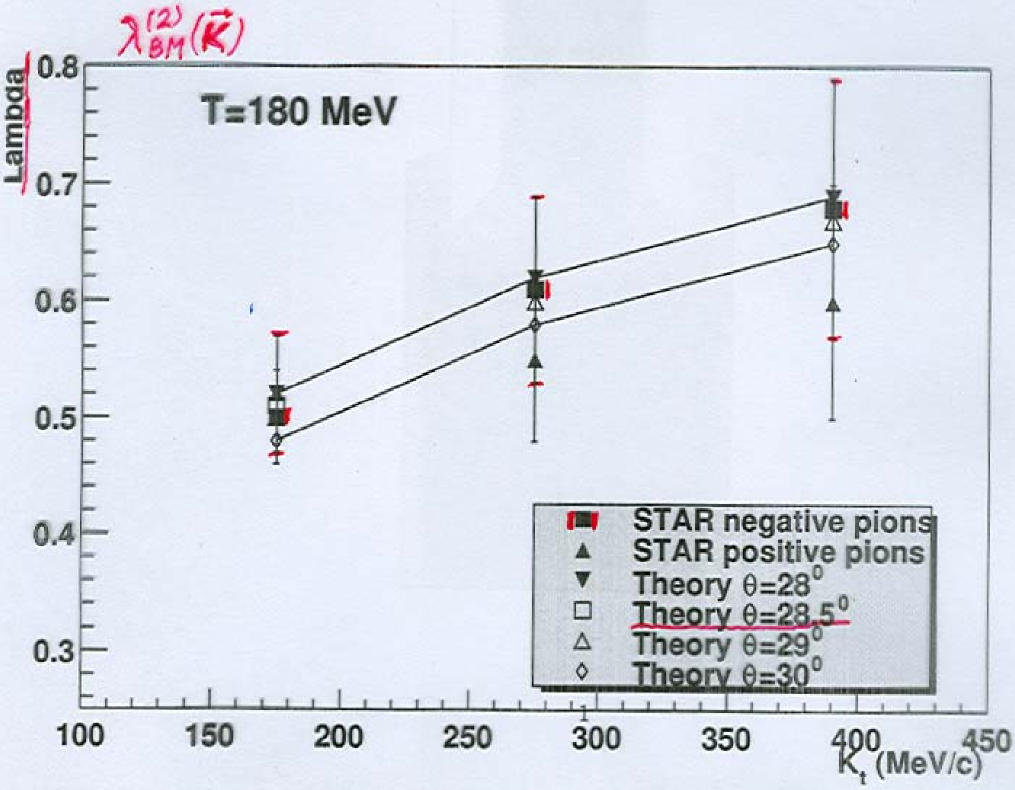
Three-pion correlations in relativistic heavy-ion collisions: a test for q,p -Bose gas model

A. M. Gavrilik
N.N.Bogolyubov Institute for Theoretical Physics,
03143, Kyiv, Ukraine

BW-2005



— Why q-deformed strcs. in phys. applics.?
- useful in a widest set of problems,
  e.g. phenom. descr. of rotat. spectra of (super) deformed nuclei;
  in phenom. descript. of static props. of hadrons

— Why q,p-deformed structures?
- q,p-deformat. contain, <u>as particular cases</u>, different versions of q-deformed ons
- if, for q-parameter, there are <u>many</u> possible phys. meanings or reasons to use, the q,p-def. <u>may combine any two</u> of them

E.g., in the context of our topic:
- interparticle interactions,
- substructure of particles,
- excluded (finite) volume -"-",
- memory effects,
- fireball is short-lived, highly non-equilibr. & complicated system
- effects from long-lived resonances (say, in core-halo model)
- possible non-chaotic components
- ...

