A non-perturbative orbifold gauge theory

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or

What if no low energy supersymmetry is discovered at the LHC

Alternative : Extra Dimension(s)

The minimal possibility is a 5D gauge theory
(looks innocent but...)

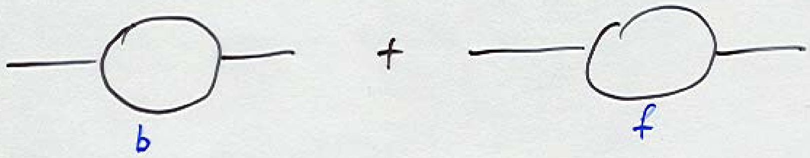
$$A_M^A = \{ A_\mu^A , \; A_5^A \equiv \phi \}$$

A_μ^A → 4D gauge field

$A_5^A \equiv \phi$ → 4D Higgs

- Correct quantum numbers? ($A \to$ adjoint rep)
- Hierarchy problem?

In supersymmetry the hierarchy is protected



$= m_0 + \dots$

↑

no quadratic cutoff dependence

A 5D gauge theory is non-renormalizable and trivial.

Start from $F^A_{MN} = \partial_M A^A_N - \partial_N A^A_M + g_0 f^{ABC} A^B_M A^C_N$,

$$S = \frac{1}{4}\int d^dx \left[b\, F^A_{MN} F^A_{MN} + c\, D_L F^A_{MN} D_L F^A_{MN} + \dots \right]$$

everything is dimensionless like on a lattice

and introduce dimensions by rescaling (RG transf.)

$$x = \Lambda x' \; ; \; A^A_M(x) = \eta A'^A_M(x') \; ; \; g_0 = \xi g_0'$$

requiring the standard 1/4 for the kinetic term,

$$S = \frac{1}{4}\int d^dx' \left[F'^A_{MN} F'^A_{MN} + \frac{c}{b}\frac{1}{\Lambda^2} D'_L F'^A_{MN} D'_L F'^A_{MN} + \dots \right]$$

question: $\frac{c}{b}$ → cut off dependent $O(1)$ number (triviality)

→ function of $g_0^2 \Lambda$

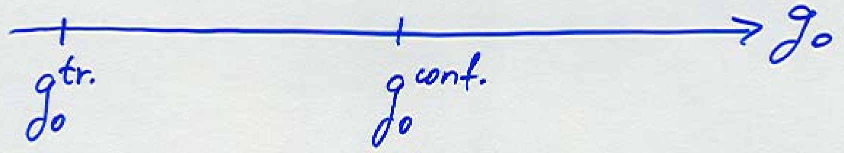
e.g. $\sum_{n=1} c_n (g_0^2 \Lambda)^n$

$d=5$: $\quad g_0^{2\,\prime} = \dfrac{g_0^2}{b\Lambda}$

An attempt to approach the continuum limit:

$$g_0' = g_0 / \sqrt{b\Lambda} \quad \hookrightarrow \quad g_0 \to \infty$$

keep fixed $\qquad\qquad$ take to ∞



the phase transition is hit $\Rightarrow$ Λ must stay finite

In order to get an intuition for the physical meaning of this, look at the 4D ϕ^4 theory

$$\ln(\Lambda/m_R) \leq A/g_R + B \ln(g_R) + \mathcal{O}(1)$$

for m_R, g_R fixed, one can not take $\Lambda \to \infty$

$\to$ the theory is trivial

Triviality: Any physical quantity "measured" in perturbation theory, will be attracted to the trivial fixed point.

