Recent Progress with Intersecting D-Brane Models

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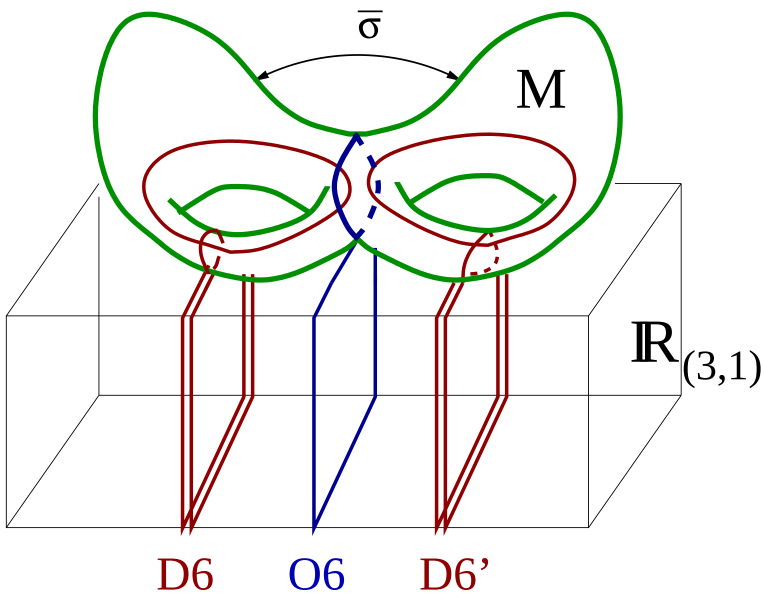
based on the review:

R. Blumenhagen, M. Cvetic, P. Langacker, G. Shiu, *Toward realistic intersecting D-brane models*, hep-th/0502005.

Contents

I. Model building with intersecting branes

Consider Type IIA $\Omega\overline{\sigma}(-1)^{F_L}$ orientifolds with $\overline{\sigma}: z_i \rightarrow \overline{z}_i$ leading to **O6-planes** with in general **intersecting D6-branes**



- **Gauge symmetry**: $U(N)$ or $SO(2N)/SP(2N)$ gauge symmetry on the D6-branes

- **Chiral matter**: localized on the intersection of the D6-branes with

$$n_f - n_{\overline{f}} = I_{ab}$$

with I_{ab} denoting the topological intersection number of two 3-cycles on M

Following:
(Blumenhagen, Braun, Körs, Lüst, JHEP **0207***, 026 (2002))*

- **Supersymmetry**: 3-cycles have to be **sLag cycles**, fixes some of the complex structure moduli

  A 3-cycle π_a is called special Lagrangian if

$$J|_{\pi_a} = 0$$
$$\Im(e^{i\varphi}\,\Omega_3)|_{\pi_a} = 0$$

  One can show that this last property implies that the **volume of the 3-cycle** is given by

$$\text{Vol}(\pi_a) = \int_{\pi_a} \Re(e^{i\varphi_a}\,\Omega_3)$$

- **R-R tadpole cancellation**: Consider the part of the **supergravity Lagrangian** where the R-R field C_7 appears

$$\begin{aligned} \mathcal{S} &= -\frac{1}{4\kappa^2}\int_{\mathrm{R}^{3,1}\times\mathcal{M}} dC_7 \wedge \star dC_7 + \mu_6 \sum_a N_a \int_{\mathrm{R}^{3,1}\times\pi_a} C_7 \\ &+ \mu_6 \sum_a N_a \int_{\mathrm{R}^{3,1}\times\pi'_a} C_7 - 4\mu_6 \int_{\mathrm{R}^{3,1}\times\pi_{\mathrm{O6}}} C_7, \end{aligned}$$

  where the ten-dimensional gravitational coupling is $\kappa^2 = \frac{1}{2}(2\pi)^7(\alpha')^4$ and the R-R charge of a D6 brane reads $\mu_6 = (\alpha')^{-\frac{7}{2}}/(2\pi)^6$.