• System of q-deformed oscillators of the AC (Arik-Coon) type: *(1976)*

$$a_i a_j^\dagger - q^{\delta_{ij}} a_j^\dagger a_i = \delta_{ij} \qquad [a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0 \qquad (1)$$

$$[\mathcal{N}_i, a_j] = -\delta_{ij} a_j \qquad [\mathcal{N}_i, a_j^\dagger] = \delta_{ij} a_j^\dagger \qquad [\mathcal{N}_i, \mathcal{N}_j] = 0 \,.$$

Here $-1 \le q \le 1$. Due to δ_{ij}, *different modes are independent.*

Vacuum state is defined by $a_i\,|0, 0, \ldots\rangle = 0$ for all i; basis state vectors $|n_1, \ldots, n_i, \ldots\rangle$ are constructed as usual, and MEs are e.g.,

$$\langle \ldots, n_i{+}1, \ldots |a_i^\dagger| \ldots, n_i, \ldots\rangle = \sqrt{\lfloor n_i{+}1 \rfloor}$$

where the "basic numbers" $\lfloor r \rfloor \equiv (1 - q^r)/(1 - q)$ are used. For an operator A, the q-bracket $\lfloor A \rfloor$ means formal series. As the q-parameter $q \to 1$, the $\lfloor r \rfloor$ resp. $\lfloor A \rfloor$ goes back to r resp. A.

For $-1 \le q \le 1$ the operators $a_i^\dagger$, a_i are conjugate to each other. Note that $a_i^\dagger a_i$ *depends on the number operator* $\mathcal{N}_i$ *nonlinearly* :

$$a_i^\dagger a_i = \lfloor \mathcal{N}_i \rfloor, \qquad (2)$$

and only at $q = 1$ the familiar equality $a_i^\dagger a_i = \mathcal{N}_i$ is recovered.

• q-Deformed oscillators of the BM (Biedenharn-Macfarlane) type. *(1989)*

$$b_i b_j^\dagger - q^{\delta_{ij}} b_j^\dagger b_i = \delta_{ij} q^{-N_j} \qquad [b_i, b_j] = [b_i^\dagger, b_j^\dagger] = 0 \qquad (3)$$

$$[N_i, N_j] = 0 \qquad [N_i, b_j] = -\delta_{ij} b_j \qquad [N_i, b_j^\dagger] = \delta_{ij} b_j^\dagger .$$

The (q-)deformed Fock space is constructed likewise, but now, instead of basic numbers, we use another q-bracket (and "q-numbers"):

$$b_i^\dagger b_i = [N_i]_q \qquad [r]_q \equiv \frac{q^r - q^{-r}}{q - q^{-1}} \,. \qquad (4)$$

The equality $b_i^\dagger b_i = N_i$ is recovered only if $q = 1$ ("no-deformation"). For consistency of conjugation, put

$$q = \exp(i\theta) \qquad 0 \le \theta < \pi \,. \qquad (5)$$

• (System of independent) generalized qp-deformed oscillators: (Ch. & J, 1991)

$$AA^{\dagger} - qA^{\dagger}A = p^{N} \qquad AA^{\dagger} - pA^{\dagger}A = q^{N} \tag{6}$$
$$[N^{(qp)}, A] = -A \qquad [N^{(qp)}, A^{\dagger}] = A^{\dagger}$$

where only one mode is shown. For qp-deformed oscillators we have

$$A^{\dagger}A = [\![N^{(qp)}]\!]_{qp}\,, \qquad [\![X]\!]_{qp} \equiv \frac{q^{X} - p^{X}}{q-p}\,. \tag{7}$$

At $p=1$ the AC-type q-bosons are recovered, while putting $p=q^{-1}$ recovers the BM-case.

Statistical q-deformed distributions

For the dynamical multi-particle (multi-pion, multi-kaon, ...) system, we adopt the model of ideal gas of q- or qp-bosons. The Hamiltonian

$$H = \sum_i \omega_i \mathcal{N}_i \qquad \omega_i = \sqrt{m^2 + \mathbf{k}_i^2} \tag{8}$$

is such that $\mathcal{N}_i$ is one of the above three versions of the number operator; the subscript $'i'$ labels different modes. This choice is the unique truly non-interacting one, with an additive spectrum. We assume the $\mathbf{3}$-momenta of particles take discrete values (i.e., the system is contained in a large finite box of volume $\sim L^3$).

To describe statistical properties, we evaluate thermal averages

$$\langle A \rangle = \frac{\mathrm{Sp}(A\rho)}{\mathrm{Sp}(\rho)} \qquad \rho = e^{-\beta H}$$

where $\beta = 1/T$, the Boltzmann constant is set equal to 1.

