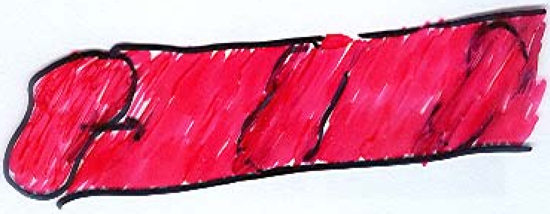
STRiNG AND FiELD THEORY LANDSCAPES

1) String theory in ten and four dimensions
2) The landscape picture of string theory
   - a phenomenological proposal: split supersymmetry
3) Moduli stabilization
4) Field theory landscapes

BW 2005
19/05/05

1) String theory in ten and four dimensions



string oscillators = particles, of mass prop. to

$2\pi T = \frac{1}{\alpha'} = M_s^2$

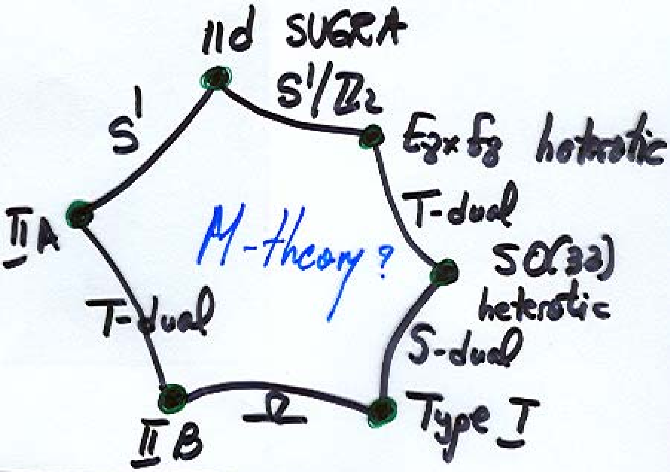
T= string tension

M_s= string scale

Stability => ten dimensional superstrings

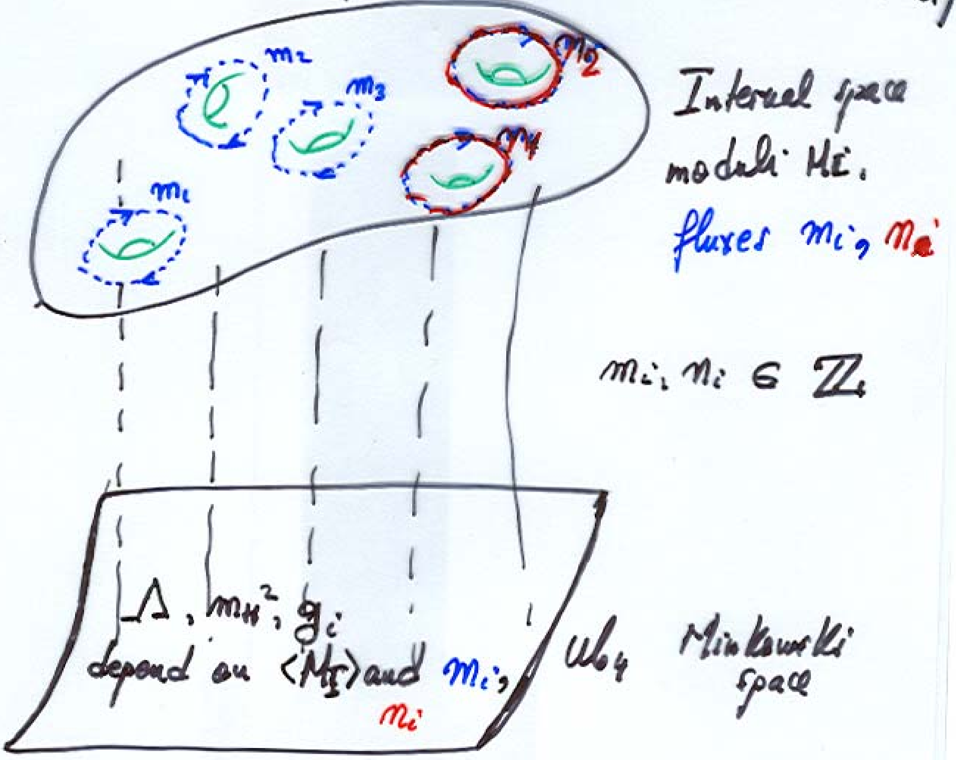
$\alpha' \to 0$ limit => supergravity (local SUSY) theory:
gravity + Yang-Mills

