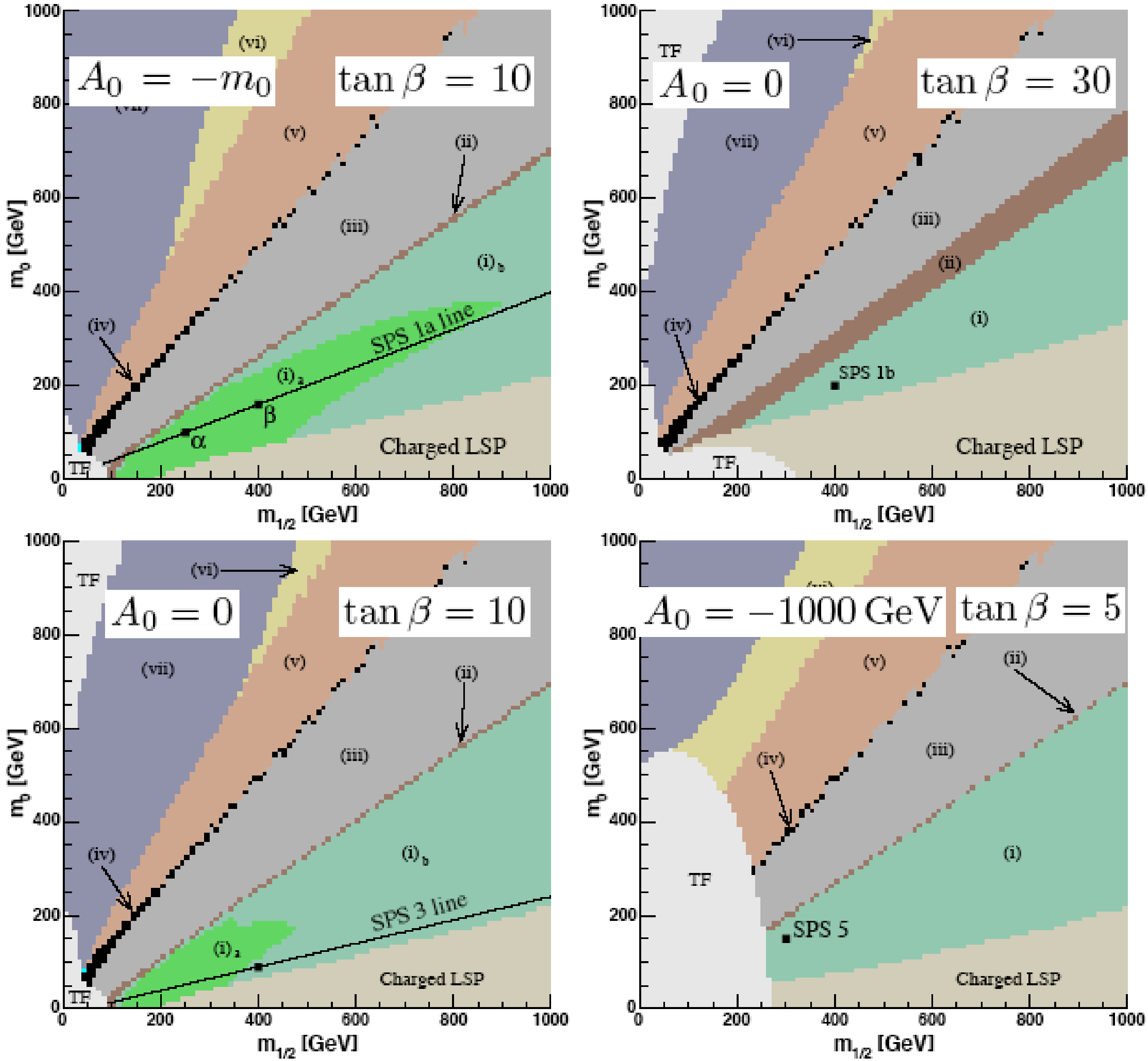
heavy neutralino

(i)	$\tilde{g} > \max(\tilde{d}_L, \tilde{u}_L, \tilde{b}_1, \tilde{t}_1)$	and	$\tilde{\chi}_2^0 > \max(\tilde{l}_R, \tilde{\tau}_1)$
(ii)	$\tilde{g} > \max(\tilde{d}_L, \tilde{u}_L, \tilde{b}_1, \tilde{t}_1)$	and	$\tilde{l}_R > \tilde{\chi}_2^0 > \tilde{\tau}_1$
(iii)	$\tilde{g} > \max(\tilde{d}_L, \tilde{u}_L, \tilde{b}_1, \tilde{t}_1)$	and	$\min(\tilde{l}_R, \tilde{\tau}_1) > \tilde{\chi}_2^0$
(iv)	$\tilde{d}_L > \tilde{g} > \max(\tilde{u}_L, \tilde{b}_1)$	and	$\min(\tilde{l}_R, \tilde{\tau}_1) > \tilde{\chi}_2^0$
(v)	$\min(\tilde{d}_L, \tilde{u}_L) > \tilde{g} > \tilde{b}_1$	and	$\min(\tilde{l}_R, \tilde{\tau}_1) > \tilde{\chi}_2^0$
(vi)	$\min(\tilde{d}_L, \tilde{u}_L, \tilde{b}_1) > \tilde{g} > \tilde{t}_1$	and	$\min(\tilde{l}_R, \tilde{\tau}_1) > \tilde{\chi}_2^0$
(vii)	$\min(\tilde{d}_L, \tilde{u}_L, \tilde{b}_1, \tilde{t}_1) > \tilde{g}$	and	$\min(\tilde{l}_R, \tilde{\tau}_1) > \tilde{\chi}_2^0$

heavy gauginos, $\tilde{g}, \tilde{\chi}_2^0$ lower right



TF
$A_0 = 0$
$\tan\beta = 30$
(vi)
(vii)
(v)
(iii)
(ii)
(i)
(iv)
SPS 1b
Charged LSP
TF
m_0 [GeV]
m_1/2 [GeV]
0
200
400
600
800
1000


$A_0 = -m_0$
$\tan\beta = 10$
SPS 1a line
Charged LSP
TF
$A_0 = 0$
$\tan\beta = 30$
SPS 1b
Charged LSP
TF
$A_0 = 0$
$\tan\beta = 10$
SPS 3 line
Charged LSP
TF
$A_0 = -1000$ GeV
$\tan\beta = 5$
SPS 5
Charged LSP
TF
m_0 [GeV]
$m_{1/2}$ [GeV]

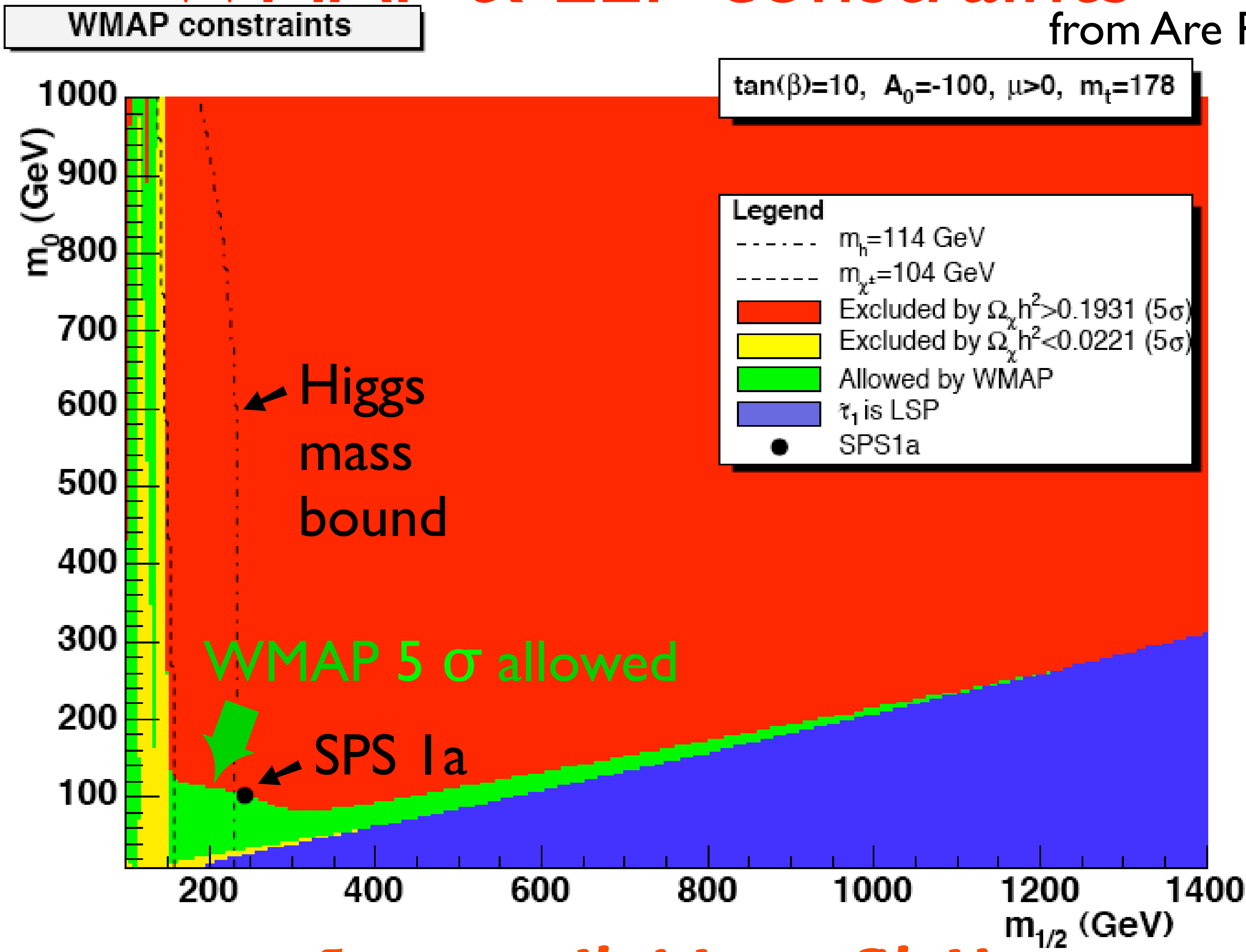
All:

$\mu > 0$

WMAP & LEP constraints

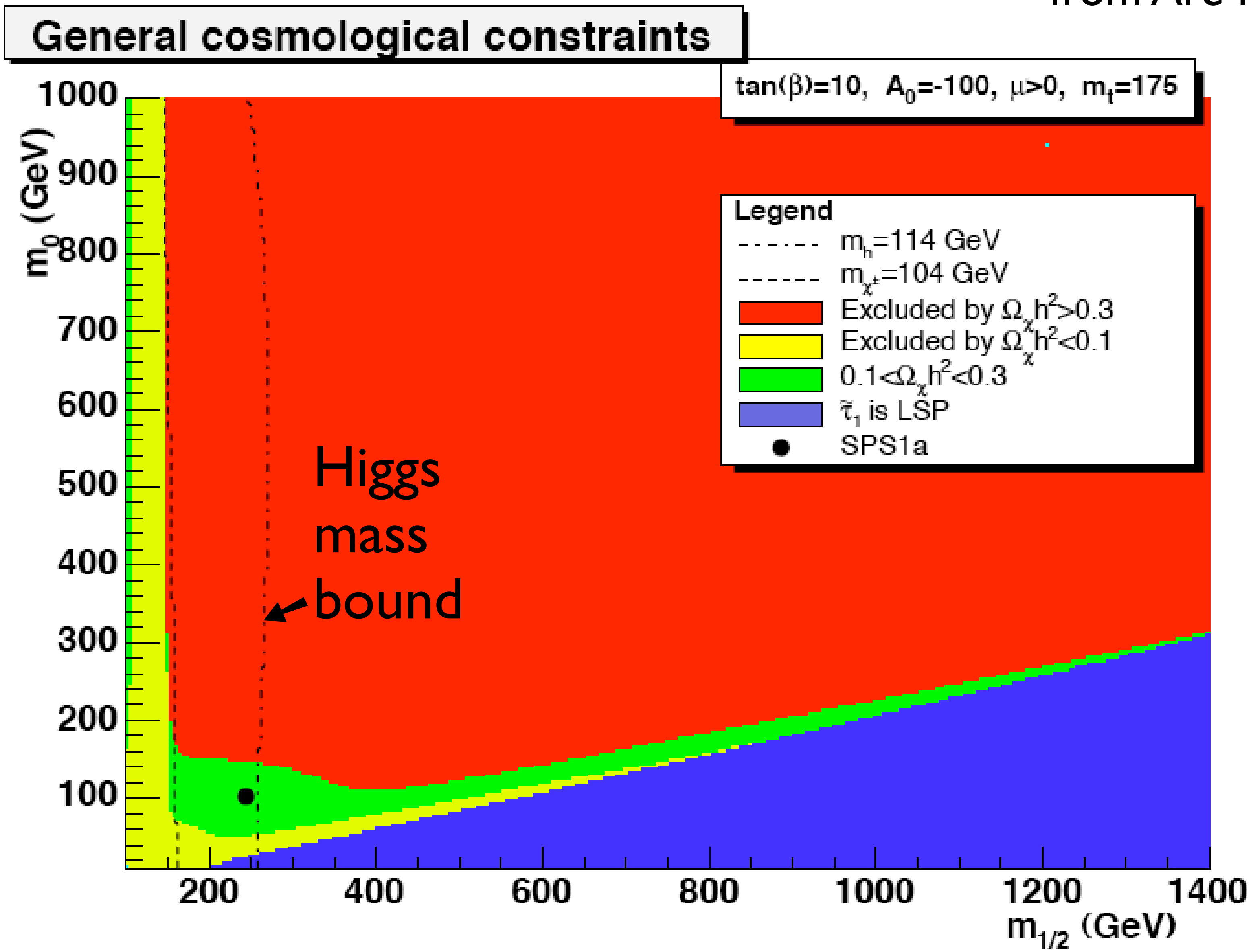
from Are Raklev

DarkSusy
ISASUSY



Ignore slight conflict!

from Are Raklev



Next question:

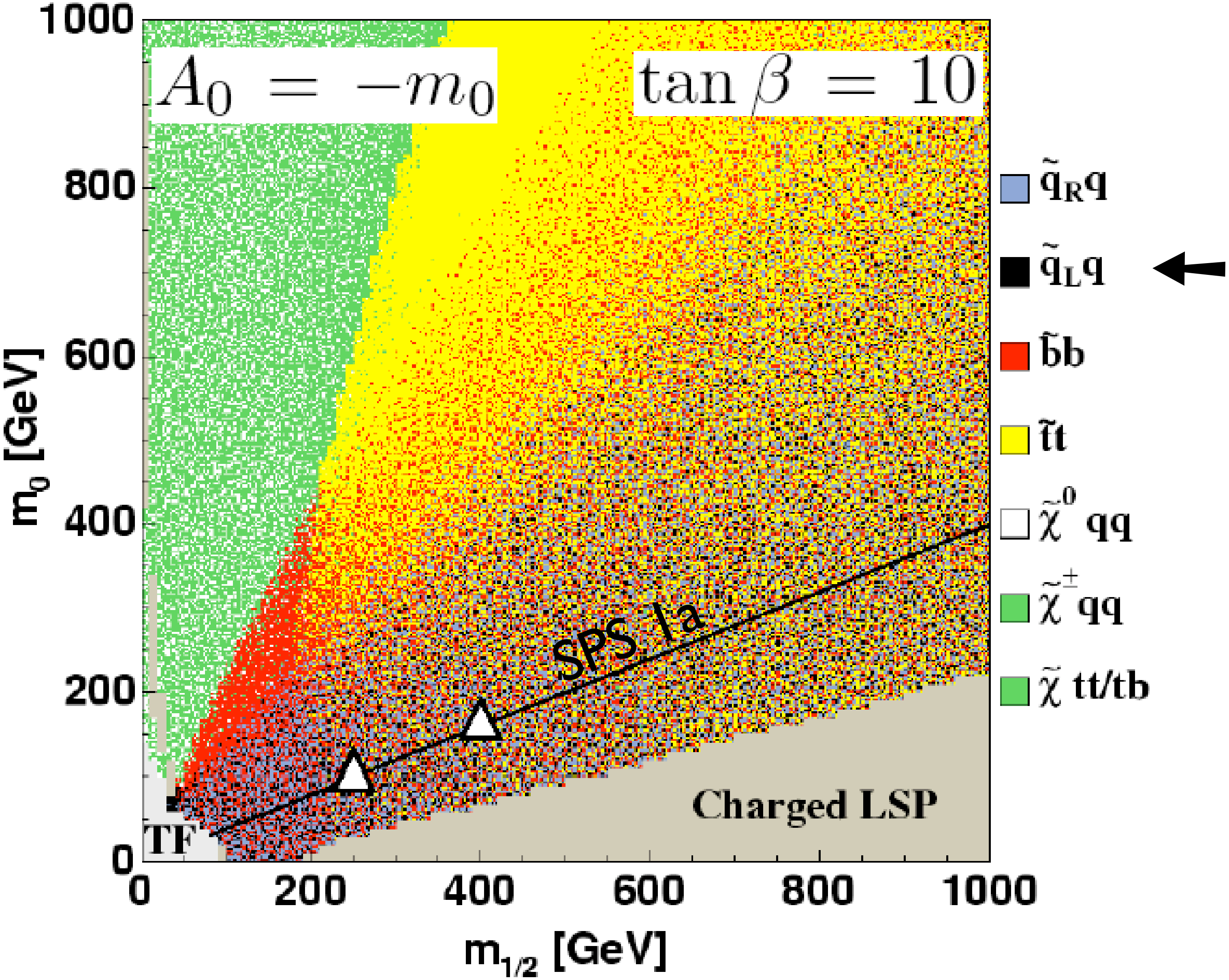
Given “correct” hierarchy,

$$m_{\tilde{g}} > m_{\tilde{q}_L} > m_{\tilde{\chi}_2^0} > m_{\tilde{l}_R} > m_{\tilde{\chi}_1^0}$$

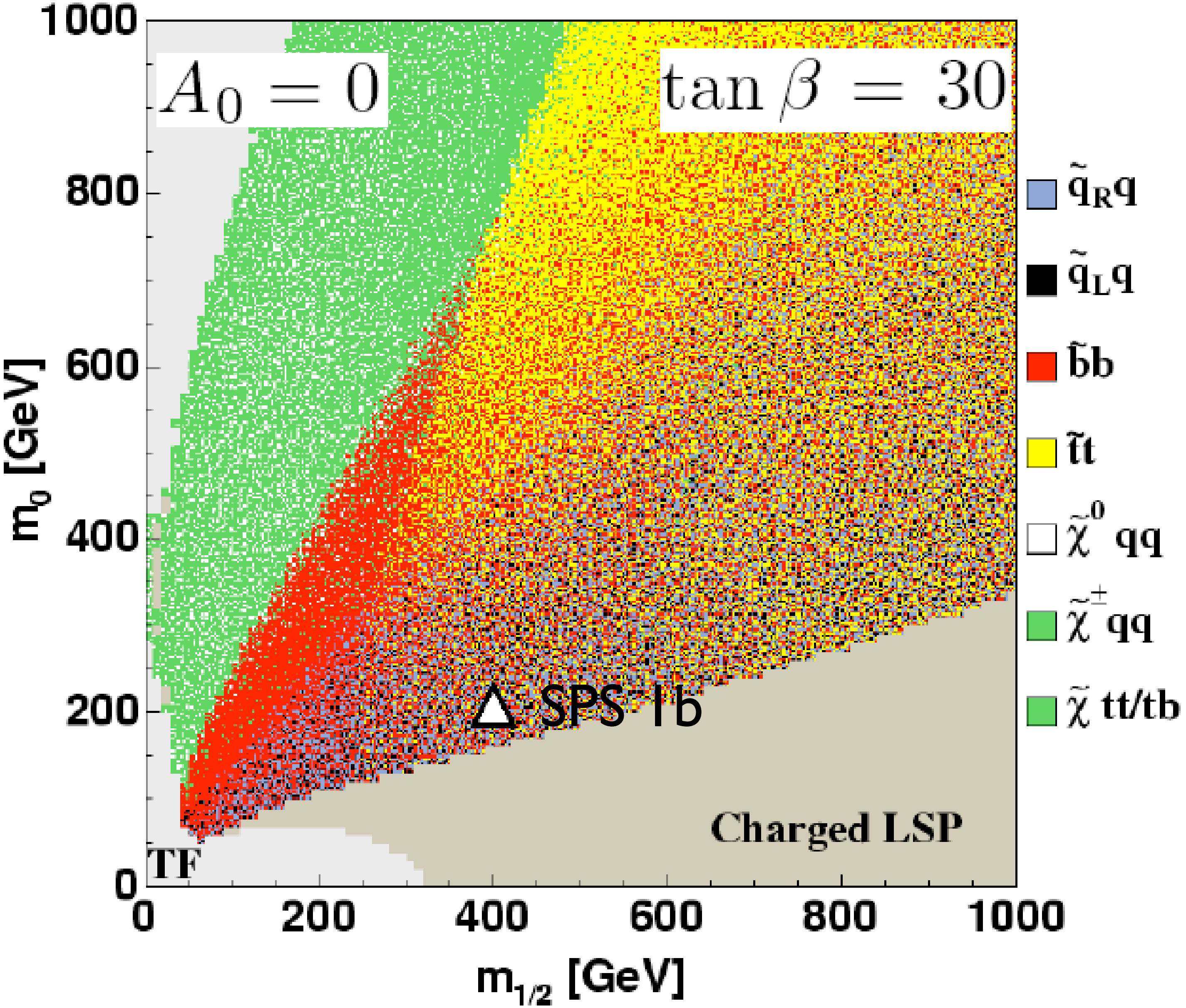
is there enough BR?

- Does the squark have significant BR to neutralino and quark?
- Does the neutralino have significant BR to slepton and lepton?

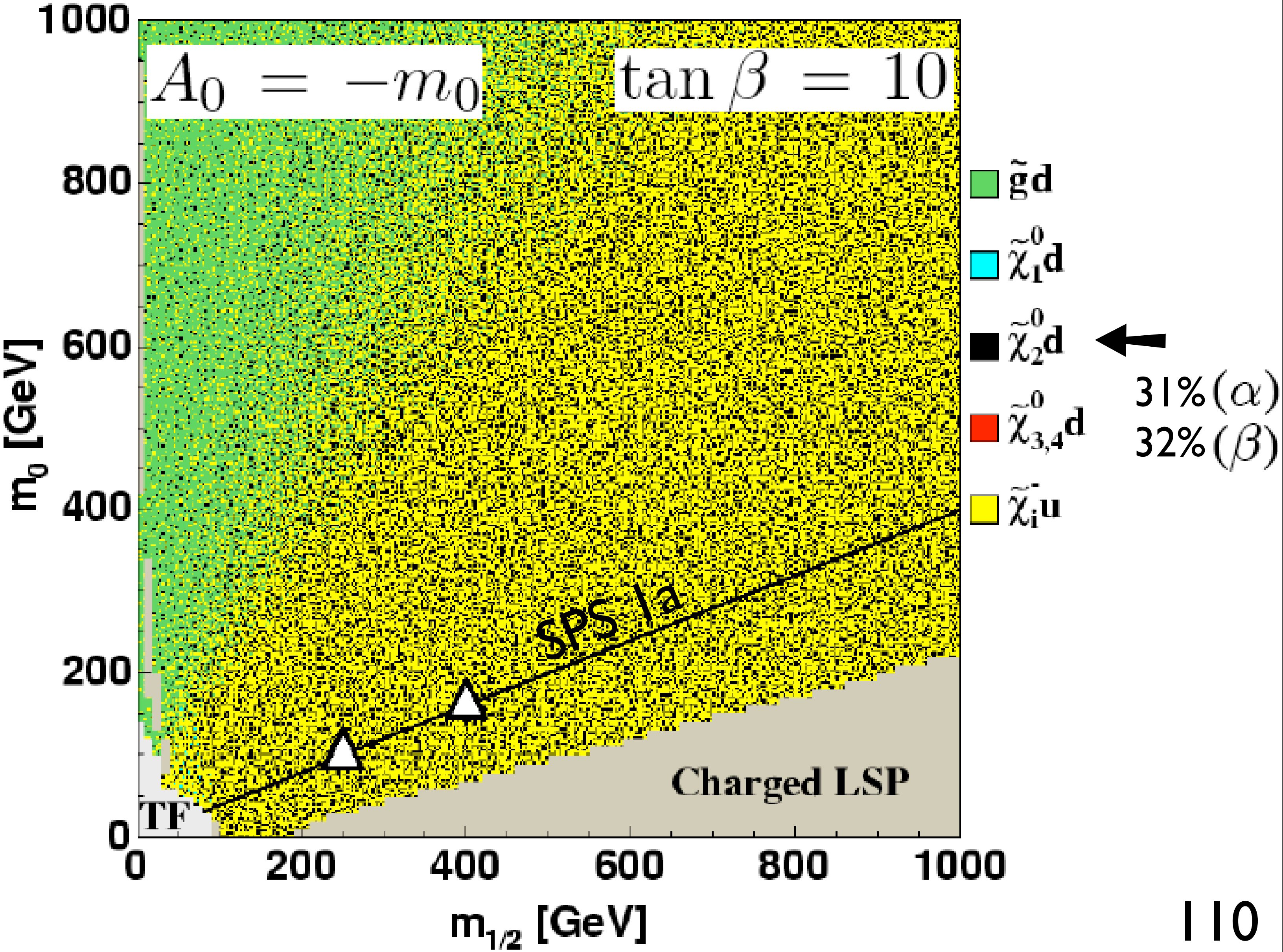
Gluino Branching Ratios ($\tilde{g}$)



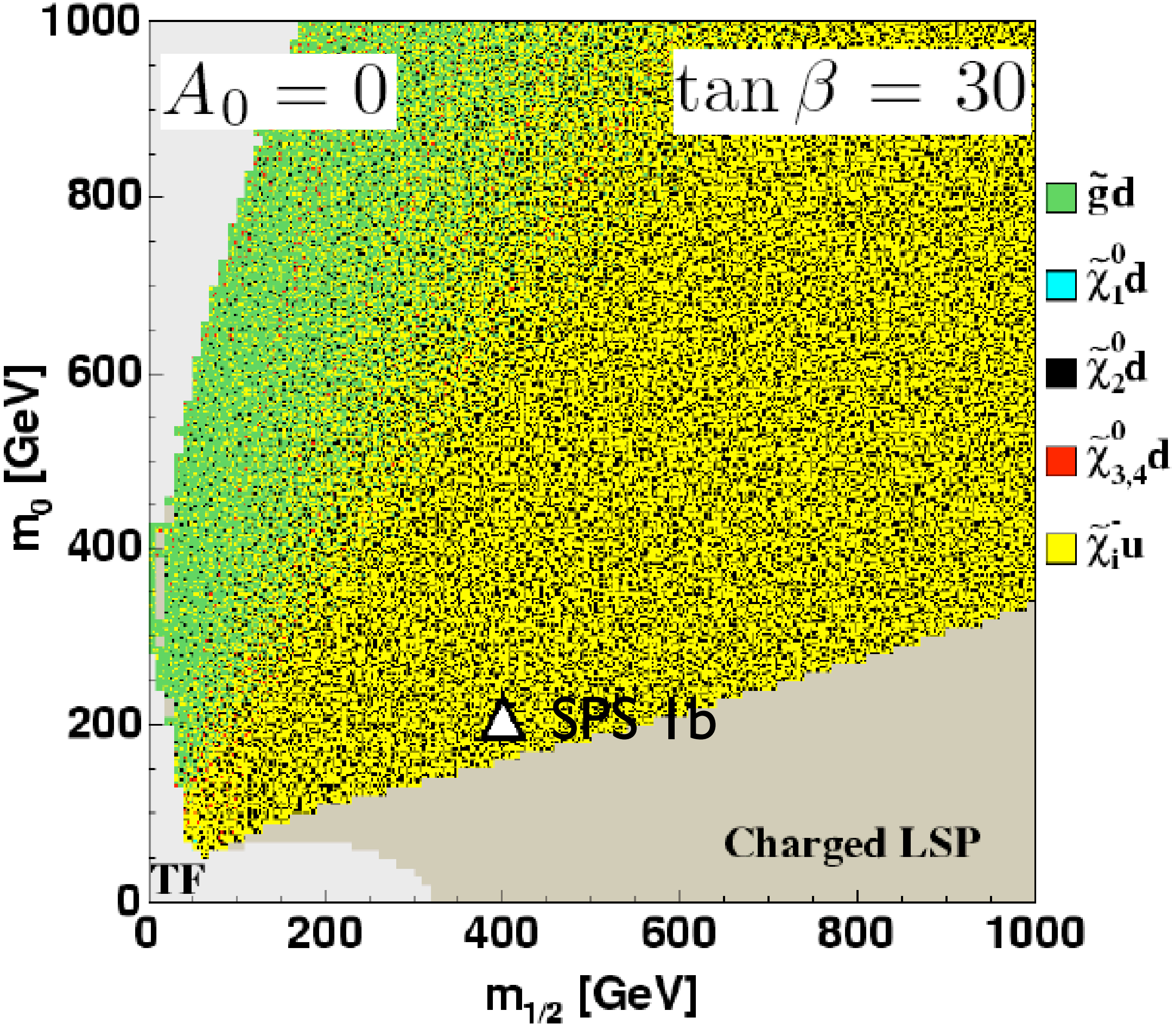
Gluino Branching Ratios ($\tilde{g}$)