Calculating, say, for AC-type q-bosons the thermal average $\langle q^{\mathcal{N}_i}\rangle$ with the chosen Hamiltonian, we obtain

Altherr & Grandou (93)
Vokos & Zachos (94)

$$\langle q^{\mathcal{N}_i}\rangle = \frac{e^{\beta\omega_i} - 1}{e^{\beta\omega_i} - q}\,.$$

The distribution function (for $-1 \leq q \leq 1$) is found to be

AC:
$$\langle a_i^\dagger a_i \rangle = \frac{1}{e^{\beta\omega_i} - q} \,. \tag{9}$$

If $q \to 1$, this is the usual Bose-Einstein distribution. At $q = -1$ or $q = 0$, the distribution function yields formally Fermi-Dirac or classical Boltzmann cases. Clearly, the defining relations (1) at $q = -1$ differ from those for the system of genuine fermions. The difference with fermions lies in the *commuting* (VS anticommuting !) *non-coinciding* modes at $q = -1$.

For BM-type of q-bosons, the Hamiltonian is taken similarly, with the relevant number operator: $H = \Sigma_i\, \omega_i N_i$.

Calculate $\langle q^{\pm N_i} \rangle$ to get $\langle q^{\pm N_i} \rangle = (e^{\beta\omega_i} - 1)/(e^{\beta\omega_i} - q^{\pm 1})$. With the formula $\langle b_i^\dagger b_i \rangle = (e^{\beta\omega_i} - q)^{-1} \langle q^{-N_i} \rangle$ the q-distribution function (with $q + q^{-1} = [2]_q = 2\cos\theta$) is then found:

BM:
$$\langle b_i^\dagger b_i \rangle = \frac{e^{\beta\omega_i} - 1}{e^{2\beta\omega_i} - 2\cos\theta\; e^{\beta\omega_i} + 1} \,. \tag{10}$$

This q-distribution function is real, for real or *complex* parameter q. The q-distribution $f(\mathbf{k}) \equiv \langle b^\dagger b \rangle(\mathbf{k})$ in (10) is suth that at $q \neq 1$ the distribution function lies in between the other two curves, Bose-Einstein one and the classical Boltzmann one. The same is true of the q-distribution function (9) of the AC-type q-bosons.

Generalized (qp-deformed) one-particle *distribution function for* qp-*bosons* in momentum space is of the form Daoud & Kibler (95)

qp-bos.:
$$\langle A_i^\dagger A_i \rangle = \frac{(e^{\beta\omega_i} - 1)}{(e^{\beta\omega_i} - p)(e^{\beta\omega_i} - q)} \,. \tag{11}$$

It contains the above q-distributions (9) resp. (10) as particular cases: at $p = 1$ resp. $p = q^{-1}$.

n-Particle correlations of qp-bosons

Let us give most general results [Adamska & A.G.] for the q,p-Bose gas model (based on qp-oscillators). With the Hamiltonian

$$H = \sum_i \omega_i \; N_i^{(qp)} \tag{12}$$

the n-particle distribution functions have been derived as

$$\langle (A_i^\dagger)^n (A_i)^n \rangle = \frac{[\![n]\!]_{qp}! \; (e^{\beta\omega_i} - 1)}{\Pi_{r=o}^n (e^{\beta\omega_i} - q^r p^{n-r})} \tag{13}$$

$$[\![m]\!]_{qp} \equiv \frac{q^m - p^m}{q - p}, \quad [\![m]\!]_{qp}! = [\![1]\!]_{qp}[\![2]\!]_{qp} \cdots [\![m-1]\!]_{qp}[\![m]\!]_{qp} .$$

From (13), we get the general result for the n-th order qp-deformed extension of the intercept $\lambda_{q,p}^{(n)} \equiv -1 + \frac{\langle A^{\dagger n} A^n \rangle}{\langle A^\dagger A \rangle^n}$ ('i' omitted):

$$\lambda_{q,p}^{(n)} = [\![n]\!]_{qp}! \; \frac{(e^{\beta\omega} - p)^n (e^{\beta\omega} - q)^n}{(e^{\beta\omega} - 1)^{n-1} \, \Pi_{k=0}^n (e^{\beta\omega} - q^{n-k} p^k)} - 1 \tag{14}$$

which constitutes our main result, being generalization to the n-th order of correlations, and to the two-parameter (qp-) deformation.