The "landscape" of 10d superstrings & dualities



$\boxed{\mathcal{M}_{10} = \mathcal{M}_4 \times K_6}$ internal space

String theory has an internal space which can support various types of fluxes (talk Behrndt)



Fluxes are quantized and typically are bounded

Ex. $m_i = n_i$ and $\boxed{\sum_i m_i^2 = |Q|_{D3}}$

$\Rightarrow$ a finite (eventually very large) number of fluxes $\longleftrightarrow$ N vacua $N \sim 10^{100} \div 10^{500}$

(M. Douglas & coll.)

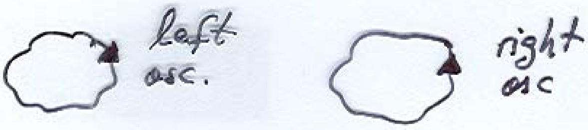
simple estimates

$\Rightarrow$ "LANDSCAPE"

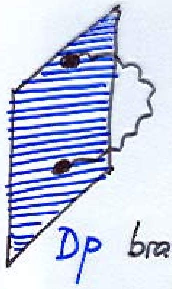
A dictionary for strings/branes

- Type II strings = 10d superstrings containing

closed strings



open strings
and D-branes



$$\mathcal{L}_{Dp} = T_p \int d^{p+1}\xi \sqrt{\det \gamma}\; e^{-\phi} + q_p \int_{Dp} A_{p+1}$$

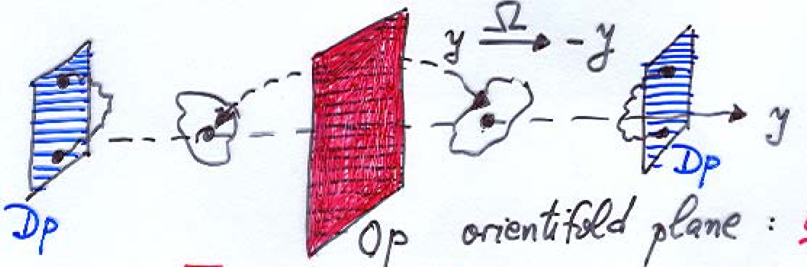
Tension > 0 dilaton Ramond-Ramond charge

$F_{p+2} = dA_{p+1}$ = generalization of $\vec{E}, \vec{B}$ in electromagnetism

- Orientifolds of type II = II / Ω

identifies left and right oscillators

and generate " mirrors "



orientifold plane : non-dynamical

$T_{O_p} < 0$, $q_{O_p} < 0$ (generically)

- a simple example of flux: internal magnetic fields
  ⟺ intersecting branes (talk Blumenhagen)

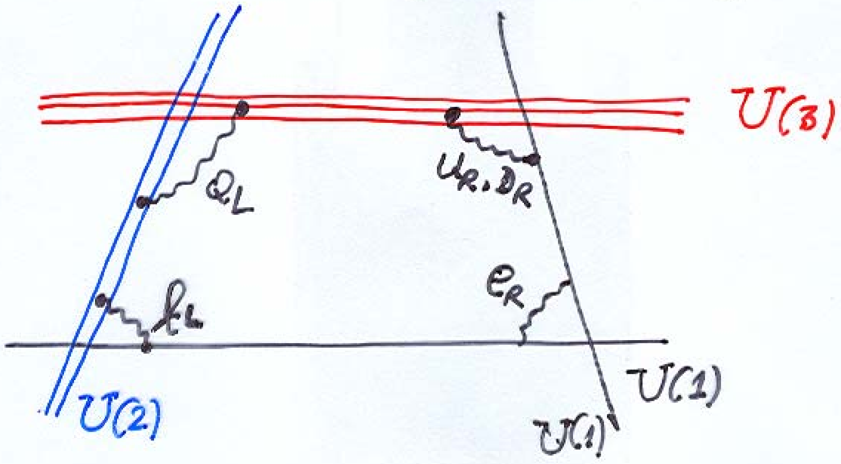
String theory can be exactly quantized in this setup: magnetic fields couple to open string end points