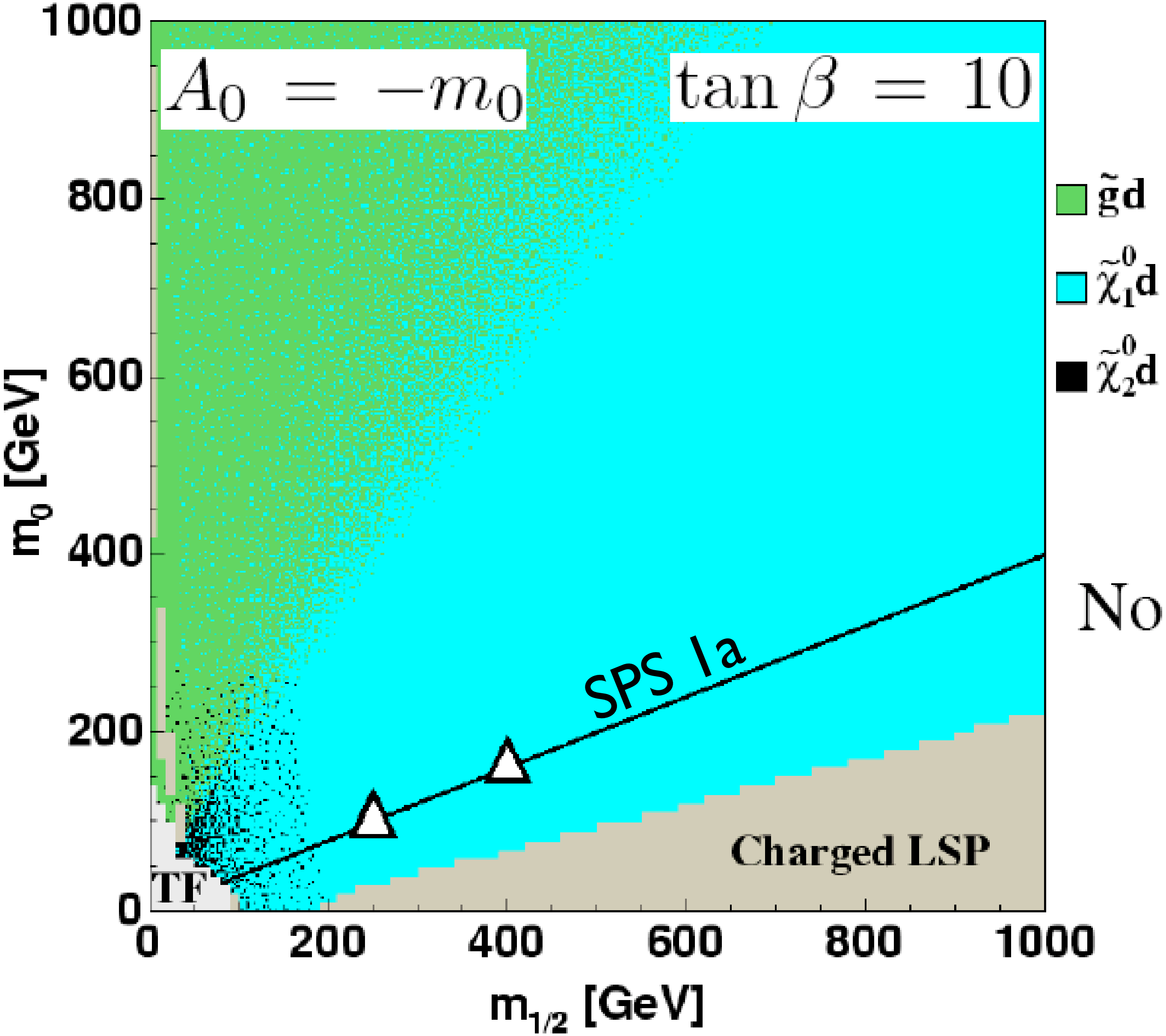
Squark Branching Ratios ($\tilde{u}_L$)



Squark Branching Ratios ($\tilde{u}_L$)

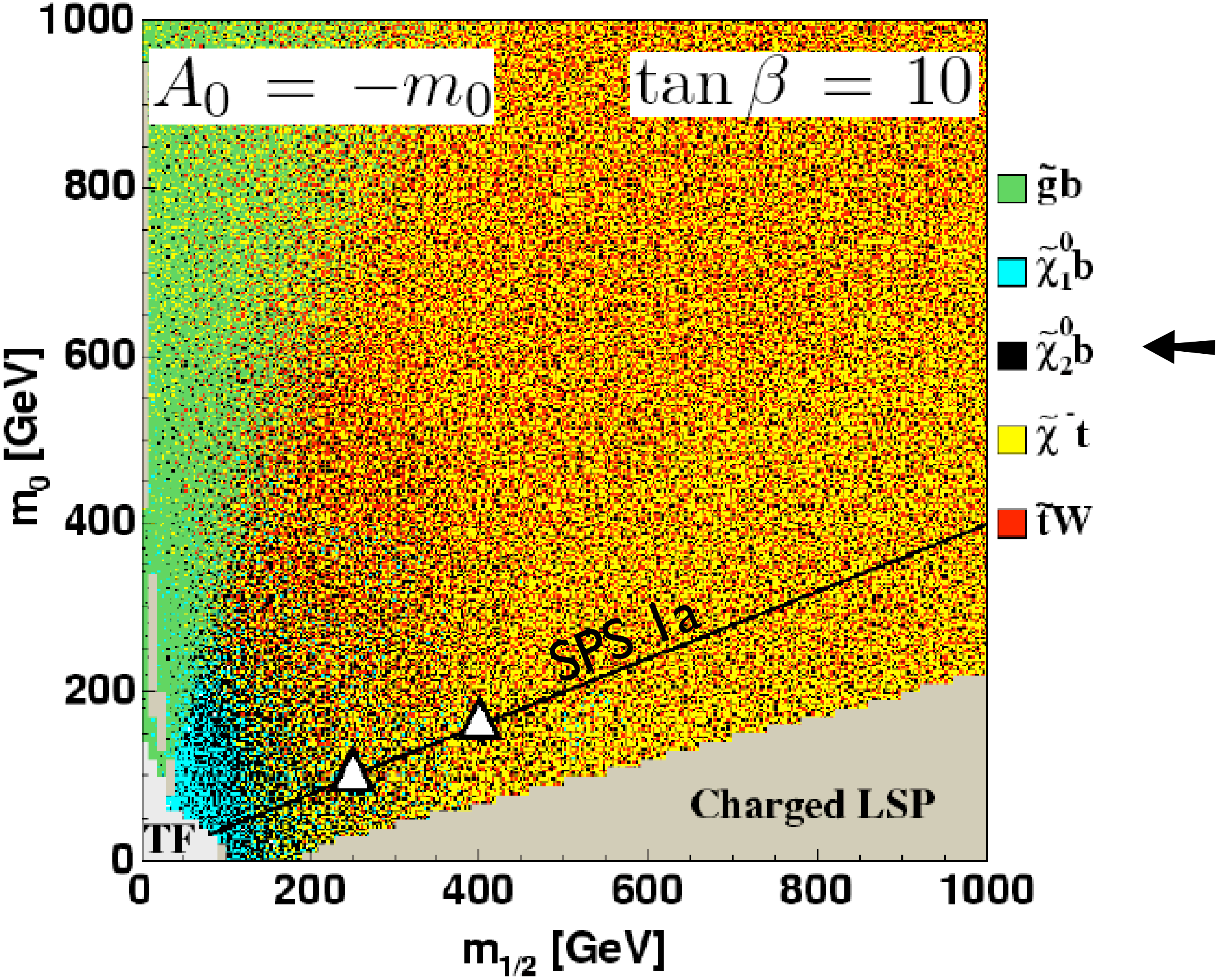


Squark Branching Ratios ($\tilde{u}_R$)

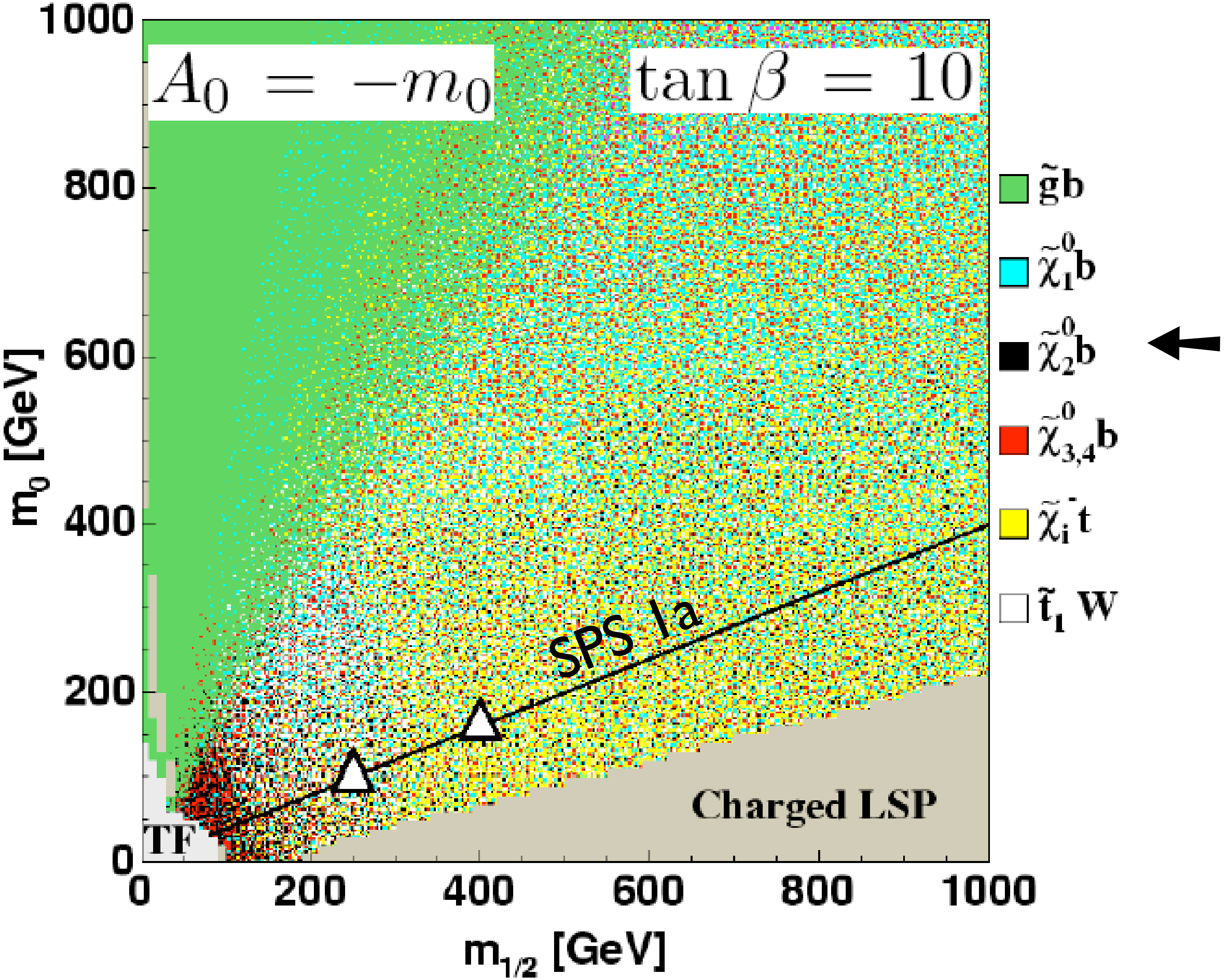


No $\tilde{\chi}_2^0$ rate

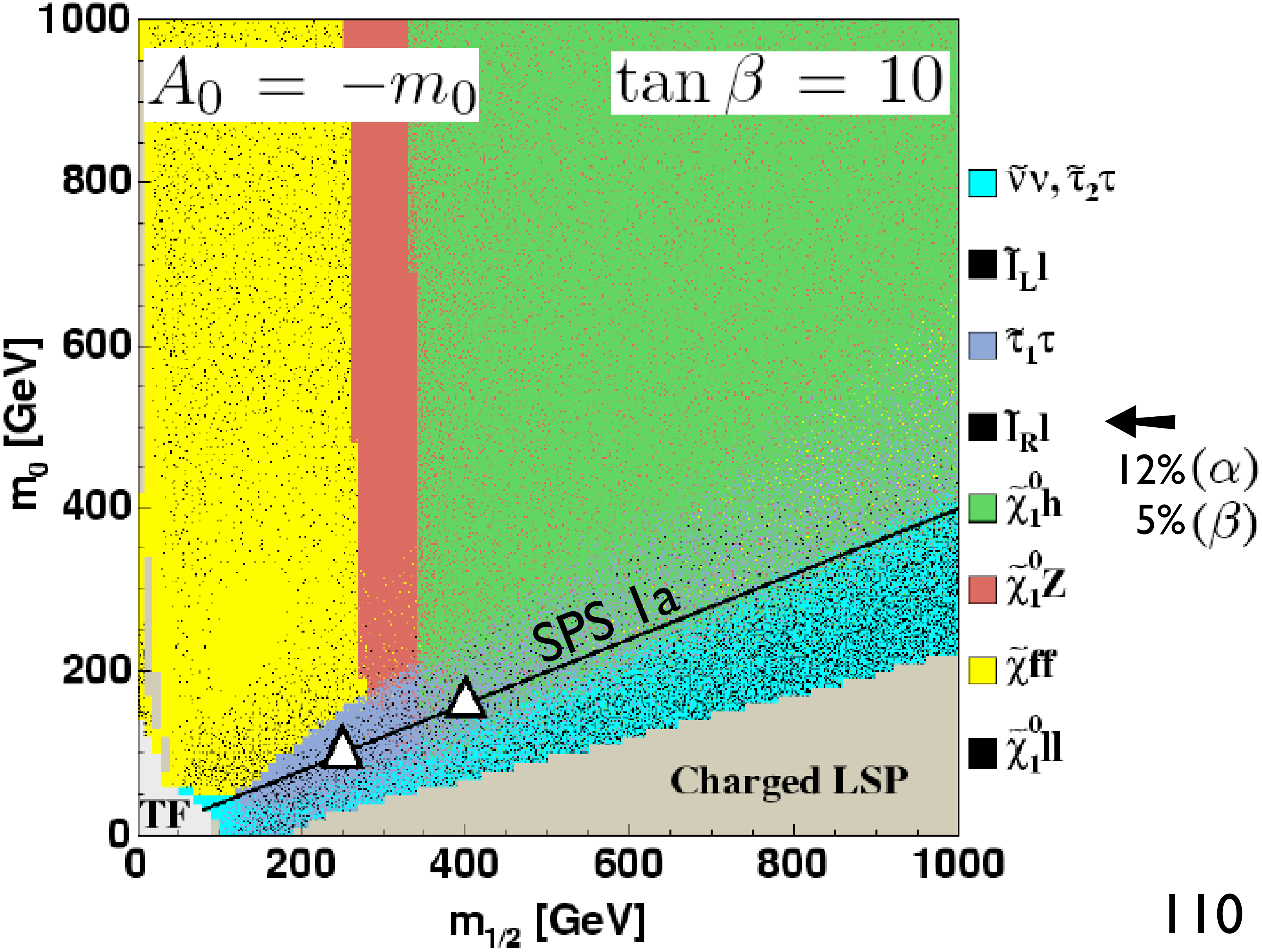
Squark Branching Ratios ($\tilde{b}_1$)