Consider the asymptotics $\beta\omega \to \infty$ (large momenta or, at fixed momentum, low temperature) of the intercepts $\lambda_{q,p}^{(n)}$:

$$\lambda_{q,p}^{(n), \text{ asympt}} = -1 + [\![n]\!]_{qp}! = -1 + \prod_{k=1}^{n-1} \Big(\sum_{r=0}^{k} q^r p^{k-r} \Big) . \tag{15}$$

For each case (AC-type q-bosons, BM-type, the qp-generalization) the asymptotics of n-th order intercept is given by the corresponding extension of the usual n-factorial (the latter yields the intercep of pure Bose-Einstein n-particle correlator).

If $n = 2$ (then, $[\![2]\!]_{qp} = p + q$) this specializes to the formula

$$\langle (A_i^\dagger)^2 (A_i)^2 \rangle = \frac{(p+q) \; (e^{\beta\omega_i} - 1)}{(e^{\beta\omega_i} - q^2)(e^{\beta\omega_i} - pq)(e^{\beta\omega_i} - p^2)} .$$

In particular, for AC type q-bosons, the n-particle distribution function

$$\langle (a_i^\dagger)^n (a_i)^n \rangle = \frac{\lfloor n \rfloor !}{\Pi_{r=1}^n (e^{\beta\omega_i} - q^r)}$$

$$\lfloor m \rfloor \equiv \frac{1-q^m}{1-q} = 1 + q + q^2 + ... + q^{m-1}$$

yields the desired formula for the intercept $\lambda^{(n)} \equiv \frac{\langle a^{\dagger n}\ a^n \rangle}{\langle a^\dagger a \rangle^n} - 1$ of n-particle correlations ('i' dropped):

$$\lambda_{\mathrm{AC}}^{(n)} = -1 + \frac{\lfloor n \rfloor !\ (e^{\beta\omega} - q)^{n-1}}{\Pi_{r=2}^n (e^{\beta\omega} - q^r)}\ . \qquad (16)$$

Asymptotically, at $\beta\omega \to \infty$ the result gets dependent solely on the q-parameter:

$$\lambda_{\mathrm{AC}}^{(n)} \overset{\text{asympt}}{=} -1 + \lfloor n \rfloor ! = -1 + \prod_{k=1}^{n-1} \Big(\sum_{r=0}^{k} q^r \Big)$$

$$= (1+q)(1+q+q^2)\cdots(1+q+\ldots+q^{n-1}) - 1. \qquad (17)$$

This remarkable fact can serve as the test one when confronting the developed approach with the numerical data for pions and kaons extracted from the experiments on relativistic heavy ion collisions.

Two- and three-pion correlations of q-bosons

Two-particle correlations of the AC-type q-bosons. From the above, we have the (monomode) formula

$$\langle a_i^\dagger a_i^\dagger a_i a_i \rangle = \frac{1+q}{(e^{\beta\omega_i} - q)(e^{\beta\omega_i} - q^2)}$$

from which the "intercept" $\lambda_i^{(2)} \equiv \frac{\langle a_i^\dagger a_i^\dagger a_i a_i \rangle}{\langle a_i^\dagger a_i \rangle^2} - 1$ follows:

$$\lambda_{AC}^{(2)} = -1 + \frac{(1+q)(e^{\beta\omega} - q)}{e^{\beta\omega} - q^2} = q \frac{e^{\beta\omega} - 1}{e^{\beta\omega} - q^2}\ . \qquad (18)$$

$\lambda_{AC}^{(2),\ \text{asymp.}} = q$

Three-particle correlations of the AC-type q-bosons.

The 3-particle monomode distribution function

$$\langle a_i^\dagger a_i^\dagger a_i^\dagger a_i a_i a_i \rangle = \frac{(1+q)(1+q+q^2)}{(e^{\beta\omega_i}-q)(e^{\beta\omega_i}-q^2)(e^{\beta\omega_i}-q^3)}$$

leads to the formula for the intercept, or "strength", $\lambda^{(3)}$ of 3-particle correlation function ('i' dropped):

$$\lambda^{(3)}_{\rm AC} \equiv \frac{\langle a^{\dagger 3} a^3 \rangle}{\langle a^\dagger a \rangle^3} - 1 = \frac{(1+q)(1+q+q^2)(e^{\beta\omega}-q)^2}{(e^{\beta\omega}-q^2)(e^{\beta\omega}-q^3)} - 1. \tag{19}$$

$$\lambda^{(3),\,\text{asymp.}}_{AC} = q(q^2+2q+2).$$

Two-particle correlations of the BM-type q-bosons.

