MASSIVE SUSY QUANTUM GAUGE THEORY

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Contents

INTRODUCTION

- The construction of the model is done directly quantum mechanically (no quantization procedure needed)

- The gauge structure is constructed in analogy to the gauge structure of the usual gauge models

- The Hilbert space of the model has a indefinite sesqui-linear form; the positivity of the scalar is imposed only on the physical subspace

- The construction of the S-matrix (verifying Bogoliubov axioms) can be done in the spirit of Epstein-Glaser construction.

THE QUANTUM VECTOR MULTIPLET

$$
\begin{aligned}
&V(x,\theta,\bar\theta) = \\
&C(x) + \theta\chi(x) + \bar\theta\bar\chi(x) + \theta^2\ \phi(x) + \bar\theta^2\ \phi^\dagger(x) \\
&+(\theta\sigma^\mu\bar\theta)\ v_\mu(x) + \theta^2\ \bar\theta\bar\lambda(x) + \bar\theta^2\ \theta\lambda(x) \\
&+\theta^2\bar\theta^2\ d(x)
\end{aligned} \quad (1)
$$

$$V^\dagger = V; \quad (2)$$

C, d are real scalar fields, ϕ complex scalar field, v_μ Hermitian vector field, χ_a, λ_a Dirac spinor fields, θ_a Grassmann parameters.

The fields live in the Hilbert space $\mathcal{H}$ with an indefinite sesqui-linear form $< \cdot, \cdot >$.

All the fields are of mass m:

$$(\partial^2 + m^2)V = 0, \quad m > 0 \quad (3)$$

Decomposition of V in transversal, chiral and anti-chiral components

$$V = \sum_{j=0}^{3} P_j V \tag{4}$$

$$P_0 \equiv -\frac{1}{8m^2}\mathcal{D}^a \bar{\mathcal{D}}^2 \mathcal{D}_a,$$
$$P_1 \equiv \frac{1}{16m^2}\mathcal{D}^2 \bar{\mathcal{D}}^2, \quad P_2 \equiv \frac{1}{16m^2}\bar{\mathcal{D}}^2 \mathcal{D}^2. \tag{5}$$

The expressions $P_j, \quad j = 0,1,2$ are projectors on the mass shell.

One can determine the generic form of the causal commutator of the vector superfield

$$[V(X_1), V(X_2)] = -iD(X_1; X_2)\ \mathbf{1}. \tag{6}$$

THE GAUGE STRUCTURE OF THE QUANTUM VECTOR MULTIPLET

Three ghost superfields $U, \tilde{U}$ Fermionic H Bosonic;

chirality condition

$$\mathcal{D}_a \; U = 0 \quad \mathcal{D}_a \; \tilde{U} = 0 \quad \mathcal{D}_a \; H = 0. \tag{7}$$

The gauge charge is uniquely determined by:

$$\begin{aligned} Q\Omega = 0 \quad Q^\dagger = Q \\ [Q, V] = U - U^\dagger \\ \{Q, U\} = 0. \\ \{Q, \tilde{U}\} = -\frac{1}{16} \; \mathcal{D}^2 \; \bar{\mathcal{D}}^2 V - i \; m \; H \\ [Q, H] = i \; m \; U. \end{aligned} \tag{8}$$

From here:

$$Q^2 = 0 \tag{9}$$

$Ker(Q)/Im(Q)$ is the physical space

The generic form for the one-particle states from the Hilbert space

$$\begin{aligned}\Psi = [\int f_1(X)V(X) \\ + \int f_2(X)U(X) + \int f_3(X)U(X)^\dagger \\ + \int f_4(X)\tilde{U}(X) + \int f_5(X)\tilde{U}(X)^\dagger \\ + \int f_6(X)H(X) + \int f_7(X)H(X)^\dagger]\Omega \end{aligned} \tag{10}$$

with $f_j, j = 1,\ldots,7$ supersymmetric test functions. The integration is over $d^4xd^2\theta d^2\bar{\theta}$. The writing is unique *iff* we require

$$P_2 f_j = 0, \quad j = 2,4,6; \qquad P_1 f_j = 0, \quad j = 3,5,7 \tag{11}$$

The test functions also verify Klein-Gordon equation.

The condition $\Psi \in Ker(Q)$ i.e. $Q\Psi = 0$

$$\begin{aligned} P_1 f_1 = 0 \quad P_2 f_1 = 0 \\ f_4 = 0, \quad f_5 = 0 \end{aligned} \tag{12}$$

The generic expression of an element from $Im(Q)$

$$\begin{aligned}\Psi' = [\int (g_1 + img_6)(X)U(X) \\ + \int (-g_1 + img_7)(X)U(X)^\dagger \\ -\frac{1}{16}\int (\bar{\mathcal{D}}^2\mathcal{D}^2 g_4 + \mathcal{D}^2\bar{\mathcal{D}}^2 g_5)(X)V(X) \\ -im\int g_4(X)H(X) + im\int g_5(X)H(X)^\dagger]\Omega. \end{aligned} \quad (13)$$

If we take the test functions $g_j, j = 1, \ldots, 7$ convenient, then we can arrange such that

$$\Psi - \Psi' = \int f_1(X)V(X)\Omega \quad (14)$$

where f_1 is a transverse supersymmetric test function:

$$P_1 f_1 = 0 \quad (15)$$

In every equivalence class from

$$Ker(Q)/Im(Q) \cap \mathcal{H}^{(1)}$$

there exists one and only one element of transverse form, i.e. the equivalence classes are indexed by supersymmetric transversal functions. Equivalently, only the transversal part of the vector superfield is producing physical states.

