Flux Compactification of Type IIB Supergravity

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1) Introduction

2) Fluxes, Scherk-Schwarz reduction
and gauged supergravity

3) Deformed geometry and G-structure

4) Fluxes in type IIB supergravity

Introduction

Major problem in string compactifications:

emergence of moduli space of string vacua

moduli appear in two guises:

(i) closed string moduli: deformations of internal cycles
(ii) open string moduli: wrapped branes not rigid

standard model of particle physics & (inflationary) cosmology

$\Longrightarrow$ These moduli have to be fixed !

If supersymmetry is broken only at fairly low energies

need fixing mechanism that preserves
some supersymmetry

So far : only fluxes seem to provide mechanism
for lifting *both* moduli while preserving some susy

fluxes = field strength of RR and NS forms
that are non-zero in the vacuum

(i) fluxes: expand (shrink) parallel (perpendicular) cycles

$\longrightarrow$ fix closed string (geometrical) moduli

But often: run-away behavior

(ii) they couple to Born-Infeld action $\longrightarrow$ generate potential

$\longrightarrow$ can also fix open string moduli
they are compact - potential has always extrema

But also : metric fluxes (twisted tori e.g.)

all fluxes :

related to generalized Scherk-Schwarz reductions

these reductions are especially interesting, because

Scherk-Schwarz reductions yield consistent truncation
on massless KK spectrum

In addition: $\rightarrow$ related to gauged supergravity
$\Rightarrow$ can calculate the potential explicitly

and hence: - KK spectrum
- number of vacua
- de Sitter extrema

($\longrightarrow$ section 2)

However: gauged supergravity does not yield the deformed (internal) geometry

$\Rightarrow$ lift to 10 dimension in most cases not clear !

To get new (internal) geometry

solve directly the 10-d equations

$\rightarrow$ uncovers embedding of the fluxes
$\rightarrow$ solution is classified by torsion classes

or the set of globally defined differential forms (G-structures)

($\longrightarrow$ section 3)

<u>remark</u>: branes are charged $\Longrightarrow$ fluxes give embedding of branes so that back reaction of the branes on geometry is under control

branes vs. fluxes? - branes = δ-function sources in eom or Bianchi identity
- fluxes "near horizon" limit of branes

unforturnately: explicit solutions available only for special cases

e.g. in massive IIA – nearly Kahler spaces

(in section 4 $\longrightarrow$ present a solution in type IIB supergravity)

after finding the spaces:

ask (the mathematicians) for moduli spaces
related to this geometry

Fluxes, Scherk-Schwarz reduction and gauged supergravity

Constraints on vacuum:

strong: 10-d metric flat and all fields are trivial

weak: only 4-d vacuum is Poincaré invariant

$\longrightarrow$ internal metric and RR/NS-fields can be non-trivial !

$\longrightarrow$ related by supersymmetry (back reaction of fluxes on geometry)

hence: internal metric and gauge potential are not constant !

BUT: in order to integrate over the internal coordinates, gauge invariant quantities (Ricci tensor, field strengths) have to independent on internal coordinates!

(generalized) Scherk-Schwarz reduction

Consider type IIB supergravity

bosonic fields :

common sector (e, B, ϕ)

RR-sector (C_0, C_2, C_4^+)

with the field strengths :

$$G_3 = \frac{F_3 - T F_3^*}{\sqrt{1-|T|^2}} \quad , \quad F_5 = dC_4 - \frac{1}{8}(C_2 \wedge dB)$$

$$dG_3 = -(P - iQ) \wedge G_3 , \quad dF_5 = -\frac{1}{8} d\, C_2 \wedge B$$

$$T = \frac{1 + i\tau}{1 - i\tau} , \quad \tau = C_0 + i e^{\phi} , \quad P = \frac{dT}{1-|T|^2} , \quad Q = \frac{\mathrm{Im}(T\, dT^*)}{1-|T|^2}$$

example: axion/dilaton coupling in type IIB supergravity

$$S \sim \int \sqrt{g}\left[R - \frac{g^{MN}\partial_M\tau\partial_N\bar{\tau}}{|\tau-\bar{\tau}|^2}\right]$$

symmetry : $\tau \rightarrow \tau + c$

Scherk-Schwarz ansatz : $\tau(x,y) = \tau(x) + m\,y$

metric ansatz : $ds^2 = e^{2\sigma}(dy + A_\mu dx^\mu)^2 + g_{\mu\nu}dx^\mu dx^\nu$