- The resulting **equation of motion** for the R-R field strength $G_8 = dC_7$ is

$$\frac{1}{\kappa^2} d \star G_8 = \mu_6 \sum_a N_a \, \delta(\pi_a) + \mu_6 \sum_a N_a \, \delta(\pi'_a) - 4\mu_6 \, \delta(\pi_{\mathrm{O6}}),$$

where $\delta(\pi_a)$ denotes the Poincaré dual 3-form of π_a. Since the left hand side is exact, the **R-R tadpole cancellation** condition boils down to just a simple condition on the **homology classes**

$$\sum_a N_a \, (\pi_a + \pi'_a) - 4\pi_{O6} = 0.$$

- **K-theory constraints**: Since we are in an orientifold, there also exist stable (tachyon-free) non-BPS branes carrying torsion-type **K-theory** charges $\rightarrow$ cancellation of **K-theory** charges.

  Inconsistency on the world-volume theory: Uncancelled K-theory charges lead to non-vanishing **Witten anomaly** on probe branes with $SP(2N)$ gauge symmetry, i.e. there appears on odd number of fundamental reps.

The Chiral Massless Spectrum

Since the chiral spectrum has to satisfy some anomaly constraints, we expect that it is given by purely **topological data** (Atiyah-Singer index theorem).

The chiral massless spectrum indeed is completely fixed by the topological **intersection numbers of the 3-cycles** of the configuration.

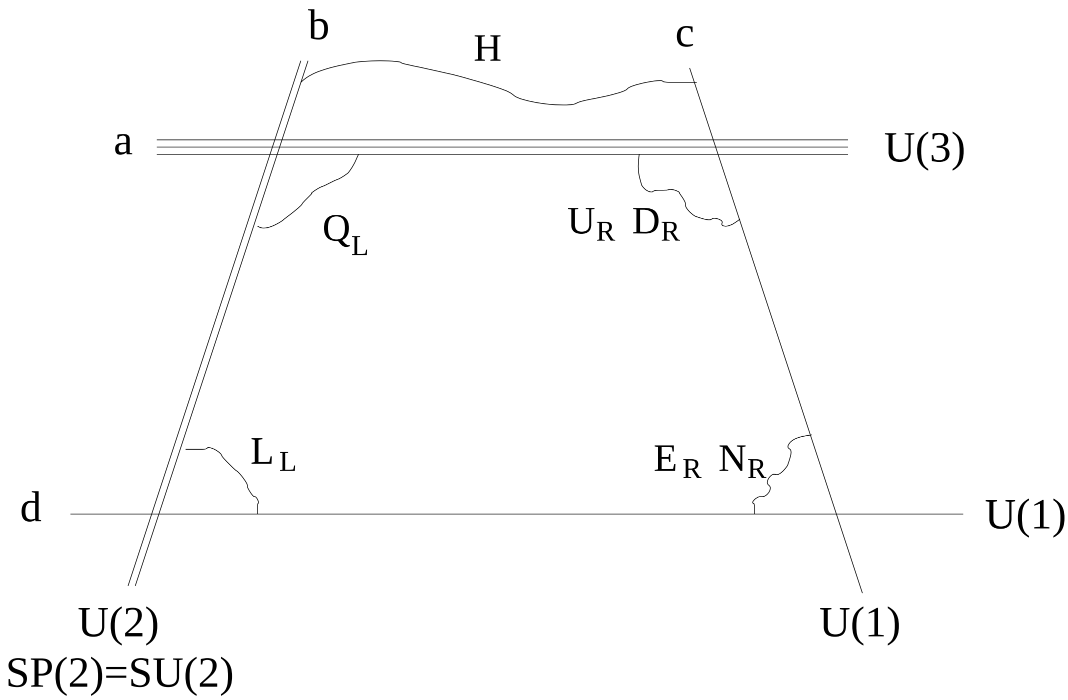
Sector	Rep.	Number
$a'\,a$	A_a	$\frac{1}{2}\,(\pi'_a \circ \pi_a + \pi_{O6} \circ \pi_a)$
$a'\,a$	S_a	$\frac{1}{2}\,(\pi'_a \circ \pi_a - \pi_{O6} \circ \pi_a)$
$a\,b$	$(\overline{N}_a, N_b)$	$\pi_a \circ \pi_b$
$a'\,b$	(N_a, N_b)	$\pi'_a \circ \pi_b$