$$\delta M^2 = \sum_i (2k_i+1)\,|\theta_L^i+\theta_R^i| + 2\sum_i(\theta_L^i+\theta_R^i)\,\underset{\uparrow}{\Sigma_i} \quad ,$$

internal helicity

where $\theta_L^i = \operatorname{arctan}(q_L H_i) \xrightarrow[\text{field limit}]{\text{weak}} q_L H_i$

Semi-realistic constructions (Berlin group; Madrid group; Cvetic, Shiu e Uranga, ...)



- Need (at least) 4 stacks of branes
- Hyperchage = linear combination of the 4 U(1)'s, the 3 others are massive
- Spectrum larger than MSSM: right-handed neutrinos, enlarged Higgs sector

Some properties of internal magnetic fields
↕
intersecting branes

- easy to generate 4d chirality, even in simplest (toroidal) compactifications

Ex: 6d spacetime = $\mathcal{M}_4 \times T^2$

internal magnetic field $H_{56} \equiv H$

The index theorem implies

$$\boxed{n_L - n_R \sim \int_{T^2} H_{56}}$$

\# massless left-handed fermions

\# massless right-handed fermions

- SUSY easy to achieve, due to the spin-magnetic field couplings $\vec{\Sigma} \cdot \vec{H}$

internal helicity

$$\boxed{m_F^2 - m_B^2 \sim (\vec{\Sigma}_F - \vec{\Sigma}_B)\,\vec{H}}$$

2) THE LANDSCAPE PICTURE OF STRING THEORY

(Bousso & Polchinski ; Douglas ; Susskind)

- Multiple 4-form fluxes and cosmological constant

$$S = \int d^4x \sqrt{g} \left\{ \frac{1}{2k^2} R - \Lambda_{bare} - \frac{Z}{2\cdot 4!} F_4^2 \right\} + e \underset{\substack{wb_3 \\ \uparrow \\ \text{membranes action}}}{\int} A_3$$

constant flux $\quad F^{\mu\nu\rho\sigma} = \underset{\substack{\uparrow \\ \text{constant}}}{c} \cdot \varepsilon^{\mu\nu\rho\sigma}$ is a solution of the eqs. of motion

$\Delta c = \frac{e}{Z}$ = jump of c across a membrane

$$\boxed{\Lambda = \Lambda_{bare} + \frac{Z}{2} c^2}$$

but c is quantized

$$c = \frac{e}{Z} \cdot \overset{\in \mathbb{Z}}{n} \equiv q \cdot n \quad , \text{ since}$$

