# A Correction to the RR Tadpole Cancellation Condition

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based on a collaboration with:

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# Type II Supergravity (Classical Case)

## Gauge Invariance

$$C_p \to C_p + d\Lambda_{p-1} + H \wedge \Lambda_{p-3}$$

Two Notions of Fieldstrength

$F_{p+1} = dC_p + H \wedge C_{p-2}$ is Gauge-Invariant

$G_{p+1} = dC_p$ is closed, and in the quantum case Quantized

We will define charges using the Bianchi identity

$$\rho_{D(7-p)\text{-brane}} = dG_{p+1} = dF_{p+1} + H \wedge G_{p-1}$$

Integrating over a compact $(p+2)$-cycle
and interpreting $H, G$ as de Rham cohomology classes

$$\boxed{Q_{D(7-p)} = H \wedge G_{p-1}}$$

used Stoke's Theorem

$F_{p+1}$ is gauge-invar $\therefore$ globally defined

$$\int_M dF_{p+1} = \int_{\partial M = \emptyset} F_{p+1} = 0$$

## The Dirac Quantization Condition:

Well-definedness of Dp-brane partition function implies

$$\int G_{p+2} \in \mathbb{Z}$$

To classify fieldstrengths if we don't care about Wilson lines or fluxes that are exterior derivatives of connections satisfying the quantization condition: Classify $G_{p+2}$ by integral $H^{p}$

$$H^{p+2}_{DR}(M) \longrightarrow H^{p+2}(M, \mathbb{Z})$$

$$\mathbb{R}^k \to \mathbb{Z}^k \oplus \bigoplus_{i=1}^{n} \mathbb{Z}_{m_i}$$

Physically this has two consequences:

(1) Only consider the subset of classical branes that have integral charges

(2) Get new torsion branes that are unstable in the classical theory but are stable in the quantum theory

Example: Type IIB on $\mathbb{RP}^3 \times \mathbb{R}^{6,1}$

$$H^3_{DR}(\mathbb{RP}^3) = H^0_{DR}(\mathbb{RP}^3) = \mathbb{Z}$$

$$H^1_{DR}(\mathbb{RP}^3) = H^2_{DR}(\mathbb{RP}^3) = 0$$

So in the classical theory one finds, for example:

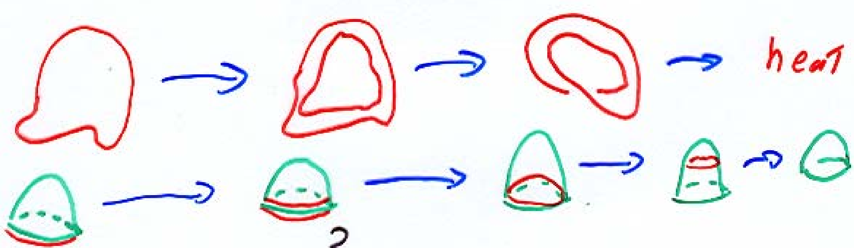
D1-branes that are a point on $\mathbb{RP}^3$ and $\mathbb{R}^{1,1} \subset \mathbb{R}^{6,1}$

D3-branes that wrap $\mathbb{RP}^3$ and extend in time

Recall that $\mathbb{RP}^3$ contains a nontrivial 1-cycle $S$ such that $2S$ is homotopic to $\emptyset$

In the classical theory a brane wrapped around $S$ is not stable ($H^1_{DR}=0$)

It can decay by turning into 2 branes of half the original charge. This is a half-brane wrapping $2S$ which is contractible, so it decays.



## The Quantum Case

$$H^0(\mathbb{RP}^3;\mathbb{Z}) = H^3(\mathbb{RP}^3;\mathbb{Z}) = \mathbb{Z}$$

$$H^1(\mathbb{RP}^3;\mathbb{Z}) = 0 \qquad H^2(\mathbb{RP}^3;\mathbb{Z}) = \mathbb{Z}_2$$

To classify branes, instead of fluxes, we need the homology groups (in the deRham case there was an isomorphism so the distinction was not important)

$$H_0(\mathbb{RP}^3;\mathbb{Z}) = H_3(\mathbb{RP}^3;\mathbb{Z}) = \mathbb{Z}$$

$$H_1(\mathbb{RP}^3;\mathbb{Z}) = \mathbb{Z}_2 \qquad H_2(\mathbb{RP}^3;\mathbb{Z}) = 0$$

So in this classification a brane wrapping S is nontrivial, but a charge 2 brane wrapping S is trivial.