The **non-abelian gauge anomalies cancel automatically** and mixed $U(1)_a - SU(N)_b^2$ anomalies are canceled by a **generalized Green-Schwarz mechanism** involving dimensionally reduced RR-forms.

II. Constructing semi-realistic models

(Blumenhagen, Görlich, Körs, Lüst, JHEP **0010***, 006 (2000))*
(Angelantonj, Antoniadis, Dudas, Sagnotti
Phys. Lett. B **489***, 223 (2000))*
(Aldazabel, Franco, Ibáñez, Rabadán, Uranga,
J. Math. Phys. **42***, 3103 (2001), JHEP* **0102***, 047 (2001))*
(Blumenhagen, Körs, Lüst, JHEP **0102***, 030 (2001))*
many others

Realization of the Standard Model: The easiest way to realize the SM matter spectrum uses **four stacks** of D6-branes with the following localization of the SM fields



The **intersection** numbers are $\pi_i \circ \pi_j = \pm 3$. In this set-up one always gets a **right handed neutrino**. Simply using intersecting branes on T^6 it was possible to realize the particle spectrum of just the **non-supersymmetric** SM.

(Ibáñez, Marchesano, Rabadán, JHEP **0111***, 0402 (2001))*

Initial **gauge group**

$$G = SU(3) \times SU(2) \times U(1)_a \times U(1)_b \times U(1)_c \times U(1)_d$$

with **chiral matter** spectrum

Intersection	Matter	Rep.	Y
(a,b)	Q_L	$(3,2)_{(1,-1,0,0)}$	1/3
(a',b)	q_L	$2 \times (3,2)_{(1,1,0,0)}$	1/3
(a,c)	U_R	$3 \times (\overline{3},1)_{(-1,0,1,0)}$	−4/3
(a',c)	D_R	$3 \times (\overline{3},1)_{(-1,0,-1,0)}$	2/3
(b',d)	L_L	$3 \times (1,2)_{(0,-1,0,-1)}$	−1
(c,d)	E_R	$3 \times (1,1)_{(0,0,-1,1)}$	2
(c',d)	N_R	$3 \times (1,1)_{(0,0,1,1)}$	0

and $Q_Y = \frac{1}{3}Q_a - Q_c + Q_d$.

Green-Schwarz mechanism $\rightarrow$ $U(1)$ factors receive a **mass**.

Supersymmetric IBWs

For susy models one has to use more general backgrounds, as for instance **toroidal orbifolds** or **algebraic CYs**(Gepner models).

- $\mathbf{Z_2} \times \mathbf{Z_2}$:

  (Cvetic, Shiu, Uranga, Nucl. Phys. B **615***, 3 (2001))*

  (Larosa, Pradisi, Nucl.Phys. B667 (2003) 261)

  (Cvetic, Li, Liu, hep-th/0403061)

- $\mathbf{Z_4}$ and $\mathbf{Z_4} \times \mathbf{Z_2}$:

  (Blumenhagen, Görlich, Ott, JHEP **0301***, 021 (2003))*

  (Honecker, Nucl. Phys. B **666***, 175 (2003))*

- $\mathbf{Z_6}$:

  (Honecker, Ott, hep-th/0404055)

- $\mathbf{Z_7}$, $\mathbf{Z_8}$, $\mathbf{Z_{12}}$: defined on non-factorisable tori. Methods to deal with these more complicated objects have been developed in

  (Blumenhagen, Conlon, Suruliz, hep-th/0404254)