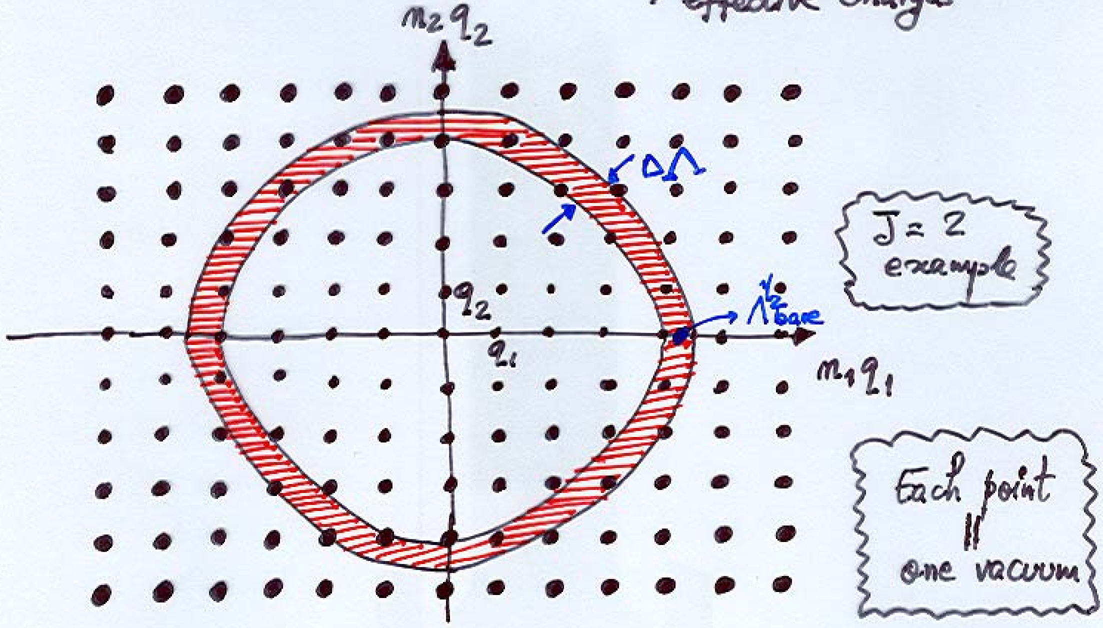
$$\int_{X_4} F_4 = \frac{2\pi n}{e} \xrightarrow[\text{condition}]{\text{Dirac quant.}} F_4 = F_0 = c = \frac{e}{Z} \cdot n$$

Generalization to multiple (= J) fluxes

$$\boxed{\Lambda = \Lambda_{bare} + \frac{1}{2}\sum_{i=1}^{J} n_i^{\,2} q_i^{\,2}}$$

$n_i \in \mathbb{Z}$

effective charges



Accommodation of a small cosm. constant ⇒ at least one point in the red area ⇒

$$\boxed{Vol_{red\ area} \sim \frac{2\pi^{J/2}}{\Gamma(\frac{J}{2})}\,|2\Lambda_{bare}|^{\frac{J}{2}-1}\,\overset{\nearrow\, 10^{-120} M_P^4}{\Delta\Lambda} \gtrsim \prod_{i=1}^{J} q_i}$$

* The bigger the radius (Λ_{bare}) and # fluxes J, the bigger the red volume ⇝ the total # of vacua is huge, eventually ~ 10^{120}

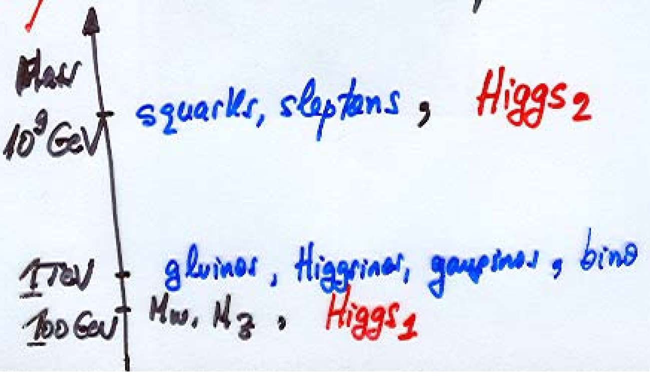
Low-energy physics

$$\left.\begin{matrix}\Lambda,\ m_H^2,\ g_a, \\ \text{quark masses}, \ldots\end{matrix}\right\} = \text{functions of } \langle M_I \rangle,\ m_i,\ n_i$$

Some questions:

- statistical distribution of Λ, m_H^2, g_a, ... in the landscape
- $\frac{N_{SM}}{N}$, where N_{SM} = number of the Standard Model like vacua
- Statistical distribution of the supersymmetry breaking scale, of discrete symmetries

This picture did inspire a new phenomenological scenario: split supersymmetry (Arkani-Hamed & Dimopoulos)



Requirements:

- large scale of SUSY
- $\tilde{m}_i^2, m_{H_1}^2, m_{H_2}^2, B\mu \gg$ other soft terms
- fine-tuning of the light Higgs

Outcome

- Gaugino, Higgsinos much lighter than the squarks and the sleptons
- Gauge coupling unification and dark matter similar to MSSM
- no flavor problems of low-energy SUSY
- gauginos & gluinos very long-lived $\Rightarrow$ exotic hadrons ?