The decay seen in the classical case is forbidden for odd-charged branes because of the Dirac quantization condition.

<u>Claim</u>: The Bianchi identity in the quantum case is:

$$Q_{D3} = G_3 \cup H + G_3 \cup G_3 + \overbrace{H \cup H}^{\text{missed by K-theory}} + P$$

for the D3-brane charge, where $P$ is independent of $G_3$ and $H$

Note that in the classical case

$$X_3 \cup X_3 = X_3 \wedge X_3 = -X_3 \wedge X_3 = 0$$

but in integral cohomology

$$X_3 \cup X_3 = -X_3 \cup X_3 \Rightarrow X_3 \cup X_3 \text{ is a } \mathbb{Z}_2\text{-class}$$

The cup product $\cup$ is just a generalization of the wedge product to integral cohomology.

The goal of this talk will be to use the Freed-Witten anomaly to prove the above claim.

Lower dim brane charges from WZ Terms

if $Q_{Dp}$ is $Dp$-charge then

$$S^{WZ}_{Dp} = Q_{Dp} \int e^{B+F} \sum_k C_{2k} = Q_{Dp} \int C_{p+1} + (B+F)\, C_{p-1} + \cdots$$

In particular if

$$\int F = a \quad \text{and} \quad Q_{Dp} = 1$$

then the $Dp$ carries $a$ units of $D(p-2)$ charge

$$S_{Dp} \supset Q_{Dp} \int F C_{p-1} = a \int C_{p-1} \subset S_{D(p-2)} \text{ (charge } a\text{)}$$

What about the $BC_{p-1}$ term? It's not quantized.

Wati Taylor has shown that it's contribution to $D(p-2)$ charge is precisely canceled by a bulk contribution $H\, C_{p-1}$ integrated over a space bounded by the brane.

But if the topology is nontrivial there are inequivalent choices of this bounded space $\Rightarrow \int H$ ambiguity

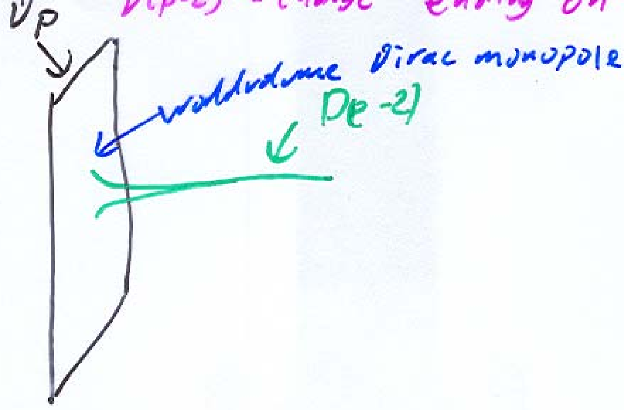
What happens when $\oint H \neq 0$ ?

$B+F$ is gauge invariant

Therefore

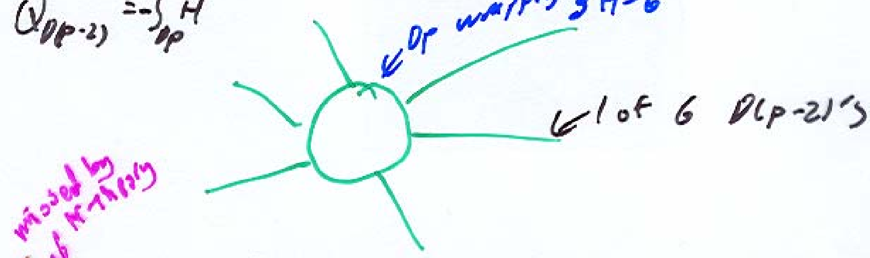
$$\int_M H + dF = \int_{\partial M_{op}} B+F = 0$$

$$\underbrace{\int dF}_{\text{D(p-2) - charge ending on Dp}} = -\int H$$




__Conclusion:__ In the classical theory a Dp wrapping K units of H flux will have K D(p-2)'s ending on it.

$$Q_{D(p-2)} = -\int_{Dp} H$$




Similarly (S-duality) an NS5 wrapping $\int G_3 = K$ will have K D3 insertions.