Squark Branching Ratios ($\tilde{b}_2$)



Neutralino Branching Ratios ($\tilde{\chi}_2^0$)



SPS 1a (line)

$$m_0 = -A_0 = 0.4\, m_{1/2}$$

$$\tan\beta = 10\,, \qquad \mu > 0$$

Two particular points on the line:

$$(\alpha): \quad m_0 = 100 \text{ GeV}\,, \qquad m_{1/2} = 250 \text{ GeV}$$

$$(\beta): \quad m_0 = 160 \text{ GeV}\,, \qquad m_{1/2} = 400 \text{ GeV}$$

Spectrum

Point	$\tilde{g}$	$\tilde{d}_L$	$\tilde{d}_R$	$\tilde{u}_L$	$\tilde{u}_R$	$\tilde{b}_2$	$\tilde{b}_1$	$\tilde{t}_2$	$\tilde{t}_1$
(α)	**595.2**	**543.0**	520.1	**537.2**	520.5	**524.6**	**491.9**	574.6	379.1
(β)	**915.5**	**830.1**	799.5	**826.3**	797.3	**800.2**	**759.4**	823.8	610.4

	$\tilde{e}_L$	$\tilde{e}_R$	$\tilde{\tau}_2$	$\tilde{\tau}_1$	$\tilde{\nu}_{e_L}$	$\tilde{\nu}_{\tau_L}$		$H^{\pm}$	A
(α)	202.1	**143.0**	206.0	133.4	185.1	185.1		401.8	393.6
(β)	315.6	**221.9**	317.3	213.4	304.1	304.1		613.9	608.3

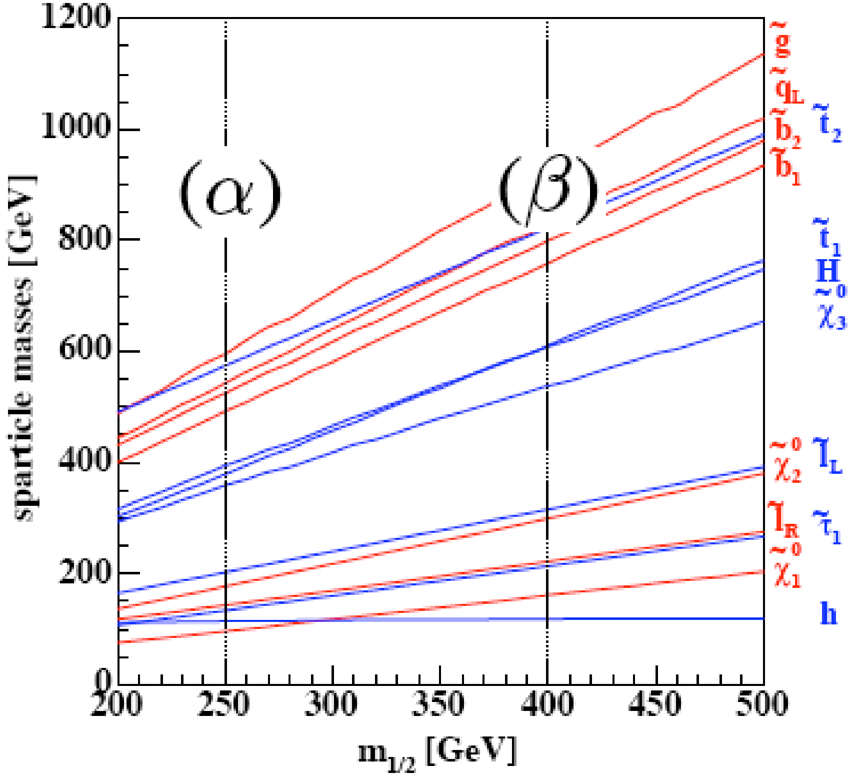
	$\tilde{\chi}_4^0$	$\tilde{\chi}_3^0$	$\tilde{\chi}_2^0$	$\tilde{\chi}_1^0$	$\tilde{\chi}_2^{\pm}$	$\tilde{\chi}_1^{\pm}$		H	h
(α)	377.8	358.8	**176.8**	**96.1**	378.2	176.4		394.2	114.0
(β)	553.3	538.4	**299.1**	**161.0**	553.3	299.0		608.9	117.9

as determined by ISASUSY 7.58 by integrating RGE's

in **bold**: particles used in study

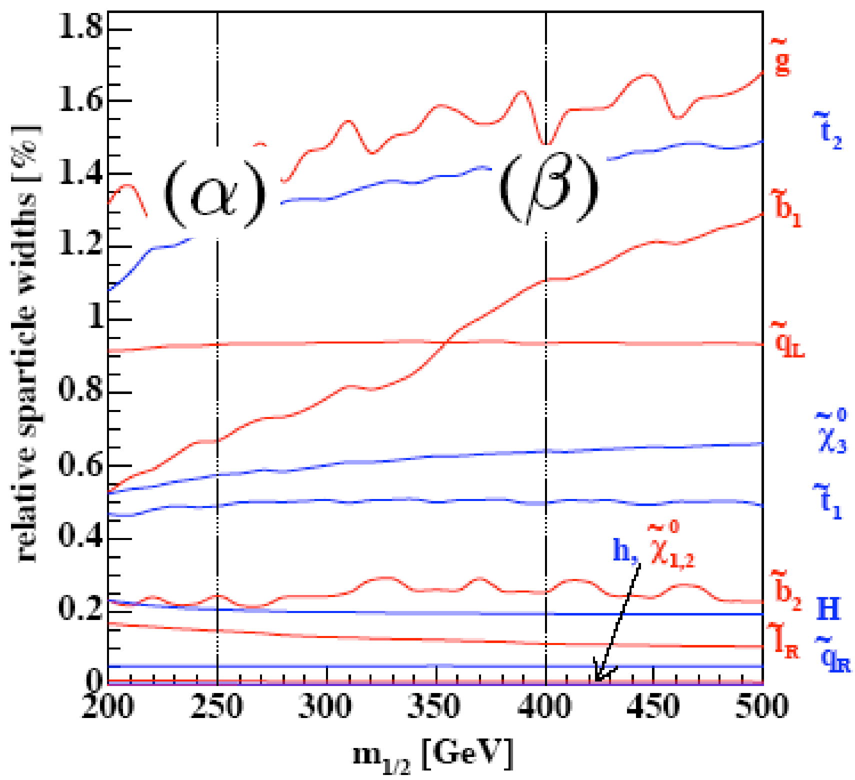
SPS 1a line

Masses



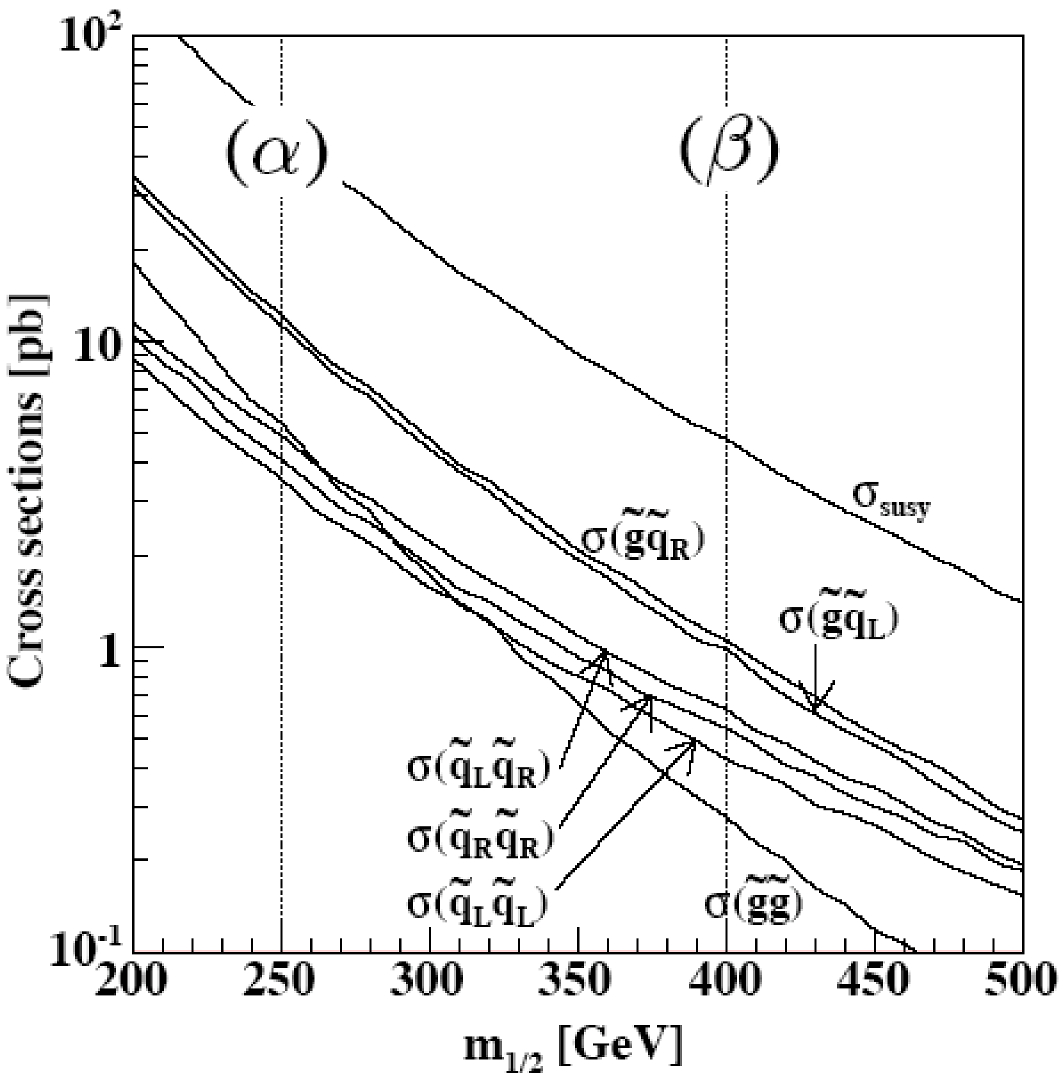
Widths

ISASUSY 7.58



~1% of mass

Cross sections:

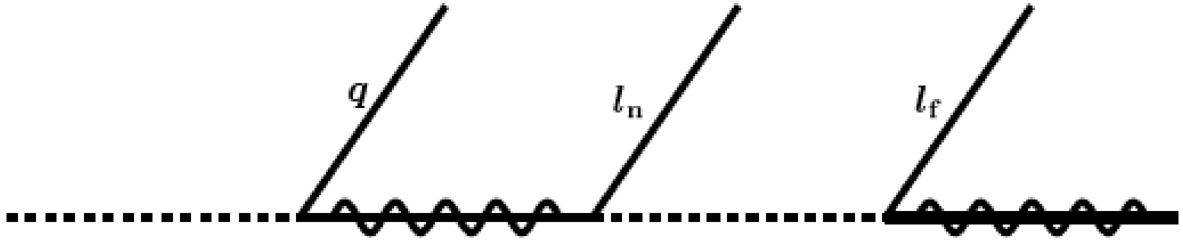


squark is often produced together with gluino, or from gluino decay

[pb]

	$\sigma(\mathrm{SUSY})$	$\sigma(\tilde{g}\tilde{g})$	$\sigma(\tilde{g}\tilde{q}_L)$	$\sigma(\tilde{g}\tilde{q}_R)$	$\sigma(\tilde{q}_L\tilde{q}_L)$	$\sigma(\tilde{q}_L\tilde{q}_R)$	$\sigma(\tilde{q}_R\tilde{q}_R)$
(α)	49.3	5.3	11.4	12.3	3.5	4.8	4.1
(β)	4.76	0.29	0.97	1.06	0.44	0.61	0.53

Quantifying the cascade:



	$\sigma = 32.8$ pb	$\tilde{q}_L$	31.4%						1245 fb
(α)	$\sigma = 7.7$ pb	$\tilde{b}_1$	35.5%	$\tilde{\chi}_2^0$	12.1%	$\tilde{l}_R$	100%	$\tilde{\chi}_1^0$	329 fb
	$\sigma = 4.3$ pb	$\tilde{b}_2$	18.0%						94 fb
									1669 fb
	$\sigma = 3.21$ pb	$\tilde{q}_L$	31.9%						54.3 fb
(β)	$\sigma = 0.50$ pb	$\tilde{b}_1$	23.6%	$\tilde{\chi}_2^0$	5.3%	$\tilde{l}_R$	100%	$\tilde{\chi}_1^0$	6.3 fb
	$\sigma = 0.31$ pb	$\tilde{b}_2$	8.8%						1.5 fb
									62.0 fb

Reduction (rate and BR) from (α) to (β)

Kinematics - lepton pair (simple example):

$$\tilde{\chi}_2^0 \to \tilde{l}_R l_{\rm n}$$

"n" = "near"

$$m^2_{\tilde{\chi}_2^0} = (p_{\tilde{l}_R} + p_{l_{\rm n}})^2 = m^2_{\tilde{l}_R} + 2p_{l_{\rm n}} \cdot p_{\tilde{l}_R}$$

Rest frame of $\tilde{l}_R$:

$$|\mathbf{p}_{l_{\rm n}}| = \frac{m^2_{\tilde{\chi}_2^0} - m^2_{\tilde{l}_R}}{2m_{\tilde{l}_R}}$$

$$\tilde{l}_R \to \tilde{\chi}_1^0 l_{\rm f}$$

"f" = "far"

$$m^2_{\tilde{\chi}_1^0} = (p_{\tilde{l}_R} - p_{l_{\rm f}})^2 = m^2_{\tilde{l}_R} - 2p_{l_{\rm f}} \cdot p_{\tilde{l}_R}$$