- Gepner models: section VI

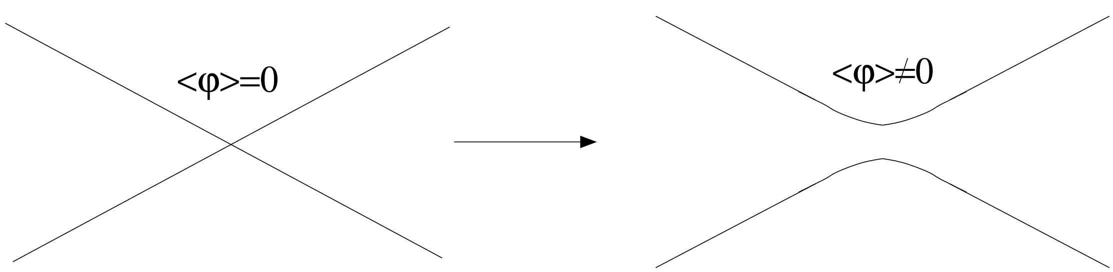
Higgs mechanism

The mass of the **bosonic** "superpartner", ϕ, of the chiral fermions localized at the intersection points of two D-branes is given by

$$M_\phi^2 = \frac{1}{2}\sum_I \Delta\Phi_I - \max\{\Delta\Phi_I\}.$$

Three different cases:

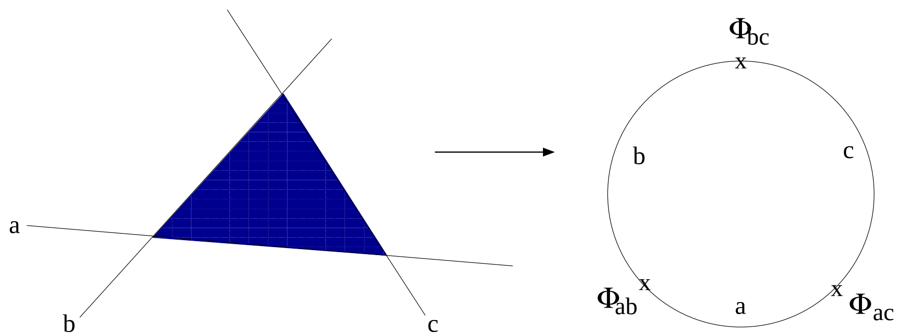
- $M_\phi = 0$: **supersymmetric** brane intersection configuration
- $M_\phi > 0$: **stable** non-BPS state (at string tree-level)
- $M_\phi < 0$: ϕ is a **tachyonic** mode $\rightarrow$ unstable brane configuration. Condensation of the tachyon corresponds to **brane recombination** process (tachyonic Higgs effect)



III. The effective low energy theory

Methods have been developed to determine the susy low energy effective: superpotential, Kähler potential, gauge couplings.

- **Yukawa couplings**: One has to compute 3-point couplings of 3 boundary changing operators at world-sheet disc level: $Y_{abc} = \langle \Phi_{ab}\, \Phi_{bc}\, \Phi_{ac} \rangle_{disc}$



Methods to compute these couplings in toroidal models have been developed:

(Cremades, Ibanez, Marchesano, JHEP 0307 (2003) 038)

(Cvetiv, Papadimitriou, Phys. Rev. D **68**, *046001 (2003))*

(Abel, Owen, Nucl. Phys. B **663**, *197 (2003))*

They are of the general form

$$Y_{abc} = Y_{quantum}(U_I) \sum_{ws-instantons} e^{-S_{cl}(T_I)},$$

which could generate hierarchical Yukawas.

The **superpotential** W is expected to depend only on **Kähler** moduli

$$Y_{abc} = (K_{ab}\, K_{bc}\, K_{ca})^{-\frac{1}{2}} e^{\frac{\mathcal{K}}{2}}\, W_{abc}. \tag{1}$$

The **mirror** symmetric Yukawas in **field theory** limit have been computed in

(Cremades, Ibanez, Marchesano, hep-th/0404229.)