The physical meaning of this:

In 5D non-compact space

$$m^2_{A^A_\mu} = m^2_{A^A_5} = 0 \quad \text{by gauge invariance}$$

$$\delta_{G_5}\left(m^2_{A^A_5} A^A_5 A^A_5\right) \neq 0$$

so this is a gauge theory without Higgs.
To have a Higgs, one must compactify: $\mathbb{R}^4 \times S^1$
Is then the Higgs massless or massive?
To answer this, one has to embed the geometry into the fields:

$$A(x_5 + 2\pi R) = A(x^5) + \text{gauge transf.}$$

Typical Kaluza-Klein set to zero

this is a gauge fixing that breaks

5D gauge invariance $\to$ 4D gauge invariance

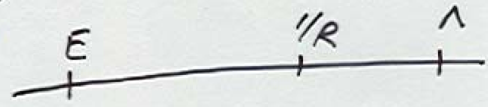
i.e. $\delta_{G_5}(m^2_{A_5^A} A_5^A A_5^A) \neq 0$ but $\delta_{G_4}(m^2_{A_5^A} A_5^A A_5^A) = 0$

so a mass term is allowed by the symmetry.
In fact, a 1-loop calculation shows that

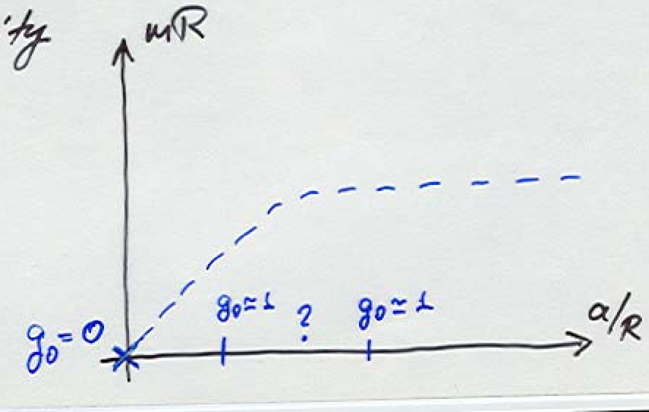
$$m^2_{A_5^A} = \frac{9}{32\pi^4} \frac{1}{\pi R} \frac{g_0^2}{R} \zeta(3)\, C_2(G)$$

- Triviality

to have appreciable m, we need small R
i.e. Λ large which forces $g_R \to 0$



- Non-renormalizability



There are arguments for the non-perturbative finiteness of m in the $\mathbb{R}^4 \times T^p$ theories.

Recall that $A_M(x_5 + 2\pi R) = U A_M(x_5) U^+$

If $U \in H' \subset G$ then gauge invariance is broken.

The form of the 1-loop induced G-breaking mass term can be seen to have the form

$$\sum_{n=1}^{\infty} A_5^+(x_5)\, W_n\, A_5(x_5 + 2\pi R n)$$

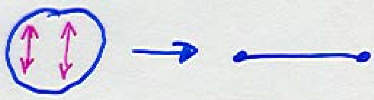
$$W_n = P\, e^{i \int_{x_5}^{x_5 + 2\pi R n} A_5(x_5')\, dx_5'}$$

i.e. a non-local coupling and hence finite.

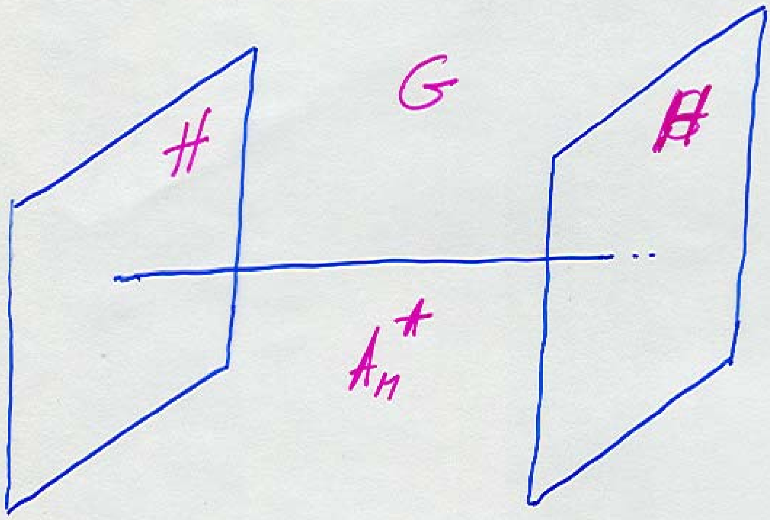
This generalizes to any order of perturbation theory.