... but explicit realizations difficult to achieve

3) MODULI STABILIZATION

Calabi-Yau compactification of Type IIB orientifolds with D3 & D7 branes with 3-form fluxes in the internal space

$$\frac{1}{2\pi\alpha'}\int F_3 \in \mathbb{Z} \quad , \quad \frac{1}{2\pi\alpha'}\int H_3 \in \mathbb{Z}$$

RR 3-form flux

NS-NS 3-form flux

$$G_3 = F_3 - i\, S\, H_3 \quad \Rightarrow$$

dilaton-axion field

$$\boxed{W(S, U_i) = \int G_3 \wedge \Omega} = A(U_i) + B(U_i)\cdot S$$

complex structure (shape) moduli

holomorphic (3,0) form of the CY space

$D_S W = D_{U_i} W = 0 \Rightarrow$ S and U_i are stabilized

(Giddings, Kachru & Polchinski)

- Fluxes can break partly or completely SUSY
- String quantization not available, SUGRA methods

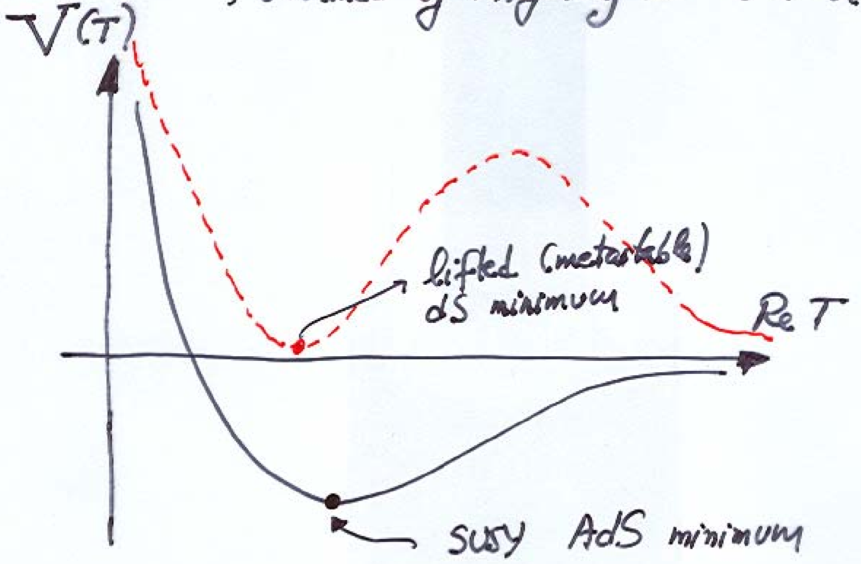
- Kähler (volume-type) moduli stabilisation needs additional ingredients

Ex: Kachru, Kallosh, Linde & Trivedi add gaugino condensation on D7 branes ⇒

$$\begin{cases} K = -3\log(T+\bar{T}) \\ W = W_0 + h\cdot e^{-b\cdot T} \end{cases}$$

gaugino condensation

obtained by integrating out S & U_i fields



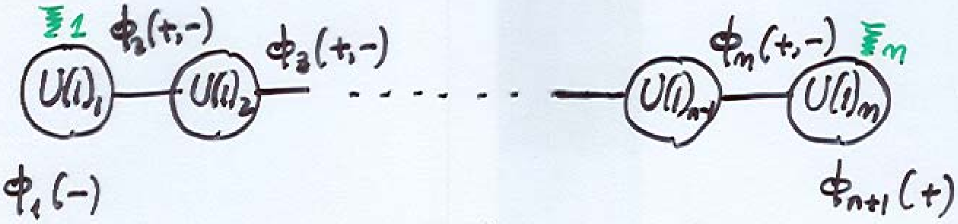
- SUGRA description valid for $T \gg 1$ ⇒ $W_0 \ll 1$ needed, need to invoke several fluxes and special choices of fluxes
- $S \ll 1$ (perturbativity) ⇒ similar requirements
- interesting framework to compute soft terms and complete phenomenology (Grana + Louis & coll.; Camara, Ibanez & Uranga; Lüst & Stieberger)