The 2-particle distribution for BM-type q-bosons is

$$\langle b_i^\dagger b_i^\dagger b_i b_i \rangle = \frac{2\cos\theta}{e^{2\beta\omega_i} - 2\cos(2\theta)e^{\beta\omega_i} + 1}\,.$$

From this, we get the intercept of 2-particle correlation function:

$$\lambda_i = -1 + \frac{\langle b_i^\dagger b_i^\dagger b_i b_i \rangle}{(\langle b_i^\dagger b_i \rangle)^2} = \frac{2\cos\theta(t_i+1-\cos\theta)^2}{t_i^2 + 2(1-\cos^2\theta)t_i} - 1 \tag{20}$$

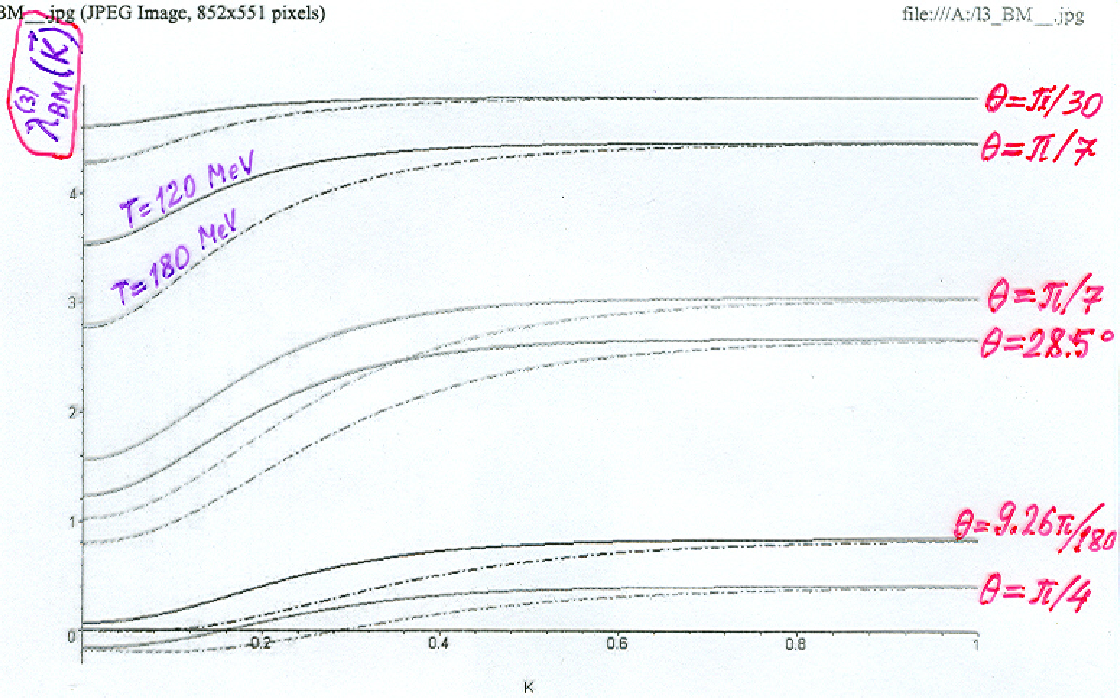
where $t_i \equiv \cosh(\beta\omega_i) - 1$, and again it is a real function.

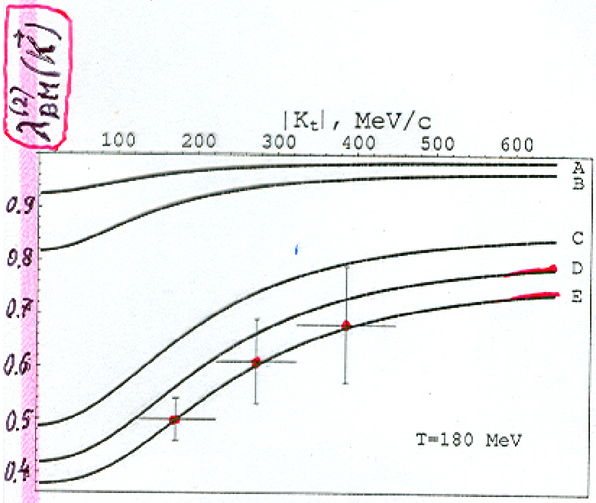
Three-particle correlations of the BM-type q-bosons.