Langrangian : independent of internal coordinate *y*

$$\frac{g^{MN}\partial_M\tau\partial_N\bar{\tau}}{|\tau-\bar{\tau}|^2} = \frac{g^{\mu\nu}D_\mu\tau(x)D_\nu\bar{\tau}(x)}{|\tau(x)-\bar{\tau}(x)|^2} + \frac{m^2}{|\tau(x)-\bar{\tau}(x)|^2}e^{-2\sigma} \quad , \quad D_\mu\tau = \partial_\mu\tau(x) - m A_\mu$$

→ local symmetry : $\tau \rightarrow \tau + c(x) \quad , \quad A \rightarrow A + \frac{dc}{m}$

≙ coordinate transformation of original metric

⟶ ie. corresponds to gauging of global shift symmetry

In general : any global symmetry can be used for

Scherk-Schwarz reduction

general procedure :

perform special (non-constant) symmetry transformation

so that, gauge invariant quantities remain independent of internal coordinates (and can be integrated out)

result : charged scalar field + potential

hope: fixes all scalar fields

note: can gauge away the charged scalar

→ mass term for the gauge field

Bergshoeff, de Roo, Green, Papadopoulos, Townsend;
Myers, Kaloper; Dabholkar, Hull; .... (Scherk, Schwarz)

In the example before: flux related to 1-form $d_y \tau = m\, dy$

Same mechanism can be applied to other forms !

(a CY-compactification e.g., has no 1-forms or isometries)

$$B + i J = u^a(x, y)\omega_a = [u^a(x) + c^a(y)]\, \omega_a$$

So that : $d(B + i J) = d\, u^a(x) \wedge \omega_a + (m^k \chi_k + cc) = du \wedge \omega + H^{flux}$

with this H-flux : $F_5 = dC_4 - \frac{1}{8} C_2 \wedge H = (\, dA - \frac{1}{8} m\, C_2^{ext}\,) \wedge \chi + cc + \ldots$

massive 2-form in 4 dimensions
~ dual to massive vector

potential comes now from $H_3^2 = G^{ab} m_a m_b = |m|^2$

Louis, Micu ; Dall`Agata

relation to gauged supergravity?

have to identify the Killing vectors

e.g. from the covariant derivative : $d\,q^u \to dq^u + k_I^u A^I$

gauged isometry : scalar dual to massive tensor $C_2^{(ext)}$

$\Rightarrow$ axionic shift universal hyper multiplet is gauged

$\longrightarrow$ straightforward to add other fluxes

result: each flux compactification certain gauging

But opposite statement is far from clear !

Relation to gauged supergravity rises the question:

Can we understand moduli stabilization within gauged sugra?

In order to generate a potential for all scalars

have to gauge isometries of M_H

(lets consider no gauging of M_V)

Killing prepotential : $P_I^x = K_{uv}^x \partial^u k_I^v$, K^x Kahler-2-forms

SU(2) valued superpotential : $W^x \sim X^I P_I^x$

symplectic section : $V = (X^I, F_I)$

Conditions for supersymmetric vacua :

fermionic variations have to vanish $\Longrightarrow$

$$\text{(i)} \quad (\nabla_A X^I) P_I^x = 0 \quad , \quad \text{(ii)} \quad X^I (\nabla_u P_I^x) = K_{uv}^x (X^I k_I^v) = 0$$

Kahler cov. derivative   SU(2) cov. Derivative

note: we ignore here massive tensors – would give symplectic inv. expressions

Condition (ii): fixed point condition of Killing vector $k = X^I k_I$

complete lift of moduli: if fixed point set = NUT (point on M_H)

$$\Longrightarrow \quad \boxed{|k| = 0 \quad , \quad det(dk) \neq 0}$$

remark: if rank of 2-form dk is not maximal – fixed point set is extended
can parallel transport Killing vector along the fixed point set

note : the full quantum moduli space might have no isometries
$\longrightarrow$ fixing of moduli becomes unclear !

$\Rightarrow$ Condition (i): relation only for scalars in vector multiplets

namely: vector $P_I = P_I(q(z))$ has to be normal to M_V

$\longrightarrow$ can happen at different isolated points

(because $\nabla_A \nabla_B V \sim g_{AB} V$)

How about supersymmetric flat space vacua ?

requires additional input : $W^x = 0$

if the rotation : $(dk)_0 \in SP(2n)$

e.g. for homogeneous spaces, Killing vector must be outside of SU(2)-R-symmetry

On other hand if: $det(dk) = 0$ potential has flat direction

$\longrightarrow$ no-scale structure is example of flat space vacuum

Deformed geometry and G-structures

Gauged supergravity approach:

Pro: obtain explicit potential (→ flat directions?)
→ address the issues of non-susy vacua, stability

Con: we dont know the 10-d geometry/fluxes
→ Are there supersymmetric cycles?
→ Can we wrap branes?