If the SM Higgs was in the adjoint representation, this would be the end of the story.

To have a Higgs in a non-adjoint representation

$$S^1 \to S^1/\mathbb{Z}_2 :$$ 

The $\mathbb{Z}_2$ projection can break in fact $G \to H$ at the boundaries.



$$A_M^A \begin{cases} \to A_\mu^a \,,\; A_5^{\hat a} \equiv \phi & (+) \\ \to A_\mu^{\hat a} \,,\; A_5^a & (-) \end{cases}$$

If we KK and compute, we find again

$$m_\phi^2 = \frac{9}{32\pi^4} \frac{1}{\pi R} \frac{g_0^2}{R} \zeta(3)\, C_2(G)$$

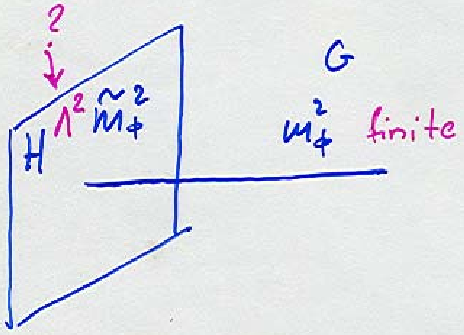
To understand this, we have to understand the general structure of the effective orbifold action.
The crucial object is the propagator:

$$\sim\!\sim\!\sim_{(S^1/\mathbb{Z}_2)} = \sim\!\sim\!\sim_{(S^1)} \left(\delta_{m-m'} + P_A^{\mathbb{Z}_2} \delta_{m+m'} \right)$$

$$\mathcal{L}_{eff}^{S^1/\mathbb{Z}_2} = \mathcal{L}_{eff}^{bulk} + \mathcal{L}_{eff}^{bound.} + \mathcal{L}_{eff}^{bulk/bound}$$

$$\boxed{\mathcal{L}_{eff}^{bound.}}$$

Symanzik: In a FT with boundaries, new, localized at the boundaries divergences appear, which induce boundary counterterms.



Since $\delta_H(\tilde{m}^2 A_5^{\hat{a}} A_5^{\hat{a}}) = 0$, it should be generated.

However, the direct 1-loop calculation showed that

$$\tilde{m}^2 A_5^{\hat{a}} A_5^{\hat{a}} = 0$$

A convincing explanation for this, is:

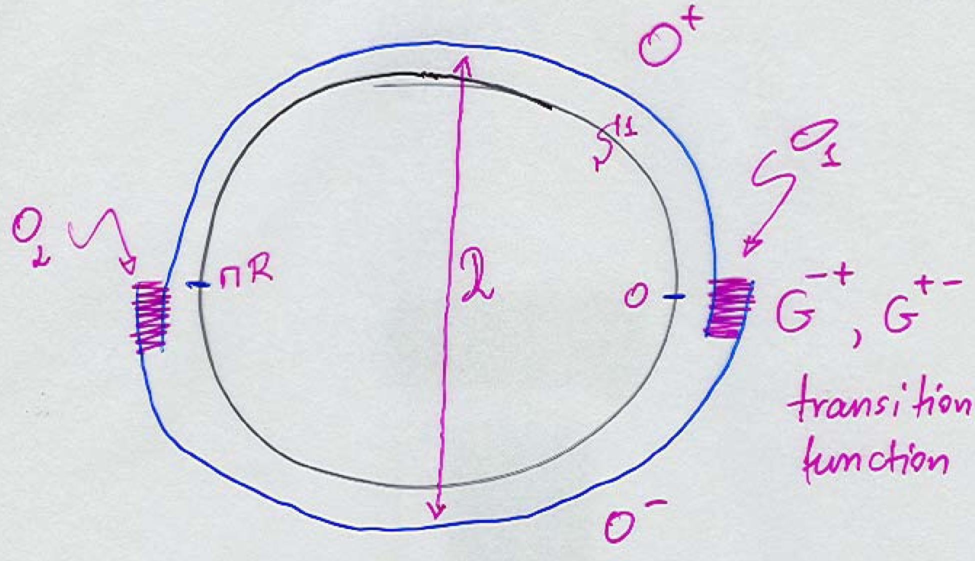
$$\delta_G \begin{cases} \to \delta_H \quad \text{(4D gauge inv.)} \to \delta_H(\tilde{m}^2 A_5^{\hat{a}} A_5^{\hat{a}}) = 0 \\ \to \delta_{G/H} \quad \text{(4D local inv.)} \begin{cases} \to \text{shift symmetry} \to \delta_{G/H}(\tilde{m}^2 A_5^{\hat{a}} A_5^{\hat{a}}) \neq 0 \\ \to \infty\text{-tower of local symmetries} \end{cases} \end{cases}$$