Finally, we specialize the obtained general result to gain the formulas for $n = 3$ case of BM type q-bosons:

$$\lambda^{(3)}_{\rm BM} = -1 + \frac{[2]_q[3]_q\,(e^{2\beta\omega} - 2e^{\beta\omega}\cos\theta + 1)^2}{(e^{\beta\omega}-1)^2(e^{2\beta\omega} - 2e^{\beta\omega}\cos(3\theta) + 1)} \tag{21}$$

$$\lambda^{(3),\,\text{asymp.}}_{\rm BM} = -1 + [2]_q[3]_q = -1 + 2\cos\theta\,(4\cos^2\theta - 1). \tag{22}$$

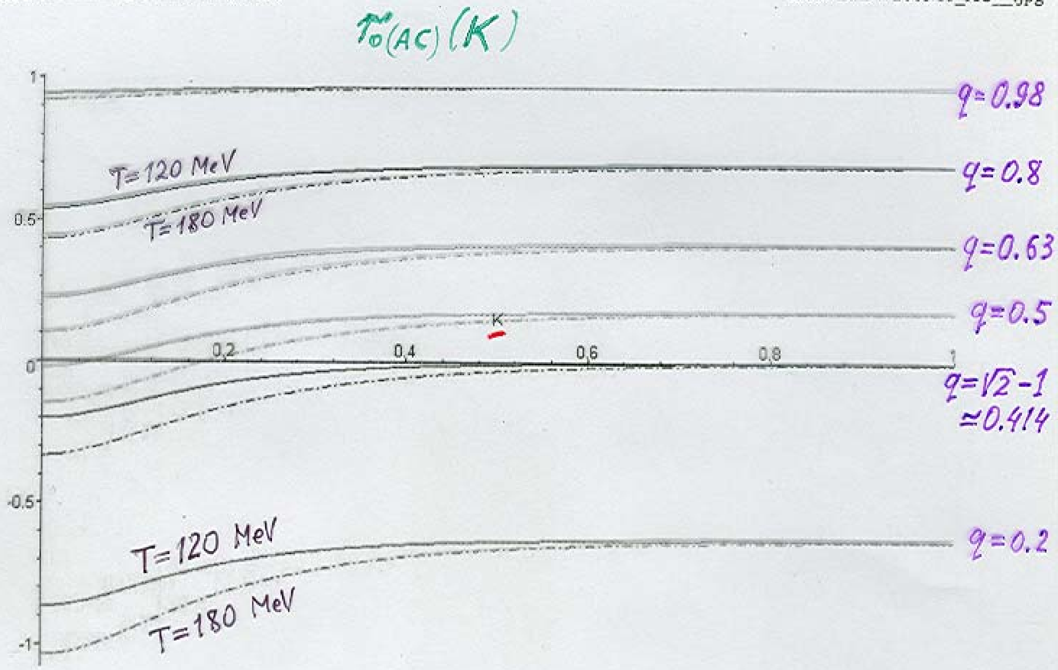


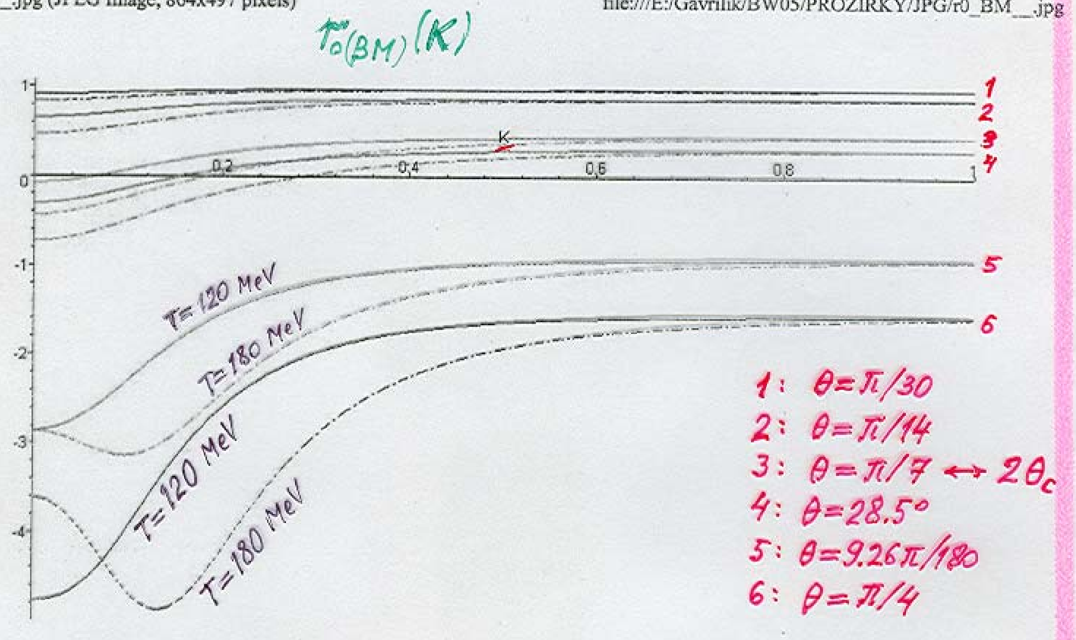


125÷225, 225÷325, 325÷450
(MeV/c)

A) $\theta = 6° = \pi/30$
B) $\theta = 10° = \pi/18$
C) $\theta = 22°$
D) $\theta = 25.7°$ (i.e. $2\theta_c$) $= \pi/7$
E) $\theta = 28.5°$

r0_AC__.jpg (JPEG Image, 803x518 pixels) file:///E:/BW2005/r0_AC__.jpg




$r_0(BM)$ (K)
1
0
-1
-2
-3
-4
0.2
0.4
0.6
0.8
1
K
T=120 MeV
T=180 MeV
T=120 MeV
T=180 MeV
1: $\theta=\pi/30$
2: $\theta=\pi/14$
3: $\theta=\pi/7 \leftrightarrow 2\theta_c$