Rest frame of $\tilde{l}_R$:

$$|\mathbf{p}_{l_{\rm f}}| = \frac{m^2_{\tilde{l}_R} - m^2_{\tilde{\chi}_1^0}}{2m_{\tilde{l}_R}}$$

Maximum di-lepton mass:

Back-to-back in $\tilde{l}_R$ Rest Frame:

$$(m_{ll}^{\max})^2 = 4|\mathbf{p}_{l_\mathrm{n}}||\mathbf{p}_{l_\mathrm{f}}| = \frac{(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}_R}^2)(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{l}_R}^2}$$

Prototype of "endpoint formulas"

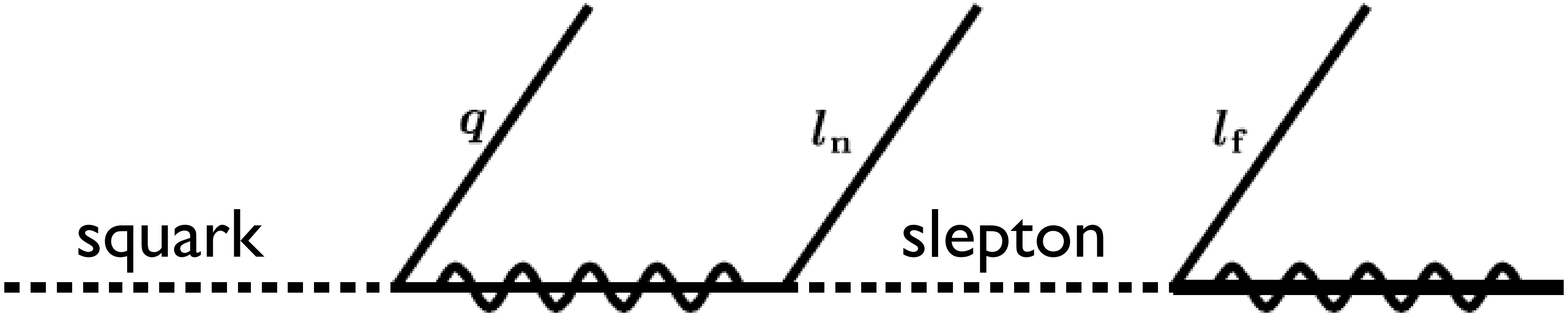
One kinematical endpt is related to various (3) masses of unstable particles:

$$m_{\tilde{\chi}_2^0} \quad m_{\tilde{l}_R} \quad m_{\tilde{\chi}_1^0}$$

Need more such formulas!

Add the squark:

$$\tilde{q}_L \to \tilde{\chi}_2^0\, q \to \tilde{l}_R\, q\, l_{\rm n} \to \tilde{\chi}_1^0\, q\, l_{\rm n}\, l_{\rm f}$$



More invariants and endpoints:

$$m_{qll} \quad m_{ql_{\rm n}} \quad m_{ql_{\rm f}} \quad m_{ll}$$

Four endpoints and *four* masses:

$$m_{\tilde{q}_L} \quad m_{\tilde{\chi}_2^0} \quad m_{\tilde{l}_R} \quad m_{\tilde{\chi}_1^0}$$

Complication 1:

The two leptons can not be distinguished

For each event, form

$$m_{ql(\text{low})} < m_{ql(\text{high})}$$ well defined

Complication 2:

For some invariants, there are multiple cases:
endpoint formula depends on mass ratios

Complication 3:

Multiple squark masses; widths

Complication 4:

Endpoints not always linearly independent

B.C. Allanach et al, hep-ph/0007009 (conditions rephrased):

$$(m_{ll}^{\max})^2 = \frac{\left(m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{l}_R}\right)\left(m^2_{\tilde{l}_R} - m^2_{\tilde{\chi}^0_1}\right)}{m^2_{\tilde{l}_R}}$$

one case

mass ratios of adjacent sparticles in chain

$$(m_{qll}^{\max})^2 = \begin{cases} \frac{\left(m^2_{\tilde{q}_L} - m^2_{\tilde{\chi}^0_2}\right)\left(m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{\chi}^0_1}\right)}{m^2_{\tilde{\chi}^0_2}} & \text{for} \quad \frac{m_{\tilde{q}_L}}{m_{\tilde{\chi}^0_2}} > \frac{m_{\tilde{\chi}^0_2}}{m_{\tilde{l}_R}} \frac{m_{\tilde{l}_R}}{m_{\tilde{\chi}^0_1}} \quad (1) \\ \frac{\left(m^2_{\tilde{q}_L} m^2_{\tilde{l}_R} - m^2_{\tilde{\chi}^0_2} m^2_{\tilde{\chi}^0_1}\right)\left(m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{l}_R}\right)}{m^2_{\tilde{\chi}^0_2} m^2_{\tilde{l}_R}} & \text{for} \quad \frac{m_{\tilde{\chi}^0_2}}{m_{\tilde{l}_R}} > \frac{m_{\tilde{l}_R}}{m_{\tilde{\chi}^0_1}} \frac{m_{\tilde{q}_L}}{m_{\tilde{\chi}^0_2}} \quad (2) \\ \frac{\left(m^2_{\tilde{q}_L} - m^2_{\tilde{l}_R}\right)\left(m^2_{\tilde{l}_R} - m^2_{\tilde{\chi}^0_1}\right)}{m^2_{\tilde{l}_R}} & \text{for} \quad \frac{m_{\tilde{l}_R}}{m_{\tilde{\chi}^0_1}} > \frac{m_{\tilde{q}_L}}{m_{\tilde{\chi}^0_2}} \frac{m_{\tilde{\chi}^0_2}}{m_{\tilde{l}_R}} \quad (3) \\ \left(m_{\tilde{q}_L} - m_{\tilde{\chi}^0_1}\right)^2 & \text{otherwise} \quad (4) \end{cases}$$

four cases

$$\left(m_{ql(\text{low})}^{\max}, m_{ql(\text{high})}^{\max}\right) = \begin{cases} \left(m_{ql_n}^{\max}, m_{ql_f}^{\max}\right) & \text{for} \quad 2m^2_{\tilde{l}_R} > m^2_{\tilde{\chi}^0_1} + m^2_{\tilde{\chi}^0_2} > 2m_{\tilde{\chi}^0_1} m_{\tilde{\chi}^0_2} \quad (1) \\ \left(m_{ql(\text{eq})}^{\max}, m_{ql_f}^{\max}\right) & \text{for} \quad m^2_{\tilde{\chi}^0_1} + m^2_{\tilde{\chi}^0_2} > 2m^2_{\tilde{l}_R} > 2m_{\tilde{\chi}^0_1} m_{\tilde{\chi}^0_2} \quad (2) \\ \left(m_{ql(\text{eq})}^{\max}, m_{ql_n}^{\max}\right) & \text{for} \quad m^2_{\tilde{\chi}^0_1} + m^2_{\tilde{\chi}^0_2} > 2m_{\tilde{\chi}^0_1} m_{\tilde{\chi}^0_2} > 2m^2_{\tilde{l}_R} \quad (3) \end{cases}$$

three cases

where

$$(m_{ql_n}^{\max})^2 = \frac{(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}_R}^2)}{m_{\tilde{\chi}_2^0}^2}$$

$$(m_{ql_f}^{\max})^2 = \frac{(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{l}_R}^2}$$

$$(m_{ql(\mathrm{eq})}^{\max})^2 = \frac{(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2)}{(2m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2)}$$

Finally:

$$(m_{qll(\theta>\frac{\pi}{2})}^{\min})^2 = \Big[(m_{\tilde{q}_L}^2 + m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}_R}^2)(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2)$$
$$- (m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)\sqrt{(m_{\tilde{\chi}_2^0}^2 + m_{\tilde{l}_R}^2)^2(m_{\tilde{l}_R}^2 + m_{\tilde{\chi}_1^0}^2)^2 - 16 m_{\tilde{\chi}_2^0}^2 m_{\tilde{l}_R}^4 m_{\tilde{\chi}_1^0}^2} +$$
$$+ 2m_{\tilde{l}_R}^2(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{\chi}_1^0}^2)\Big](4m_{\tilde{l}_R}^2 m_{\tilde{\chi}_2^0}^2)^{-1}$$

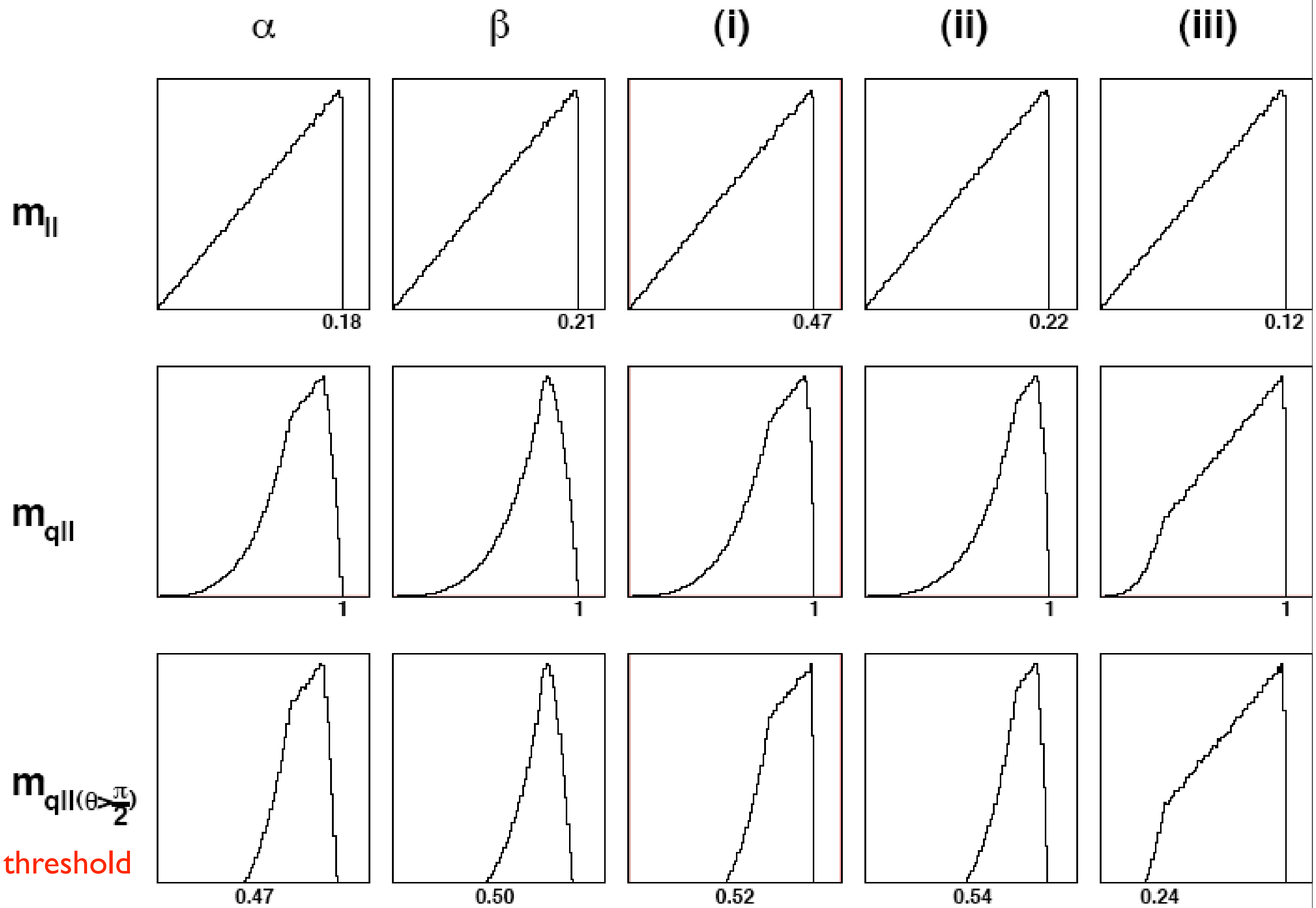
one case

θ is opening angle between leptons in $\tilde{l}_R$ rest frame

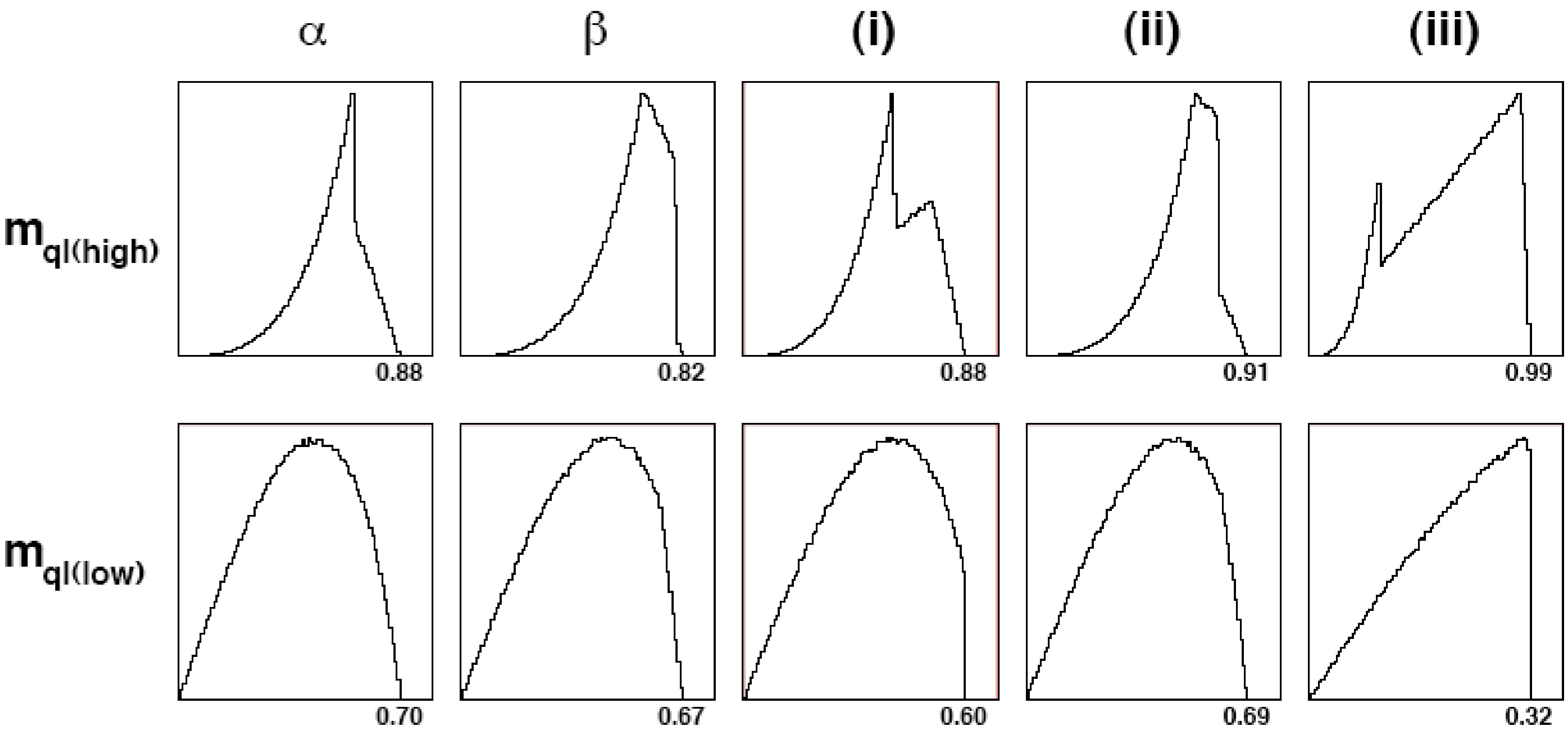
Over-all: 4×3 cases, denoted (1,1), (1,2), etc.