Is this structure preserved by Quantum corrections?

Not clear.

The reason is (again) because we are KK-ing so we have broken gauge invariance.

To be able to construct a proof about pure boundary counterterms, we have to construct a 5D gauge invariant argument (even though it is broken anyway at the boundaries)



$$O^+ = (-\epsilon, \pi R + \epsilon), \quad A_M^+, \quad \underline{\varrho}^+$$

$$O^- = (-\pi R - \epsilon, \epsilon), \quad A_M^-, \quad \underline{\varrho}^-$$

On overlaps, the gauge fields are realated by a gauge transformation

$$A_M^- = G^{-+} A_M^+ G^{-+-1} + G^{-+} \partial_M G^{-+-1}$$

and covariance requires

$$G^{-+} \rightarrow \underline{\varrho}^- G^{-+} \underline{\varrho}^{+-1}$$

$$G^{+-} G^{-+} = \pm 1 \quad \text{gluing condition}$$

A covariant derivative for the transition function can be constructed:

$$D_M G^{-+} = \partial_M G^{-+} + A_M^{-} G^{-+} - G^{-+} A_M^{+} \equiv 0$$

Orbifolding

$$\mathcal{R} : \quad z = (x_\mu, x_5) \longrightarrow \bar{z} = (x_\mu, -x_5)$$

$$\text{i.e.} \quad \mathcal{R} z = \bar{z}$$

also, $\mathcal{R} A_M(z) = \alpha_M A_M(\bar{z})$, $\alpha_\mu = +1$, $\alpha_5 = -1$

$$\left.\begin{aligned} \mathcal{R} A_M^{+} &= A_M^{-} \\ \mathcal{R}\, \underline{O}^{+} &= \underline{O}^{-} \end{aligned}\right\} \quad \text{on} \quad O^{+} \cap O^{-} = O_{i=1,2}$$

the above implies that

$$\mathcal{R} A_M^{+} = G^{-+} A_M^{+} G^{-+-1} + G^{-+} \partial_M G^{-+-1}$$

and self consistency of this requires that

$$(\mathcal{R} G^{-+})\, G^{-+} = \pm 1$$

i.e. at $x_5 = 0, \pi R$: $\quad (G^{-+})^2 = \pm 1$

outside O_i, simply : $\mathcal{R} A_M^{+}(x_5) = A_M^{-}(-x_5)$

Simplify the notation a bit:

$$A_M^{+} \to A_M \quad , \quad G^{-+} = G \equiv \text{"Spurion"}$$

$A^M(z)$ is defined everywhere

$G(z)$ is defined on O_i and $(RG)G = \pm 1$

$$x_5 = 0, \pi R : \quad G^2 = \pm 1$$

also, on O_i:

- $R A_M = G A_M G^{-1} + G \partial_M G^{-1}$
- $G \to (R\underline{o}) G \underline{o}^{-1}$
- $D_M G = 0$

locality of gauge transformations is ensured, if

- $R\underline{o} = \underline{o}$

Now, take $\epsilon \to 0$:

$$G(0) = G(\pi R) \longrightarrow g \qquad \text{where } g^2 = \text{const.} = \pm 1$$

$$\partial_5^p G|_0 = \partial_5^p G|_{\pi R} \longrightarrow 0 \qquad g = e^{-2\pi i V \cdot H}$$

at $0, \pi R$ only $[\mathcal{G}, g] = 0$ gauge transformations survive

also, the whole breaking pattern is exactly reproduced.

$$\alpha_M A_n = g A_n g$$

$$\alpha_M \partial_5 A_n = -g \partial_5 A_n g$$

$$\alpha_M \alpha_N F_{MN} = g F_{MN} g$$

$$\vdots$$

We can now classify all boundary counterterms:
all 5D gauge invariant terms containing G:

$$\mathrm{tr}\{G F_{MN} F_{MN}\} \longrightarrow \mathrm{tr}\{g F_{MN} F_{MN}\} \longrightarrow Z_1 \left[C_2(H) - \tfrac{1}{2} C_2(G)\right] F_{\mu\nu}^{a} F_{\mu\nu}^{a}$$

$$\mathrm{tr}\{G F_{MN} G F_{MN}\} \longrightarrow \mathrm{tr}\{g F_{MN} g F_{MN}\} \longrightarrow Z_2\, C_2(G) \left[F_{\mu\nu}^{a} F_{\mu\nu}^{a} - F_{\mu\nu}^{\hat{a}} F_{\mu\nu}^{\hat{a}}\right]$$

explicit 1-loop calculations have found:

$Z_1 \sim \ln \Lambda$ $\qquad\qquad$ $Z_2 \sim ?$

1-loop "brane kinetic term" $\qquad\qquad$ appears at 2-loops

A Higgs mass is contained in

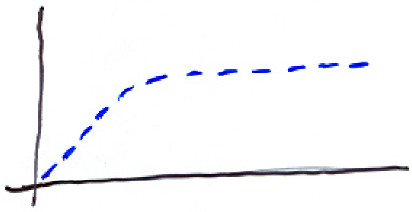
$$(D_M G)(D_M G) = 0 \quad !!!$$

{No boundary Higgs mass to all orders of p.t.}

Bulk – Brane mixing

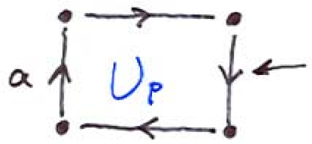
The finiteness of $m_\phi^{2\,\text{1-loop}}$ is spoiled at two loops.

scaling is again an issue



The only way to give a definite answer is to formulate the theory non-perturbatively.

Lattice formulation



$$\leftarrow U(z,M) \in SU(N) = e^{a A_M(z)}$$

$$U(z,M) \to \Omega(z)\, U(z,M)\, \Omega^{+}(z + a\hat{M})$$

Orbifold

$$R\, U(z,\mu) = U(\bar{z},\mu) \;; \quad R\, U(z,5) = U^{+}(\bar{z} - a\hat{5}, 5)$$

$$T_g\, U(z,M) = g\, U(z,M)\, g$$

$\mathbb{Z}_2$ projection: $\Gamma \equiv R\, T_g$

$$\frac{1-\Gamma}{2}\, U(z,M) = 0$$

on $I_0 = \left\{ M_\mu, \quad 0 \le m_5 \le N_5 = \frac{\pi R}{a} \right\}$

$$S_W^{\text{orb.}} = \frac{\beta}{2N} \sum_p w(p)\, \mathrm{tr}\,(1 - U(p))$$

$$w(p) = \begin{cases} 1/2 & p \in \partial \\ 0 & \text{otherwise} \end{cases}$$

The actual simulation is in progress

with B. Bunk & F. Knechtli (Berlin)