This is the (quantum) *Wess-Zumino gauge*.

We impose the condition of positivity of $< \cdot, \cdot >$ only in the physical Hilbert space.

In particular, the solution

$$D(X_1; X_2) = -D_2(X_1; X_2) = \\ (\theta_1 - \theta_2)^2 (\bar{\theta}_1 - \bar{\theta}_2)^2 \; D_m(x_1 - x_2) \tag{16}$$

for the causal commutator is acceptable from the point of view of the positivity.

A MASSIVE SUPERSYMMETRIC GAUGE INVARIANT COUPLING

We consider the following interaction Lagrangian:

$$
\begin{aligned}
T = f^{(1)}_{jkl} \left[: V_j (\mathcal{D}^a V_k)(\bar{\mathcal{D}}^2 \mathcal{D}_a V_k) : - H.c. \right] \\
+ f^{(2)}_{jkl} : V_j \ (U_k + U_k^\dagger)(\tilde{U}_l + \tilde{U}_l^\dagger) : \\
+ f^{(3)}_{jkl} \ : (H_j + H_j^\dagger) \ (U_k - U_k^\dagger)(\tilde{U}_l + \tilde{U}_l^\dagger) : \\
+ f^{(4)}_{jkl} \ : (H_j + H_j^\dagger) \ (H_k - H_k^\dagger) V_l : \\
+ f^{(5)}_{jkl} \ : (H_j + H_j^\dagger) \ V_k \ D'_l \\
+ f^{(6)}_{jkl} \ : (H_j + H_j^\dagger) \ V_k \ V_l : \\
+ f^{(7)}_{jkl} \ : (H_j + H_j^\dagger) \ (H_k - H_k^\dagger) D'_l :
\end{aligned}
\quad (17)
$$

where $f^{(j)}_{jkl}, j = 1, \ldots, 7$ are some constants to be determined later. We can suppose that

$$f^{(7)}_{jkl} = -f^{(7)}_{kjl} \quad f^{(6)}_{jkl} = f^{(6)}_{jlk}; \qquad (18)$$

here D' is defined by:

$$V_0 = P_0 V = -\frac{2}{m^2} D' \qquad (19)$$

From renormalizability considerations it can be showed that we can take

$$f^{(5)}_{jkl} = 0. \tag{20}$$

The interaction Lagrangian should follow from a supersymmetric expression after integration:

$$t(x) \equiv \int d\theta^2 d\bar{\theta}^2 T(x, \theta, \bar{\theta}) \tag{21}$$

The previous expression is suggested by the fact that after integrating out the Grassmann variables one gets coupling terms of the same form as in ordinary gauge theory (at least from the first two terms).

We impose the *supersymmetric gauge invariance* condition

$$[Q, T(x, \theta, \bar{\theta})] = \mathcal{D}^a T_a(x, \theta, \bar{\theta}) + \bar{\mathcal{D}}_{\bar{a}} \bar{T}^{\bar{a}}(x, \theta, \bar{\theta}) \tag{22}$$

This leads to a solution determined by the completely antisymmetric constants

$$f_{jkl} \equiv f^{(2)}_{jkl} \tag{23}$$

as follows:

$$f^{(1)}_{jkl} = -\frac{1}{16}\ f_{jkl}, \qquad f^{(3)}_{jkl} = \frac{i}{m_j} f_{jkl},$$

$$f^{(4)}_{jkl} = -\frac{m_k}{m_j}\ f_{jkl}$$

$$f^{(6)}_{jkl} = \frac{i}{2m_j}(m_k^2 - m_l^2)\ f_{jkl},$$

$$f^{(7)}_{jkl} = -\frac{1}{m_j m_k} f_{jkl}. \tag{24}$$

CONCLUSION

This is our massive supersymmetric quantum gauge model. Let us also remark that the doubling of the number of ghost fields cannot be avoided. The only way of halving this number would be to consider that the super-ghost multiplets are of Wess-Zumino type. However, this assumption leads to a contradiction.

The supersymmetric gauge invariance condition is the true supersymmetric extension of first order perturbative gauge invariance from the literature. It expresses gauge invariance in terms of the chronological products using the gauge structure of the free asymptotic fields only; other formulations of gauge invariance (classical or quantum) deal with interacting fields.

It determines the theory essentially unique; one can obtain very simply a Fermi coupling.

On the other hand the theory is not yet completely specified because there are still some free parameters in the commutations rules of V and the ghost fields.

To prove second order gauge invariance one expects the necessity of a Higgs superfield as in ordinary massive gauge theory.

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