(9 of 12 are realized)

Theory curves (phase space)



Theory curves (phase space) cont.



α and β are on the SPS 1a line, other points "random"

Conclusions:
Many different shapes!
Edges not linear!

values in units of $m_{qll}^{\max}$

Inverting endpoint formulas

Endpoint formulas can be inverted

Complications:

- nonlinear (rational, sqrt)
- several cases (mass regions)

$m_{qll}^{\max}$ four cases

$m_{ql(\text{low})}^{\max}, m_{ql(\text{high})}^{\max}$ three cases

Example:

Region (1,1):

$m_{qll}^{\max}$

$m_{ql(\text{low})}^{\max}, m_{ql(\text{high})}^{\max}$

$$m^2_{\tilde\chi^0_1} = \frac{(b^2-d^2)(b^2-c^2)}{(c^2+d^2-b^2)^2}a^2$$

$$m^2_{\tilde l_R} = \frac{c^2(b^2-c^2)}{(c^2+d^2-b^2)^2}a^2$$

$$m^2_{\tilde\chi^0_2} = \frac{c^2d^2}{(c^2+d^2-b^2)^2}a^2$$

$$m^2_{\tilde q_L} = \frac{c^2d^2}{(c^2+d^2-b^2)^2}(c^2+d^2-b^2+a^2)$$

$$a = m_{ll}^{\max}, \qquad b = m_{qll}^{\max}, \qquad c = m_{ql(\text{low})}^{\max}, \qquad d = m_{ql(\text{high})}^{\max}.$$

Inverting endpt formulas, cont

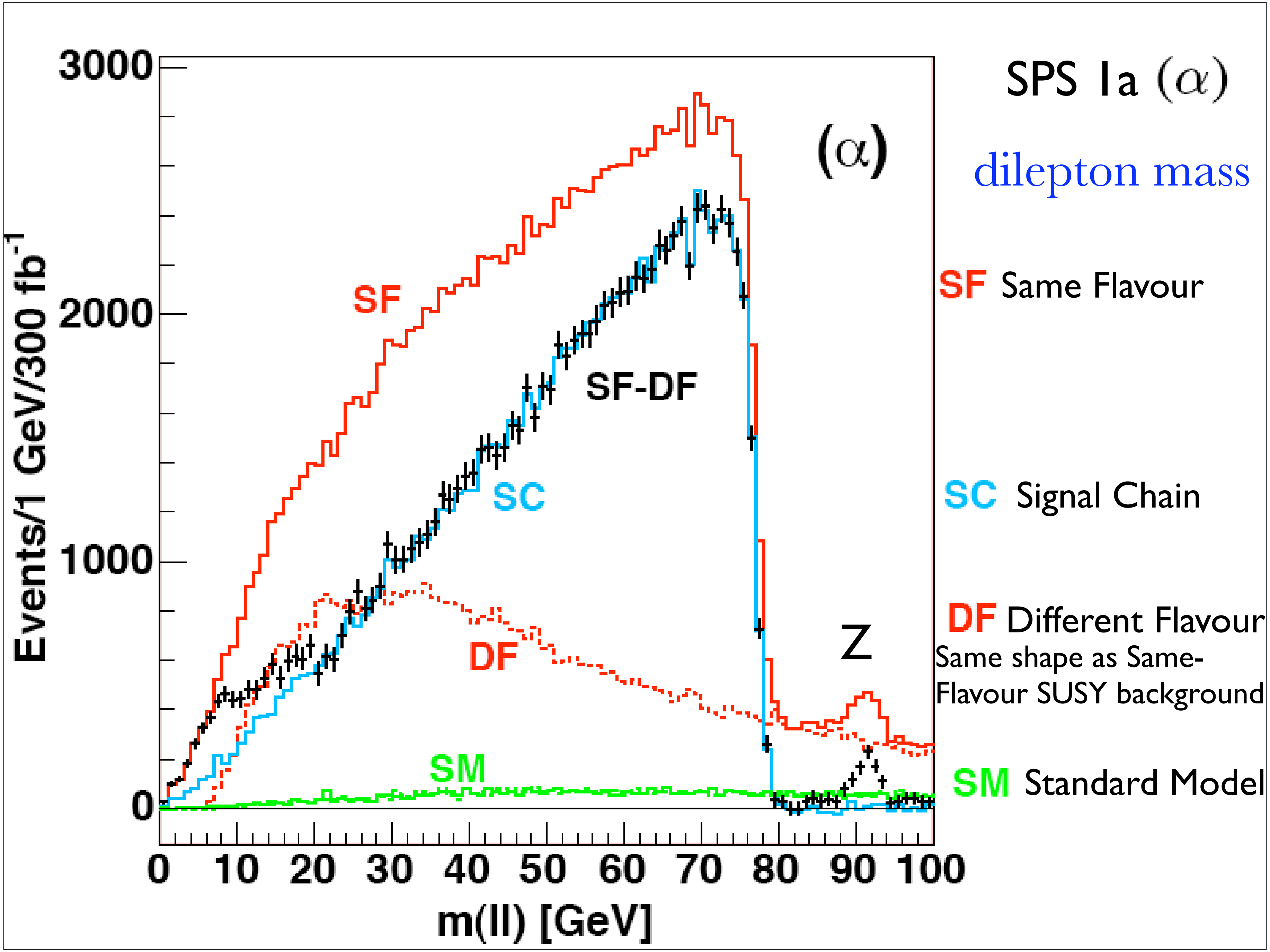
- If 4 endpts & 4 masses, (if linear) unique solution
- May have more endpts, system overconstrained
- Endpts have (different) uncertainties
- Use inversion formulas for start point of fit
- Composite formulas: multiple solutions!

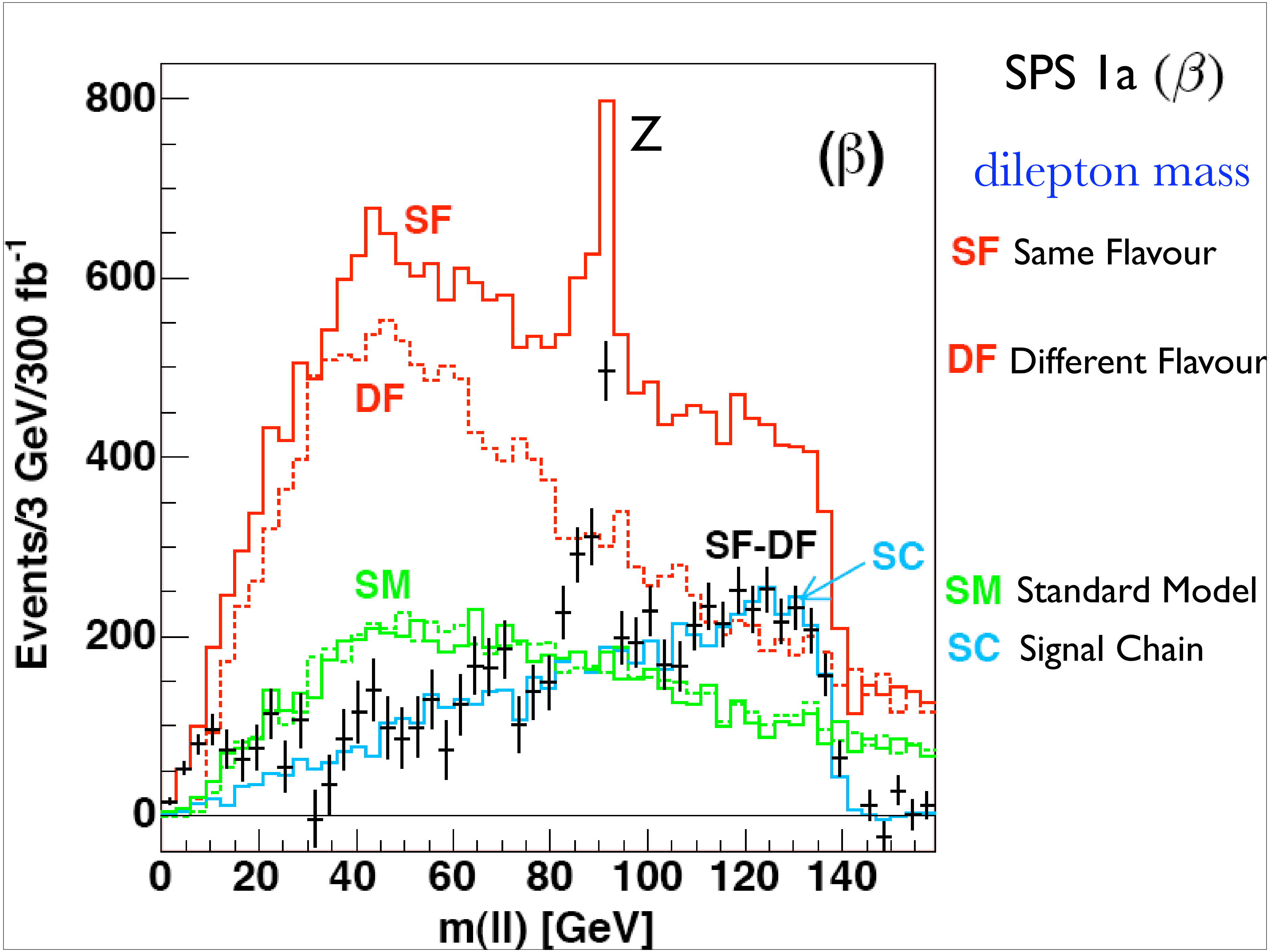
LHC simulation

- ISAJET 7.58 defines low-energy model
- PYTHIA 6.2 with CTEQ 5L: Monte Carlo sample
- ATLFAST 2.60 simulates ATLAS detector
- precuts:
  - At least three jets, satisfying: $p_T^{\rm jet} > 150, 100, 50$ GeV
  - $E_{T,\rm miss} > \max(100 \text{ GeV}, 0.2 M_{\rm eff})$
    with $M_{\rm eff} \equiv E_{T,\rm miss} + \sum_{i=1}^{3} p_{T,i}^{\rm jet}$
  - Two isolated opposite-sign same-flavour leptons (e or μ),
    satisfying $p_T^{\rm lep} > 20, 10$ GeV