- **Kähler potential**: The Kähler potential (to quadratic order) for the charged massless matter has been computed in

(Lüst, Mayr, Richter, Stieberger, hep-th/0404134)

$$K = \frac{1}{4\pi}\left[\prod_{I=1}^{3}(T_I + T_I^*)^{-\nu_{ab}^I}\sqrt{\frac{\Gamma(\nu_{ab}^I)}{\Gamma(1-\nu_{ab}^I)}}\right]\, \Phi_{\nu_{ab}}\, \Phi^*_{\nu_{ab}}.$$

- **Gauge couplings**: Each gauge factor comes with its **own gauge coupling**

$$f_a = \frac{M_s^3}{(2\pi)^4}\left[e^{-\varphi}\int_{\pi_a}\Re(\Omega_3) + 2i\int_{\pi_a} C_3\right].$$

IV. Flux compactifications

Type IIB string theory contains so-called **fluxes** $F_3 = dC_2$ and $H_3 = dB_2$, which can also be turned on.

- **New tadpole**: The CS coupling

  $$\int C_4 \wedge F_3 \wedge H_3.$$

  induces a D3-brane tadpole once 3-form fluxes are turned on.

- **Scalar potential**:

  Kinetic terms for F_3 and H_3

  $$V = \frac{1}{4\kappa_{10}^2 \mathrm{Im}\tau} \int_{\mathcal{M}} G_3 \wedge \star_6 \bar{G}_3$$

  induce a 4D **Scalar potential**

  $$V = \frac{1}{2\kappa_{10}^2 \mathrm{Im}\tau} \int_{\mathcal{M}} G_3^- \wedge \star_6 \bar{G}_3^- - \frac{i}{4\kappa_{10}^2 \mathrm{Im}\tau} \int_{\mathcal{M}} G_3 \wedge \bar{G}_3,$$

  which can be derived from a **superpotential**

  $$W = \int \Omega_3 \wedge G_3 \qquad \text{with } G_3 = F_3 - \tau H_3.$$

  The scalar potential vanishes if G_3 is **ISD**

  $$\star G_3 = i G_3$$

$$G_3 = G_3^{(2,1)} + G_3^{(0,3)}$$

Supersymmetry $\rightarrow$ $D_\rho W \simeq W = 0$, i.e. $G_3 = G_3^{(2,1)}$. Therefore, fluxes can **freeze** some of the complex structure moduli and the dilation (KKLT extension $\rightarrow$ de Sitter vacua).

- **Chiral models**: Combination of these ideas with IBWs: Introduce not only $D3$-branes but $D9$-$\bar{D}9$ **pairs with magnetic fluxes** (T-dual resp. mirror symmetric to intersecting $D6$-branes).

  For the $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifold **explicit construction** in

  (Blumenhagen,Lüst,Taylor, Nucl. Phys. B **663**,*319 (2003))*

  (Cascales, Uranga, JHEP **0305**, *011 (2003))*

  Explicit examples of **MSSM like**, chiral brane configurations with frozen moduli

  (Marchesano, Shiu, hep-th/0408059, hep-th/0409132)

  (Cvetic, Liu, hep-th/0409032)

V. Phenomenological issues

- **Threshold corrections**: One-loop threshold corrections to gauge couplings for toroidal models have been computed.

  (Lüst, Stieberger, hep-th/0302221)

- **Proton decay**: Possible enhancement factors of $\alpha_{GUT}^{-\frac{1}{3}}$ of proton decay amplitudes via dimension six operators have been evaluated in

  (Klebanov, Witten, Nucl. Phys. B **664***, 3 (2003))*

  (in searching for the smoking gun of string theory!)