<u>The Quantum Theory</u>: $Z_{F\text{-string}} \supset e^{i(B+F+w_2)}$

So well-definedness of the partition function implies:

$$\int_{Dp} d(B+F+w_2) = \int_{Dp} H + dF + w_3 = Q_{D(p-2)} + \int_{Dp} H + w_3$$

$$= \int_{\partial(Dp)=\emptyset} B+F+w_2 = 0$$

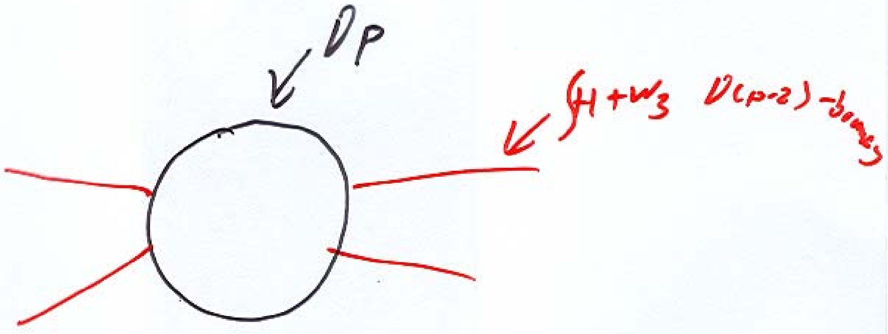
← well, it needs to be integral, but the classical argument kills the integral part

Therefore in the quantum case

$$Q_{D(p-2)} = -\int_{Dp} H + W_3$$

And by s-duality

$$Q_{D3} = -\int_{NS5} G_3 + w_3$$



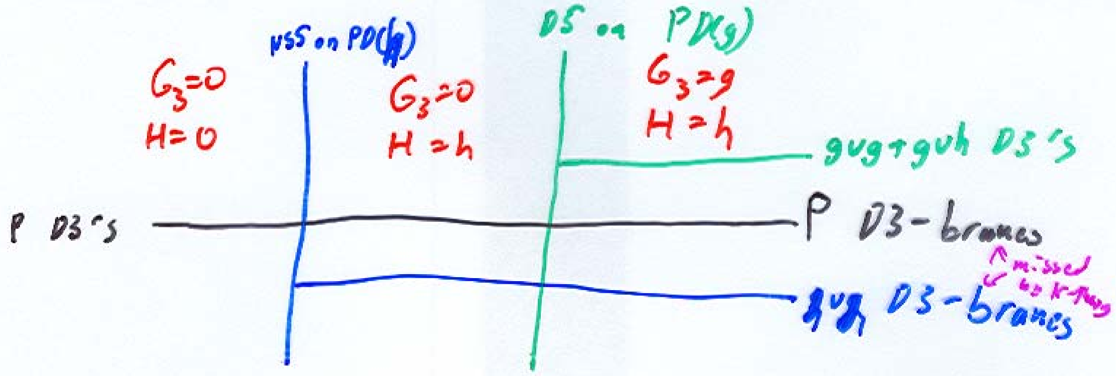
Strategy: Consider IIB on $\mathbb{R} \times M^9$ (not necessary)

Turn on 3-form fluxes using dual 5-branes

These 5-branes will have FW anomalies

which will be canceled by D3 insertions

which change the D3 charge.




Used the fact from topology

$$\int_{PD(x_3)} W_3 \cup Y = \int_{PD(x_3 \cup x_3)} Y$$

or 'equivalently

$$i : PD(x_3) \to M^9$$

$$x_3 \cup x_3 = Sq^3 x_3 = i_* W_3(N\,PD(x_3))$$

# Generalizations

Sometimes the above $i_*$ has a kernel
$\Rightarrow$ anomaly is canceled by D(p-4)-branes

There is no known explicit form for
the D(p-4)-charge

If we can find this charge, we
can work out generalizations to lower
dim D-brane charges

Generalization to cases with orientifolds

and with orbifold fixed points is

probably nontrivial but also unknown.

For CY 3-fold $H^6(CY^6) = \mathbb{Z}$
no $\mathbb{Z}_2$ torsion so our corrections
are trivial. Maybe nontrivial in
orientifold case.