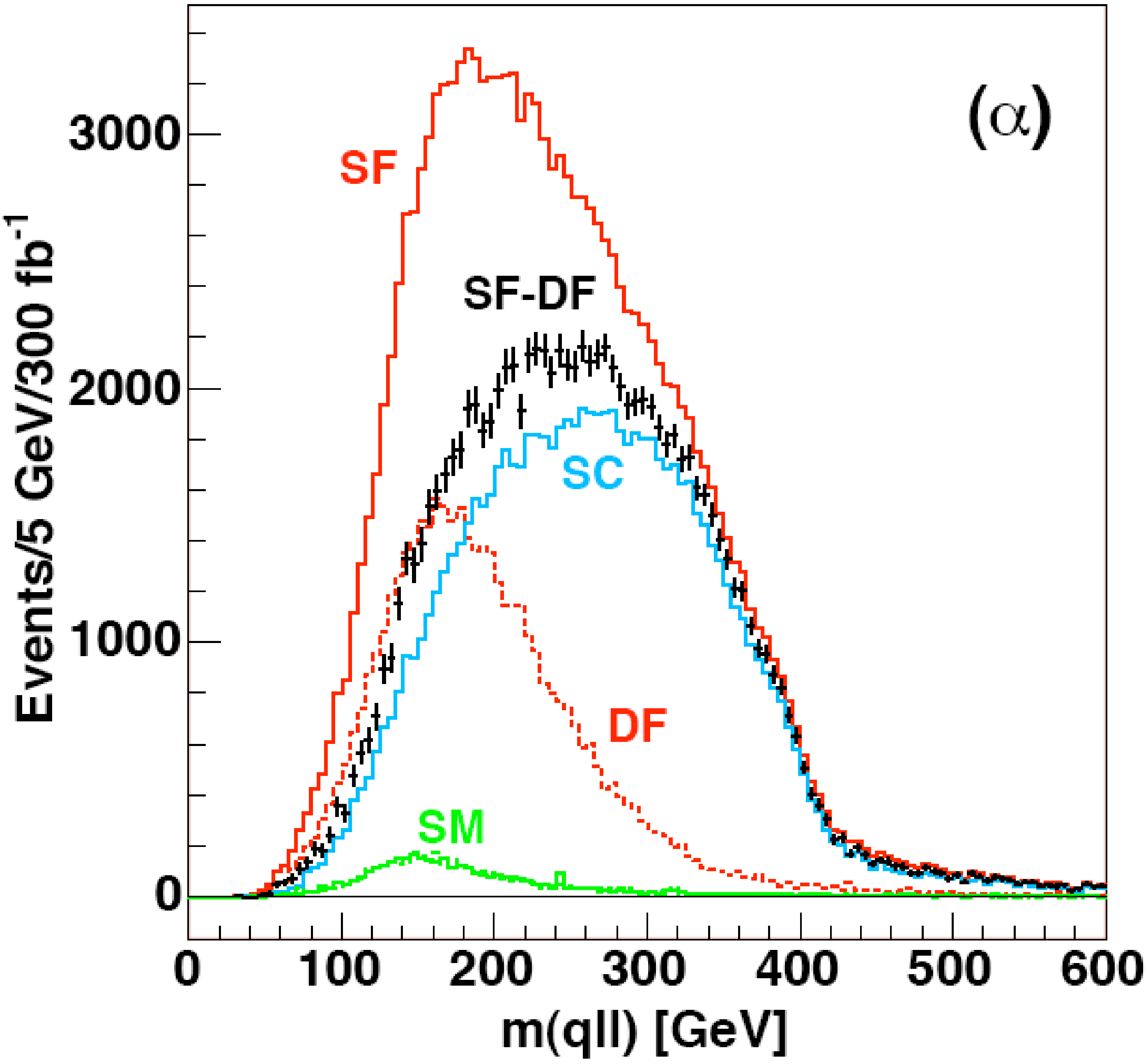
SM background: 95% $t\bar{t}$

Aim: determine/study expected accuracy

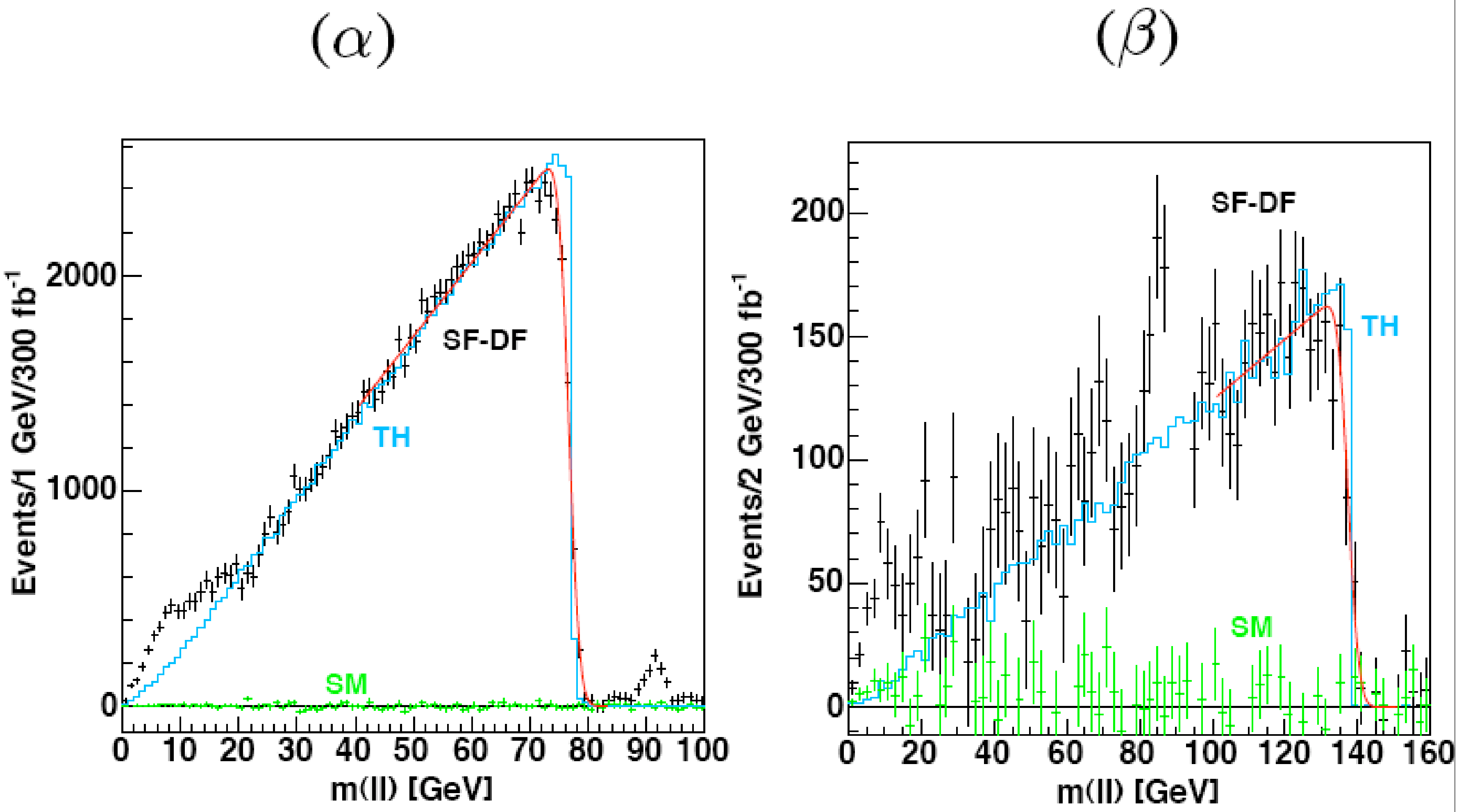

SPS 1a (α)
(α)
dilepton mass
SF Same Flavour
SC Signal Chain
DF Different Flavour
Same shape as Same-Flavour SUSY background
SM Standard Model
SF
SF-DF
SC
DF
SM
Z
Events/1 GeV/300 fb^{-1}
m(ll) [GeV]
0
1000
2000
3000
0 10 20 30 40 50 60 70 80 90 100



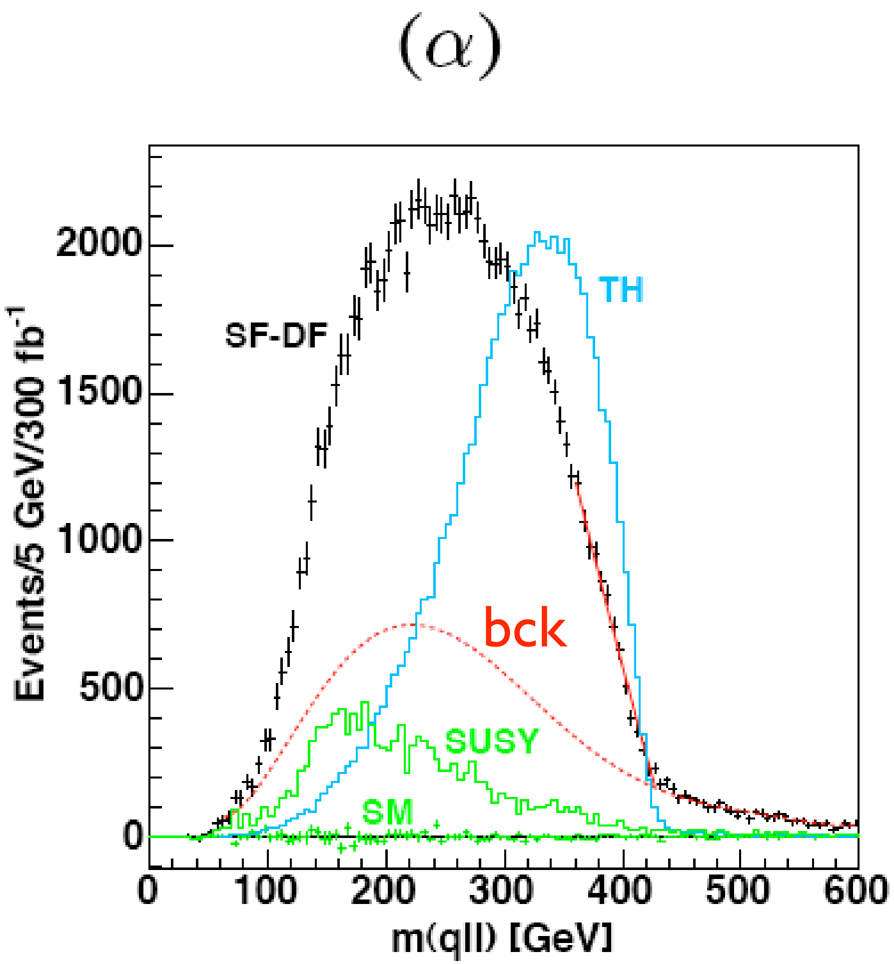
SPS 1a (β)
dilepton mass
SF Same Flavour
DF Different Flavour
SM Standard Model
SC Signal Chain
Events/3 GeV/300 fb^{-1}
m(ll) [GeV]
Z
(β)
SF
DF
SM
SF-DF
SC
800
600
400
200
0
0
20
40
60
80
100
120
140

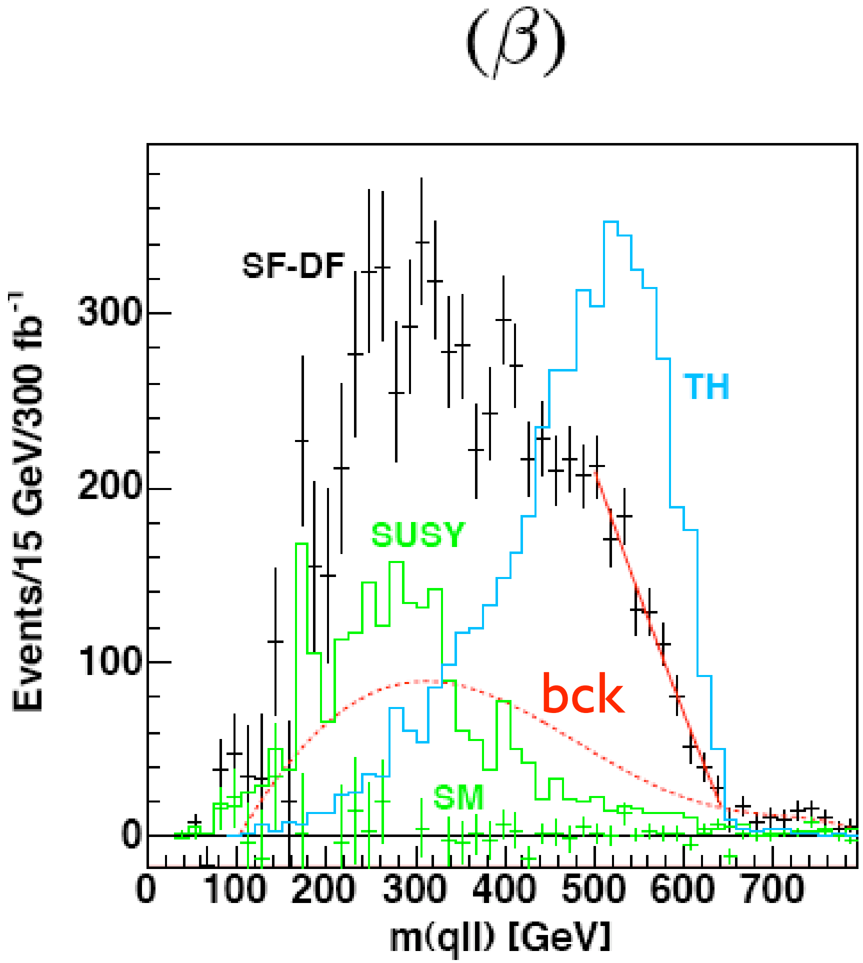

(α)
SF
SF-DF
SC
DF
SM
3000
2000
1000
0
Events/5 GeV/300 fb^{-1}
0
100
200
300
400
500
600
m(qll) [GeV]

Fitting dilepton edge



Fitting qll edge





background estimated from mixed events

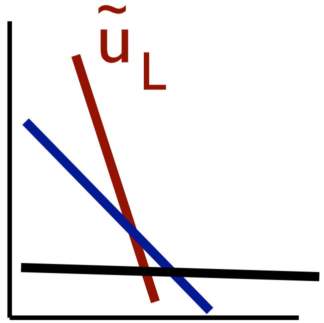
Fit results (endpoints) 300 fb^{-1}

Edge	Nominal Value [GeV]	Fit Value [GeV]	Energy Scale Error (σ^{scale}) [GeV]	Statistical Error (σ^{stat}) [GeV]	Syst. Fit Error [GeV]
(α)					
$m_{ll}^{\max}$	77.07	76.72	0.08	0.04	0.1
$m_{qll}^{\max}$	425.9	427.7	2.1	0.9	0.5
$m_{ql(\text{low})}^{\max}$	298.5	300.7	1.5	0.9	0.5
$m_{ql(\text{high})}^{\max}$	375.8	374.0	1.9	1.0	0.5
$m_{qll(\theta>\frac{\pi}{2})}^{\min}$	200.7	-	1.0	2.2	2.0
$m_{bll(\theta>\frac{\pi}{2})}^{\min}$	183.1	-	0.9	4.5	4.0
(β)					
$m_{ll}^{\max}$	137.9	137.4	0.14	0.5	0.1
$m_{qll}^{\max}$	649.1	647.0	3.2	5.0	3.0
$m_{ql(\text{low})}^{\max}$	436.6	443.0	2.2	6.3	4.0
$m_{ql(\text{high})}^{\max}$	529.9	520.5	2.6	5.5	3.0
$m_{qll(\theta>\frac{\pi}{2})}^{\min}$	325.7	-	1.6	13.0	10.0

Multiple squark masses:

	$\tilde{b}_1$	$\tilde{b}_2$	$\tilde{u}_L$	$\tilde{d}_L$
SPS 1a (α)	491.9	524.6	537.2	543.0
SPS 1a (β)	759.4	800.2	826.3	830.1

two endpoints



Higher mass: higher endpoint
but lower rate - background -
blurring of endpoint

Extraction of masses

- simulate 10,000 ATLAS ‘experiments’
- focus on statistical uncertainty
- each endpoint: gaussian distribution
- invert endpoint formulas, fit masses
- what is chance of finding correct minimum?