- **FCNC**: For toroidal IBW models, lower bounds on the string scale have been extracted from FCNC suppression in non-universal string theoretic 4-point couplings

  (S. Abel, M. Masip, J. Santiago, JHEP 0304 (2003) 057)

  (S. Abel, O. Lebedev, J. Santiago, hep-ph/0312157)

- **Soft-susy breaking terms**: For Type IIB with G_3 flux and $D3$ and $D7$ branes the flux induced soft terms in the world-volume theory on the branes has been determined by

  - expanding the **DBI action** to lowest order in the coordinates transverse to the branes

    (Camara,Ibanez,Uranga, hep-th/0311241, hep-th/0408036)

    (Grana,Grimm Jockers, Louis, hep-th/0312232)

  - employing the standard **supergravity** formalism, where the soft terms are parameterized by the VEVs of the auxiliary F-components of the chiral supermultiplets

    (Lüst, Reffert, Stieberger, hep-th/0406092, hep-th/0410074)

  Susy breaking scale: $M_{susy} = \frac{M_s^2}{M_{pl}}$ in the intermediate regime.

VI. Orientifolds of Gepner models

Final aim: The construction of MSSM-like models with D-branes on more general Calabi-Yau manifolds.

Obstacle: Not very much is known about sLAG 3-cycles on concrete CY spaces.

Rescue: At specific points in moduli space the non-linear sigma models on CY_3 are exactly solvable and given by a rational SCFT $\rightarrow$ **Gepner models**. Consider **orientifolds** of Gepner models

Construction of fully fledged **Gepner model orientifolds**

(Angelantonj, Bianchi, Pradisi, Sagnotti, Stanev, Phys. Lett. B **387***, 743 (1996))*

(Blumenhagen, Wisskirchen, Phys. Lett. B **438***, 52 (1998))*

(Aldazabal, Andres, Leston, Nunez, JHEP **0309***, 067 (2003))*

(Blumenhagen, JHEP **0311***, 055 (2003))*

(Brunner, Hori, Hosomichi, Walcher, hep-th/0401137)

(Blumenhagen, Weigand, JHEP **0402***, 041 (2004))*

(Dijkstra, Huiszoon, Schellekens, hep-th/0403196)

(Aldazabal, Andres, Juknevich, hep-th/0403262)

(Blumenhagen, Weigand, hep-th/0403299)

Orientifold projections

Due to **mirror symmetry** the following relations hold for orientifold models

Sector	**Type IIB**	**Type IIA**
proj.	Ω	$\Omega\overline{\sigma}$
B-type	Z_D	Z_C
	M	W
A-type	Z_C	Z_D
	W	M

so that one can only consider **Type IIB** models. For the A/B-type model it turns out that the resulting **tadpoles** can be canceled by introducing **A/B-type RS-boundary states**.

In

(Dijkstra, Huiszoon, Schellekens, hep-th/0403196)

a systematic search for susy MSSM like models has been carried out and many models have been found.

Generic features: Additional vector-like matter, additional hidden sectors, non-hierarchical Yukawas.

VII. Statistics of String Vacua

- **Huge number of flux vacua** $\simeq 10^{500}$

- We have to be very lucky to find **the** realistic string vacuum.

- A **statistical approach** was proposed by M. Douglas

  (Douglas, hep-th/0303194), which might allow for:

  - estimates for number of Standard-like models
  - statistical "solutions" of **fine tuning problems**: $\Lambda, M_H, \rightarrow$ phenomenological models of **split supersymmetry**

    (Arkani-Hamed, Dimopoulos, hep-th/0405159),
  - prospect of **falsifying** string theory
  - **statistical correlations** providing evidence for string theory.