4) Field theory landscapes

A large N example

- large number of $U(1)$'s with a large number of fields naturally generates a large number of vacua

Our example below has a higher-dimensional interpretation



$$W = \lambda \prod_{i=1}^{n+1} \phi_i$$

$$V = \frac{1}{2} \sum_{a=1}^{n} g_a^2 D_a^2 + \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 , \quad \text{where}$$

$$D_a = \sum_{i=1}^{n+1} q_i^{(a)} |\phi_i|^2 + \xi_1 \delta_{a1} + \xi_n \delta_{an}$$

Field eqs.

$$0 = \phi_i \left(q_i^{(1)} \xi_1 + q_i^{(n)} \xi_n + \sum_{a=1}^{n} q_i^{(a)} \sum_{j=1}^{n+1} q_j^{(a)} |\phi_j|^2 + |\lambda|^2 \sum_{j \neq i} |\phi_1 \cdots \hat{\phi}_i \cdots \hat{\phi}_j \cdots \phi_{n+1}|^2 \right)$$

- No flat directions
- Total number of extrema is

$$\boxed{2^{n+1}-1} \qquad \begin{tikzcd} & 0 \\ \langle \phi_i \rangle \arrow[ur] \arrow[dr] & \\ & v_i \end{tikzcd}$$

The stable extrema are

i) $\xi_n > 0 \ , \quad \xi_1 + \xi_n > 0$

$\{1, \dots, n\}$
$$\begin{cases} v_1^2 \propto \xi_1 + \xi_n \\ v_2^2, \dots, v_n^2 \simeq \xi_n \\ v_{n+1} = 0 \end{cases}$$

$\cancel{\text{SUSY}}$ scale very small

$$\sim \left(\frac{\xi_n}{M_p^2}\right)^{n-2} \left(\frac{\lambda}{M_p^{n-2}}\right)^2$$

ii) $\xi_1 < 0 \ , \quad \xi_1 + \xi_n < 0$

$\{2, \dots, n+1\}$
$$\begin{cases} v_1 = 0 \\ v_2^2, \dots, v_n^2 \simeq -\xi_1 \\ v_{n+1}^2 \simeq -(\xi_1 + \xi_n) \end{cases}$$

$\cancel{\text{SUSY}}$ scale very small

$$\sim \left(\frac{\xi_1}{M_p^2}\right)^{n-2} \left(\frac{\lambda}{M_p^{n-2}}\right)^2$$

iii) $\xi_1 > 0 \ , \quad \xi_n < 0$

$\{1, n+1\}$
$$\begin{cases} v_1^2 = \xi_1 \\ v_{n+1}^2 = -\xi_n \\ v_2 = \dots = v_n = 0 \end{cases}$$

SUSY vacuum
R-symm. preserving

There is an exponentially large number of other extrema

- $\{2,3,\dots,n\}$: $$\begin{cases} v_1 = v_{n+1} = 0 \\ v_k^2 = \frac{1}{n}\left[(\xi_1+\xi_n)(k-1) - n\xi_1\right] \qquad k=\overline{2,\dots,n} \\ \langle D_1\rangle = \dots \langle D_n\rangle = \frac{\xi_1+\xi_n}{n} \end{cases}$$

\- High-scale SUSY , unbroken R-symmetry, unbroken U(1) generator $Q_1 + \dots + Q_n$

- $\{2,3,\dots n-1, n+1\}$: $$\begin{cases} v_1 = v_n = 0 \quad , \quad v_{n+1}^2 = -\xi_n \\ v_k^2 = \frac{k-n}{n-1}\,\xi_1 \qquad k = 2,\dots,n-1 \\ \langle D_1\rangle = \dots = \langle D_{n-1}\rangle = \frac{\xi_1}{n-1} \quad , \quad \langle D_n\rangle = 0 \end{cases}$$

etc ....