Following Allanach et al, each endpt E_i^{exp} taken as:

$$E_i^{\text{exp}} = E_i^{\text{nom}} + A_i \sigma_i^{\text{stat}} + B \sigma_i^{\text{scale}}$$

A, B picked from gaussian distribution, mean 0, width 1

One A for each endpoint,
one B for m_{ll}, other B for endpoints involving jets

determine masses

Minimize:

$$\Sigma = [\mathbf{E}^{\text{exp}} - \mathbf{E}^{\text{th}}(\mathbf{m})]^T \mathbf{W} [\mathbf{E}^{\text{exp}} - \mathbf{E}^{\text{th}}(\mathbf{m})]$$

$\mathbf{W}$ inverse error/correlation matrix

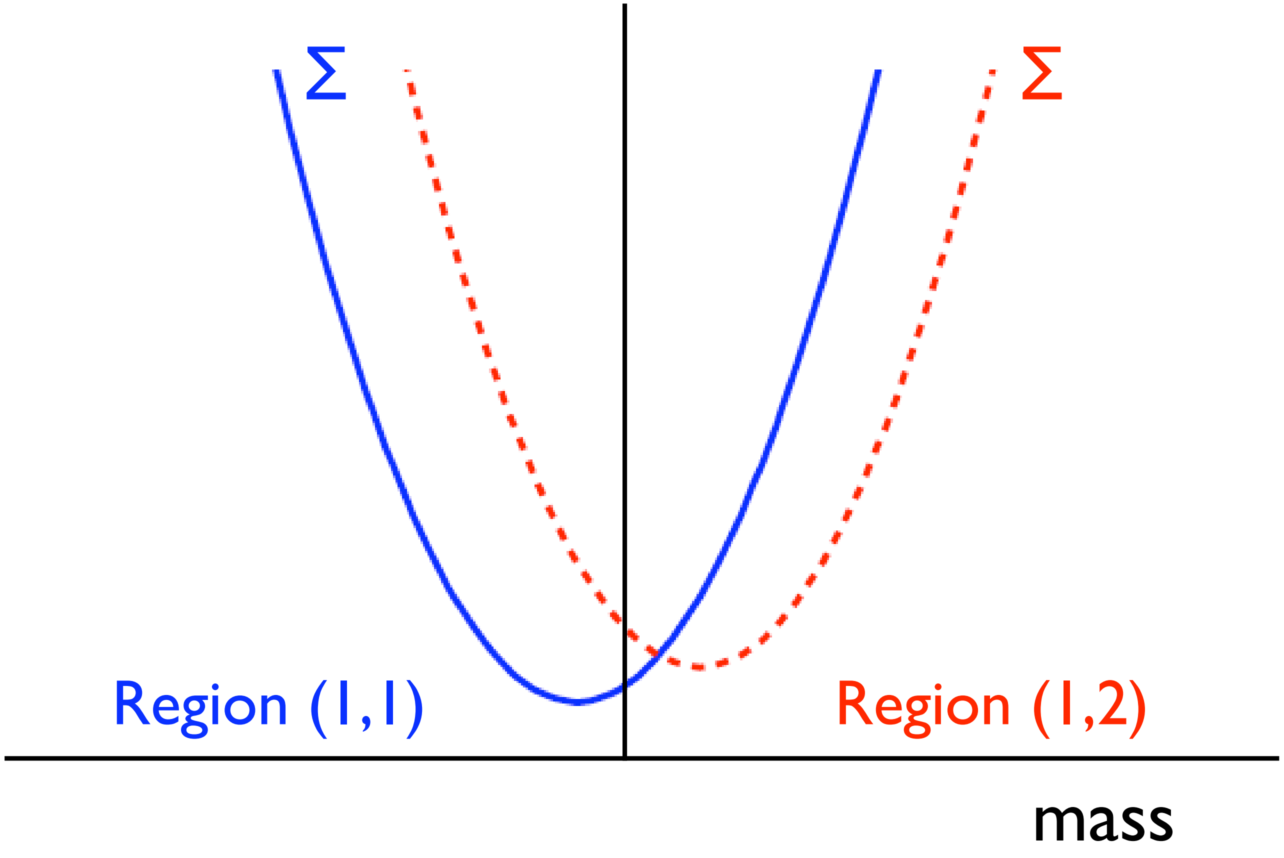
SPS 1a (α) $\Delta\Sigma \leq 1$

	nominal	correct fit			false fit		
		(1,1)			(1,2)		
	Nom	$\langle m \rangle$	σ	γ_1	$\langle m \rangle$	σ	γ_1
$m_{\tilde{\chi}_1^0}$	96.1	96.3	3.8	0.2	85.3	3.4	0.1
$m_{\tilde{l}_R}$	143.0	143.2	3.8	0.2	130.4	3.7	0.1
$m_{\tilde{\chi}_2^0}$	176.8	177.0	3.7	0.2	165.5	3.4	0.1
$m_{\tilde{q}_L}$	537.2	537.5	6.1	0.1	523.2	5.1	0.1
$m_{\tilde{b}_1}$	491.9	492.4	13.4	0.0	469.6	13.3	0.1
$m_{\tilde{l}_R} - m_{\tilde{\chi}_1^0}$	46.9	46.9	0.3	0.0	45.1	0.7	-0.2
$m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$	80.8	80.8	0.2	0.0	80.2	0.3	-0.1
$m_{\tilde{q}_L} - m_{\tilde{\chi}_1^0}$	441.2	441.3	3.1	0.0	438.0	2.7	0.0
$m_{\tilde{b}_1} - m_{\tilde{\chi}_1^0}$	395.9	396.2	12.0	0.0	384.4	12.0	0.1

Note: Three lightest masses are very correlated

Problem due to compositeness of formulas:

If masses are close to border of 'region', may find a similar-quality or better minimum in 'other' region



How likely is a false minimum?

Depends on cut $\Delta\Sigma$ (level of confidence)

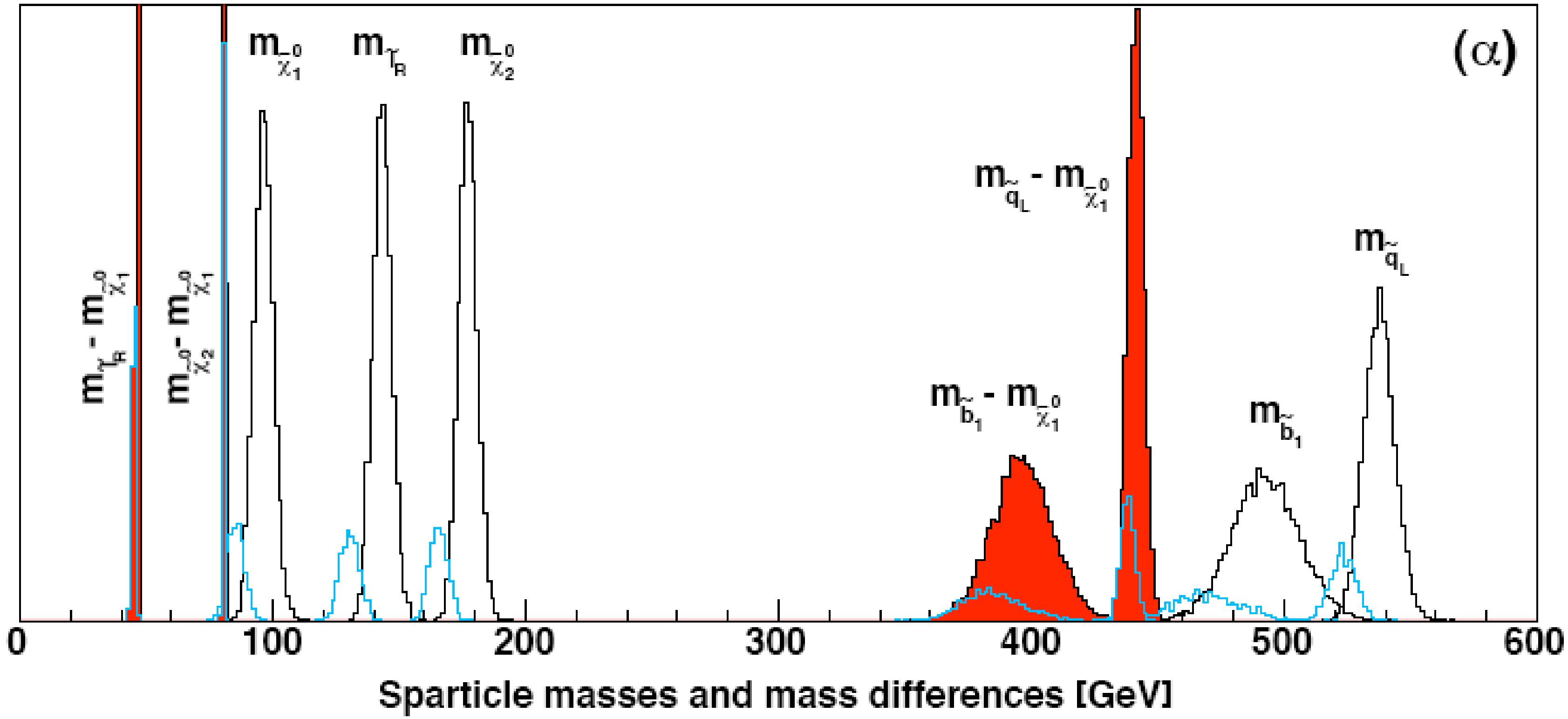
SPS 1a (α)

	# Minima	*(1,1)*	*(1,2)*
$\Delta\Sigma \leq 0$	1.00	90%	10%
$\Delta\Sigma \leq 1$	1.12	94%	17%
$\Delta\Sigma \leq 3$	1.30	97%	33%
$\Delta\Sigma \leq \infty$	1.88	99%	88%

Example: $\Delta\Sigma \leq 3$

30% chance of finding *two* minima

Spread

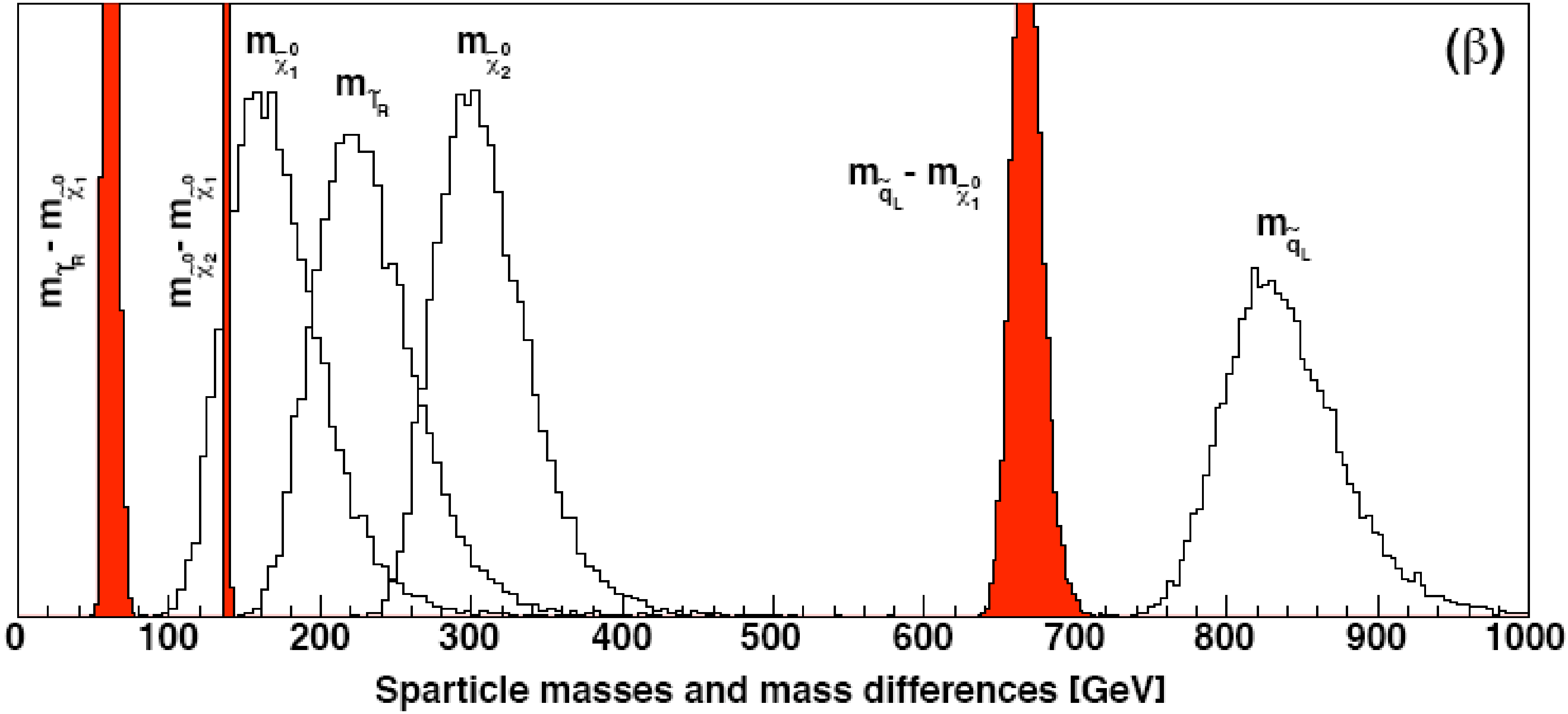


black: correct solution

red: mass differences

blue-green: false solution (area prop to probability)

Spread



false solutions not shown

LC input ("fixing" LSP mass)

SPS 1a (α)

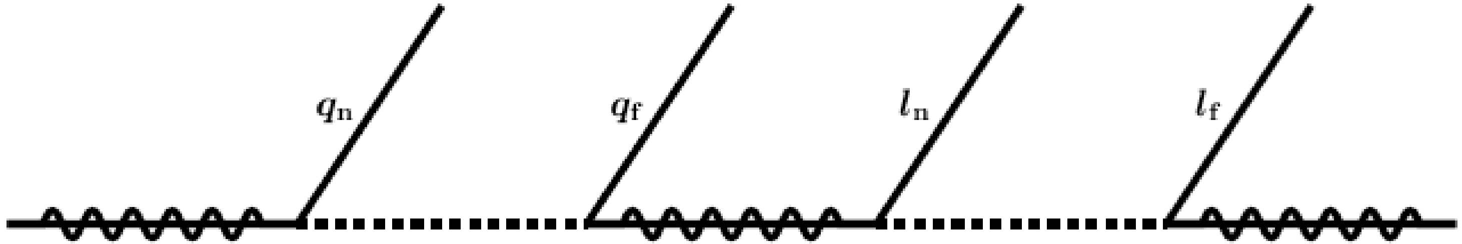
	Nom	(1,1) $\langle m \rangle$	(1,1) σ
$\tilde{\chi}_1^0$	96.05	96.05	0.05
$\tilde{l}_R$	142.97	142.97	0.29
$\tilde{\chi}_2^0$	176.82	176.82	0.17
$\tilde{q}_L$	537.25	537.2	2.5
$\tilde{b}_1$	491.92	492.1	11.7

Masses in GeV

SPS 1a (β)

		1 solution		2 solutions			
		(1,2)/(1,3)/B		(1,2)		(1,3)	
	Nom	$\langle m \rangle$	σ	$\langle m \rangle$	σ	$\langle m \rangle$	σ
$\tilde{\chi}_1^0$	161.02	161.02	0.05	161.02	0.05	161.02	0.05
$\tilde{l}_R$	221.86	221.15	3.26	222.22	1.32	217.48	1.01
$\tilde{\chi}_2^0$	299.05	299.15	0.57	299.11	0.53	299.05	0.52
$\tilde{q}_L$	826.29	826.1	6.3	825.9	5.8	828.6	5.5

Gluino cascade chain



$\sigma = 35.4$ pb	$\tilde{g}$	23.0%	$\tilde{q}_L$	31.4%					309 fb
		17.0%	$\tilde{b}_1$	35.5%	$\tilde{\chi}_2^0$ 12.1%	$\tilde{l}_R$	100%	$\tilde{\chi}_1^0$	259 fb
		9.7%	$\tilde{b}_2$	18.0%					75 fb
									643 fb

SPS 1a numbers

Several new kinematical edges involving q_n

Only *one* new mass, need (minimum) only *one* more edge

Summary (SPS 1a)

- SPS 1a SUSY masses can be determined with precision 4-10 GeV
- Non-zero probability of fitting wrong minimum
- LC input on LSP mass removes ambiguity and increases precision significantly
- Gluino mass can be obtained using two b jets
- Better understanding of backgrounds very beneficial
- Understand better systematics of fitting (detector & theory)