- Up to now: only flux sector analysed

- Phenomenologically important **gauge sector** should also be considered

- Hope to find **statistical correlations** between observables

- How does the statistics depend on **coupling to fluxes**

  (Blumenhagen, Gmeiner, Honecker, Lüst, Weigand, hep-th/0411173)

Stringy consistency conditions:

- D-branes wrap sLag 3-cycles
- Symplectic basis: (α_I, β_I) of $H_3(M, \mathbb{Z})$, where $\alpha_I \in H_3^+(M)$ and $\beta_I \in H_3^-(M)$
- **O6-planes**

$$\pi_{O6} = \frac{1}{2} \sum_{I=1}^{b_3/2} L_I \, \alpha_I$$

- **D6-branes**:

$$\begin{aligned} \pi_a &= \sum_{I=1}^{b_3/2} (X_{a,I} \, \alpha_I + Y_{a,I} \, \beta_I), \\ \pi_a' &= \sum_{I=1}^{b_3/2} (X_{a,I} \, \alpha_I - Y_{a,I} \, \beta_I) \end{aligned}$$

- **Tadpole cancellation**: $b_3/2 = 1 + h_{21}$ conditions

$$\sum_{a=1}^{k} N_a \, X_{a,I} = L_I - L_{I,flux}$$

- I_{ab} **chiral multiplets** in the bifundamental $U(N_a) \times U(N_b)$ representation

$$I_{ab} = \sum_I X_{a,I} \, Y_{b,I} - Y_{a,I} \, X_{b,I}$$

- Brute force **computer** classification

- Use **saddle point method** to determine asymptotic expansion of integrals

$$P(L) = \frac{1}{2\pi i}\oint dq \frac{g(q)}{q^{L+1}} = \frac{1}{2\pi i}\oint dq\, e^{f(q,L)}$$

with $f(q,L) = \log g(q) - (L+1)\log q$.

- In the next to leading order SAP it can be approximated as ($f'(q_0) = 0$)

$$\mathcal{N}^{(2)}(L) = \frac{1}{\sqrt{2\pi}} \frac{e^{f(q_0)}}{\sqrt{\frac{\partial^2 f}{\partial q^2}|_{q_0}}}$$

Total number of 8D models

- Tadpole condition in 8D

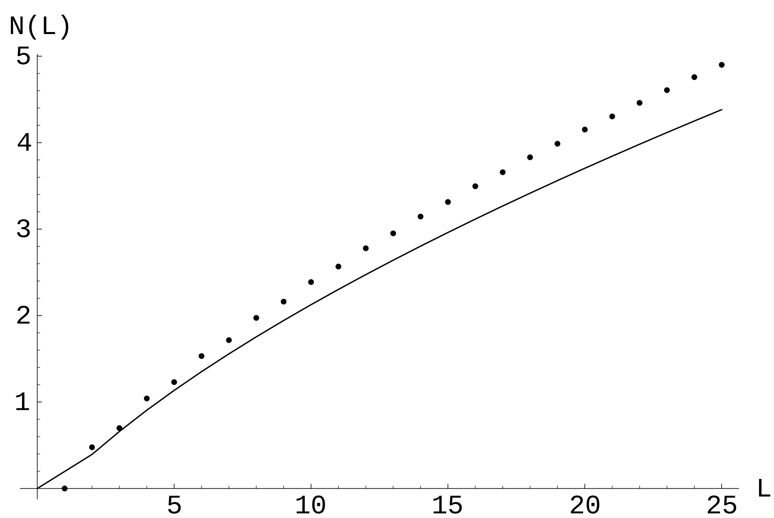
$$\sum_{a=1}^{k} N_a X_a = L$$

- **Saddle point method** gives

$$\mathcal{N}(L) = \frac{1}{2\pi i} \oint dq \frac{1}{q^{L+1}} \exp\left(\sum_{X=1}^{L} \frac{q^X}{1-q^X} \right)$$