* All new extrema have a linear profile in the site coordinate k ($\rightarrow$ linear in extra dimension), magnetic flux interpretation

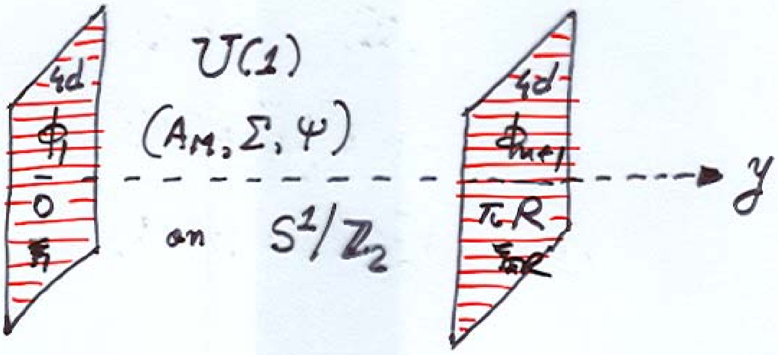
* The new extrema are unstable (tachyons) now, but become stable by adding SUGRA interactions

$$\rightarrow \quad V_{soft} = \sum_{i=1}^{n+1} \tilde{m}_i^2 |\phi_i|^2$$

and generate a landscape picture

[illegible]) Higher-dimensional flux interpretation

- $U(1)^n$ with bifundamental fields in vacua $i)$ and $ii)$ has a well-known five-dimensional interpretation



- Kaluza-Klein spectrum of $(A_M, \Sigma) \rightarrow$ eigenvalues of the $(n+1)\times(n+1)$ mass matrices for (A_μ, ϕ_i) with

$$\begin{tikzcd} & \frac{m}{\sqrt{\xi_n}} \\ R \arrow[ur, "\text{vacuum } i)"] \arrow[dr, "\text{vacuum } ii)"'] & \\ & \frac{m}{\sqrt{|\xi_1|}} \end{tikzcd}$$

$$\mathrm{Re}\,\phi_i\,,\ \mathrm{Im}\,\phi_i \longleftrightarrow \Sigma\,,\ A_5$$

- What about the other vacua ?

Ex:

$$\begin{cases} v_1 = v_{n+1} = 0 \\ v_k^2 = \frac{1}{m}\left[(\xi_1 + \xi_n)(k-1) - m\,\xi_1\right] & k = 2 \dots n \\ \langle D_1 \rangle = \langle D_2 \rangle = \dots = \langle D_m \rangle = \frac{\xi_1 + \xi_n}{m} \end{cases}$$

5d **SUSY description**
gauge multiplet
(A_M, Σ, ψ)

$$\begin{cases} D(y) = \partial_5 \Sigma + (-|\phi_1|^2 + \xi_1)\,\delta(y) + (|\phi_{n+1}|^2 + \xi_n)\,\delta(y - \pi R) \\ \boxed{\partial_5 D = 0} \quad (\Sigma \text{ field eq.}) \Rightarrow \\ \Sigma(y) = \frac{\xi_1 + \xi_n}{2\pi R}\, y - \frac{\xi_1}{2}\,\varepsilon(y) \\ D(y) = \frac{\xi_1 + \xi_n}{2\pi R} = \text{cst.} \end{cases}$$

$\downarrow$ 6d uplift

$$\Sigma = A_6$$

$$\boxed{F_{56} = \partial_5 A_6 - \partial_6 A_5 = \frac{\xi_1 + \xi_n}{2\pi R} - \xi_1\,\delta(y) - \xi_n\,\delta(y - \pi R)}$$

constant flux

compensate localized sources (standard)

- The 4d model is calculable and allows to address quantitatively various physical questions
- Most vacua have flux interpretation

(internal magnetic fields ⟷ intersecting brane constructions

tachyons ⟷ Nielsen-Olesen instabilities)

CONCLUSIONS AND PROSPECTS

- Intersecting D-brane models are closer and closer to the Standard Model spectrum and interactions
- Problem of SUSY in string theory still with us. Flux compactifications admit only a SUGRA description
Computations with NS tadpoles necessary
- Good progress in moduli stabilisation. Detailed phenomenology of flux compactifications very useful
- The emerging landscape picture can produce new phenomenological scenarios which are testable. but calculability need to be improved. Field theory realisations welcome.