4: $\theta=28.5^\circ$
5: $\theta=9.26\pi/180$
6: $\theta=\pi/4$

Comparison with experimental data

To confront the obtained results with the existing data for 3-particle correlations of pions or kaons produced and registered in the experiments on relativistic heavy ion collisions, usually the following combination is taken [Heinz & Zhang - 97]:

$$r^{(3)}(p_1,p_2,p_3) \equiv$$

$$\equiv \frac{C^{(3)}(p_1,p_2,p_3)-C^{(2)}(p_1,p_2)-C^{(2)}(p_2,p_3)-C^{(2)}(p_3,p_1)+2}{2\sqrt{(C^{(2)}(p_1,p_2)-1)\,(C^{(2)}(p_2,p_3)-1)\,(C^{(2)}(p_3,p_1)-1)}}$$

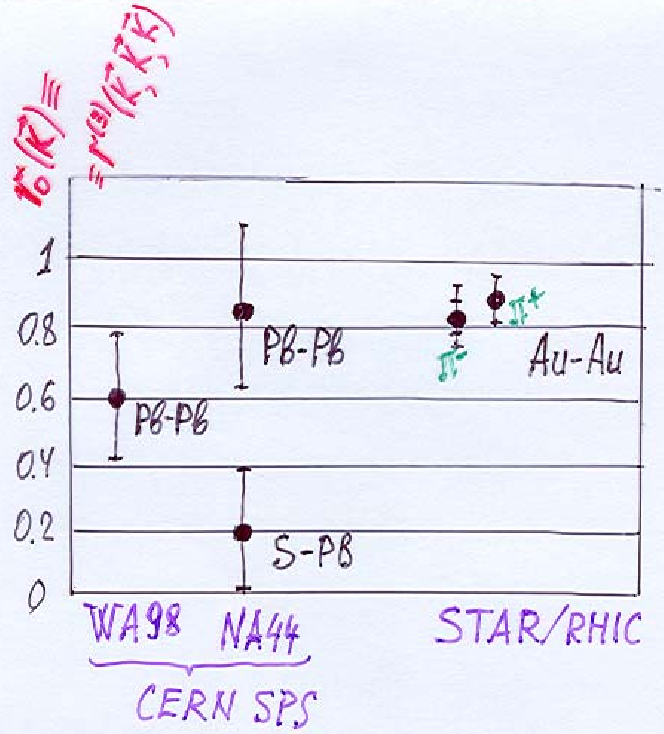
as well as the characteristic quantity formed from intercepts (set $p_1 = p_2 = p_3 = K$)

$$r_0(K) \equiv r^{(3)}(K,K,K) = \frac{1}{2}\,\frac{\lambda^{(3)}(K) - 3\lambda^{(2)}(K)}{\left(\lambda^{(2)}(K)\right)^{3/2}}, \qquad (23)$$

where

$$\lambda^{(3)}(K) = C^{(3)}(K,K,K)-1, \qquad \lambda^{(2)}(K) = C^{(2)}(K,K)-1,$$