- Scaling

$$\mathcal{N}(L) \simeq e^{2\sqrt{L \log L}}$$

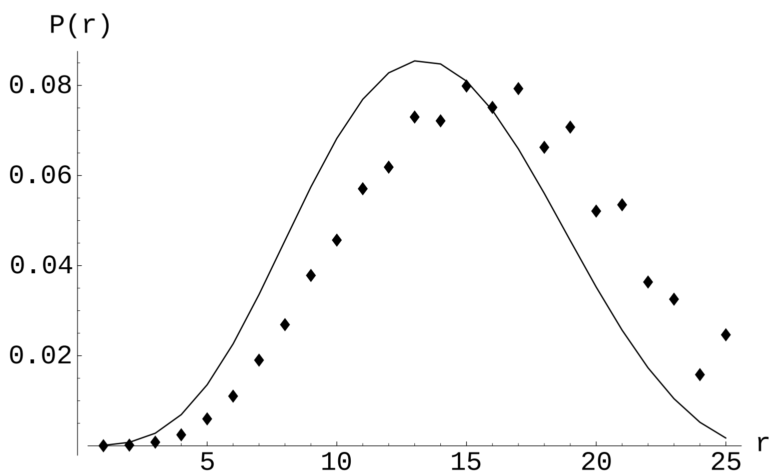


Dots: Exact results, Line: SPA

8D Results - rank distribution

- **Rank distribution**, i.e. frequency to get a gauge group of rank r

$$P(r) \simeq \frac{1}{2\pi i \mathcal{N}(L)} \oint dq \frac{1}{q^{L+1}} \oint dz \frac{1}{z^{r+1}} \exp\left(\sum_{X=1}^{L} \frac{z\, q^X}{1 - z\, q^X} \right) \qquad (2)$$



$L = 25$, Dots: Exact results, Line: SPA

Results

- Similar results in 6D and 4D

- Frequency of $SU(N)$ **gauge** factor

$$P(N) \simeq e^{-\sqrt{\frac{\log L}{L}}N}$$

- Frequency of **number of families**

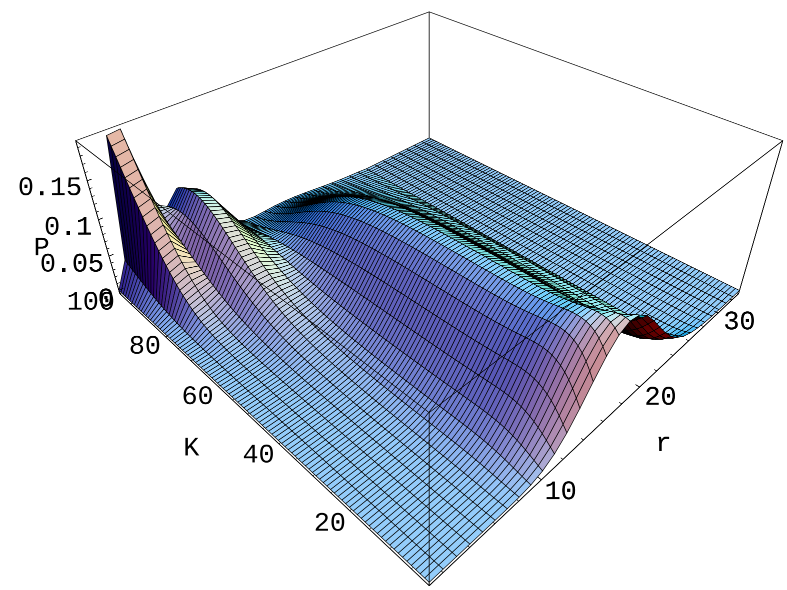
$$P(\chi) \simeq e^{-\kappa\sqrt{\chi}}$$

- **Correlation** between rank and chirality

4D Results - inclusion of fluxes

Example: rank distribution:

$$\overline{P}(r) = \frac{1}{N_{norm}} \sum_{N_{flux}=0}^{N_{flux}^{max}} (N_{flux}+1)^K \, \mathcal{N}(r; L_0 - N_{flux}, L_1, L_2, L_3)$$



VIII. Conclusions

- Compactifications with intersecting D-branes (orientifolds) provide an interesting arena for **string model building** .

- **Systematic** search for **MSSM** like IBW models.

- **Statistical approach** to string vacuum problem

- Uncovering the more detailed **phenomenological** consequences of these models