Alternative: solve the equations directly in 10 dimensions !

⟶ Killing spinor equations

$$0=\delta\psi_M = \left(D_M-\frac{i}{2}Q_M+\frac{i}{480}F\Gamma_M\right)\epsilon-\frac{1}{96}\left(G\Gamma_M+6G_M\right)\epsilon^*$$

$$0=\delta\lambda = iP\epsilon^*-\frac{i}{24}G\epsilon$$

<u>notation :</u> $G^{(3)} = G^{(3)}_{MNP}\Gamma^{MNP}$ $\quad etc.$

ansatz for the bosonic fields (arbitrary function on internal coordinates)

$$ds^2 = e^{-2V(y)}(g_{\mu\nu}dx^\mu dx^\nu + h_{mn}dy^m dy^n)$$
$$P = P_m dy^m \quad , \quad G = G_{mnp}dy^m \wedge dy^n \wedge dy^p$$
$$F = 5\, e^{-4V} vol_4 \wedge dZ + Hodge-dual$$

external space : AdS $\Lambda < 0$ or Minkowski $\Lambda = 0$

ansatz for the fermions : $\epsilon = a\,\theta \otimes \eta + b^*\,\theta^* \otimes \eta^*$

With a single internal spinor – can define SU(3) structure

$$\eta\gamma_{mn}\eta^* = i\,e^{2\alpha}J_{mn} \quad , \quad \eta\gamma_{mnp}\eta = i\,e^{2(\alpha+i\beta)}\Omega_{mnp}$$

Note: Killing spinor is singlet under structure group

Fluxes ~ deformation described by torsion classes

$$\delta\psi_m = 0 \qquad \Rightarrow \qquad (\nabla_m - \frac{1}{4}\tau_m{}^{pq}\gamma_{pq})\eta = 0$$

intrinsic torsion components : 6x15 = 6 x (1+6+8)

SU(3) singlet spinor

42 $\rightarrow$ (1+1) + (8+8) + (6+6) + (3+3) + (3+3) = $W_1 + W_2 + W_3 + W_4 + W_5$

scalar — (1,1) form — (2,1) form — vector — vector — SU(3)- structures

(Chiossi, Salamon)

$$dJ = \frac{3i}{4}(W_1 \Omega^* + W_1^* \Omega) + W_3 + J \wedge W_4$$

$$d\Omega = W_1 J \wedge J + J \wedge W_2 + \Omega \wedge W_5$$

non-complex :
- $\tau \in W_1$ (nearly Kahler)
- $\tau \in W_2$ (almost Kahler)

complex :
- $\tau \in W_3$ (special hermitian)
- $\tau \in W_5$ (Kahler)
- $\tau = 0$ (Calabi Yau)

Fluxes in type IIB supergravity

Solving the Killing spinor equations:

external part	$\delta\psi_\mu = 0 \quad , \quad \delta\lambda = 0$	constraints on fluxes
internal part	$\delta\psi_m = 0$	differential eq. for spinor

A-type vacuum

$$\epsilon = a\left(\theta \otimes \eta + \theta^* \otimes \eta^*\right)$$

(Majorana spinor)

results: - only NS flux, all RR fields vanish
- common sector e.g. NS5-brane solution

B-type vacuum

$$\epsilon = a\,\theta \otimes \eta$$

results: - holomorphic axion/dilaton e.g. D3 and D7-branes
- internal space is still Kaehler or Calabi-Yau.

General case: $\epsilon = a\ \theta \otimes \eta + b^*\ \theta^* \otimes \eta^*$

results: $W_1 = W_2 = 0\ ,\ \Lambda = 0$ $\qquad b + i\,a = e^{-V/2 + i\alpha}$

- internal space: complex (can use compl. coordinates z^i)
- external space: flat Minkowski