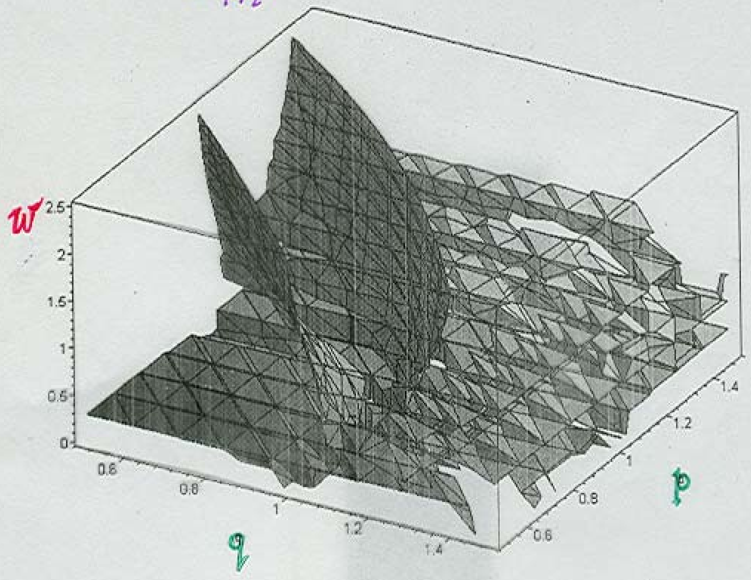
either of AC-, or BM-type, or qp-deformed version.



$r_0(\vec{K}) \equiv$
$\equiv r^{(3)}(\vec{K}, \vec{K}, \vec{K})$
1
0.8
0.6
0.4
0.2
0
Pb-Pb
Pb-Pb
S-Pb
π^+
π^-
Au-Au
WA98
NA44
CERN SPS
STAR/RHIC

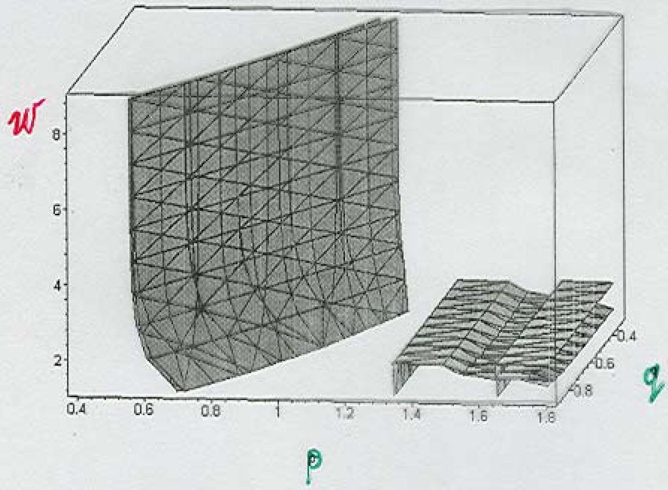
$$\lambda^{(2)}_{p,q}(w) = 0.44 \pm 0.04$$

$$\lambda^{(3)}_{p,q}(w) = 1.35 \pm 0.12$$



$\lambda^{(2)}_{p,q}(w) = \underline{0.44} \pm 0.04$

$\lambda^{(3)}_{p,q}(w) = \underline{1.35} \pm 0.12$



Refs.

Quant. algebras in hadron phenomenology:

A.G., Nucl. Phys. B (Pr. Suppl.) 102, 298 (2001) [hep-ph/0103325]

Proposal to use model of q-Bose gas in the context of 2-pion/-kaon correlations (observed non-Bose kind of behavior) in r.h.i.c's:

1) D.Anchishkin, A.G., N.Iorgov, Eur. Ph. J. A7 229 (2000)
" " " MPL A15 1637 (2000)

2) D.Anchishkin, A.G., S.Panitkin, Ukr. J.Ph. (2004 [hep-ph/0112262]

qp-generalization:

3) L.Adamska, A.G., J. Phys. A (2004) [hep-ph/0312390]