3-form:

$$G = \frac{1}{4} e^{-2V}\ J \wedge \left(\cot\alpha\ P_i\, dz^i + \tan\alpha\, P_{\bar{i}}\, dz^{\bar{i}}\right) + G^{prim}$$

with $\quad J \wedge G^{prim} = \Omega \wedge G^{prim} = \bar{\Omega} \wedge G^{prim} = 0$

remaining fields can be written as :

$$Q = d\theta - \frac{d\tau}{2\tau_2}\ ,\quad P = i\, e^{2i\theta} \frac{d\tau}{2\tau_2}\ ,\quad G_3 = i \frac{e^{i\theta}}{\sqrt{\tau_2}} (d A_2 - \tau\, dB_2)$$

$$F_5 \sim dZ \wedge vol_4 + Hodge\, dual$$

with $(\tau\ ,\ \theta\ ,\ Z)$ given by

$$\tau = c_0 + i\, e^{\phi_0} \frac{|f|^2 \cos 2\alpha}{f \sin^2\alpha + \bar{f} \cos^2\alpha}$$

$$e^{-4V} = \frac{\mathrm{Re}\, f}{4|f|^2} \frac{\sin^2 2\alpha}{\cos 2\alpha}$$

$$Z = \frac{|f^2|}{\mathrm{Re}\, f} \frac{\cos^2 2\alpha}{\sin 2\alpha}$$

$$\tan\theta = -\frac{\mathrm{Im}\, f}{\mathrm{Re}\, f} \cos 2\alpha$$

where: $f = \dfrac{\tilde{Z}}{\cos 2\alpha + i \tan\theta} = f(z^i)$ $\qquad \tilde{Z} = Z \dfrac{\sin^2 2\alpha}{\cos 2\alpha}$

<u>Remark:</u> Supersymmetry leaves one function un-fixed !

$\longrightarrow$ fixed by equations of motion or Bianchi identities

here: α other possibility: Z can be fixed by $dF_5 = \Delta Z = \dfrac{5i}{12} G \wedge \bar{G}$

Discussion

$$f = constant$$

Apart from trivial solution, includes the flow type solution

$\longrightarrow$ fields depend only on one real coordinate

for $G \wedge \bar{G} = 0$ Z harmonic $\Leftrightarrow$ is monoton (and singular!)

UV-regime $\Leftrightarrow$ poles of Z (or warp factor) $\Leftrightarrow$ $\alpha=0$ $\Leftrightarrow$ internal space becomes CY

geometry can be regular $\rightarrow$ $AdS_5 \otimes S^5$

IR regime $Z \simeq 0$ or $\cos 2\alpha \simeq 0$
singular, NOT CY ! However, regular iff

$G \wedge \bar{G} \neq 0$ $\rightarrow$ internal space is always non-Kaehler is necessary for flow to flat space

$$f \neq constant$$

Non-constant holomorphic function on compact space

$\leftrightarrow$ has zeros and poles

zeros : $\sin 2\alpha \simeq 0$

$\rightarrow$ B-type vacuum:
- Kaehler geometry (e.g. Calabi Yau)
- axion/dilaton is holomorphic
- prototype sugra solution: D3/D7-branes

poles : $\cos 2\alpha \simeq 0$

$\rightarrow$ A-type vacuum:
- all RR fields are trivial
- prototype sugra solution: NS5-branes

Summary

to fix the moduli = one of the important issues in string theory

to get contact to particle physics and cosmology

fluxes ~ only possibility that preserves some supersymmetry (?)

two approaches : (i) gauged supergravity

→ gives explicit potential
but lift to 10 dimensions unclear
vacuum: fixed point of Killing vector

presented: criteria when all moduli lifted

(ii) solving (Killing spinor) equations in D=10

→ gives explicit geometry (defined by its torsion)
but there is no explicit potential (moduli spaces?)

presented: example on IIB side (complex spaces)
open questions: all moduli fixed, chiral matter ?

The End.

Moduli from Kaluza-Klein reduction

In low energy approximation:

type II string theory = type II supergravity

with N=2 supersymmetrie in D=10 supersymmetry

fermionic sector : two gravitinos and two dilatinos

opposite chirality $\longrightarrow$ type IIA (non-chiral)

equal chirality $\longrightarrow$ type IIB (chiral)

Lets assume we have a consistent truncation

then : moduli = zero modes of KK scalar fields

Scalars come from internal metric and RR/NS forms.

And moduli are in one-to-one correspondence to harmonic diff. forms of internal space.

Example: if $\omega_n \neq d\,\omega_{n-1}$, $d\,\omega_n = d^*\,\omega_n = 0$

$\longrightarrow$ get scalar field from $C_n = \phi(x)\,\omega_n + \ldots$

because $F = dC = 0$ for $\phi = \phi_0 = \text{constant}$

$\longrightarrow$ one modulus (consequence of gauge symmetry)

in the following denote :

complete basis of 3-forms : $\{\, \chi^k, \tilde{\chi}_k \,\} \in H^{(3)}(Y)$

complete basis of 2-forms : $\{\, \omega_a \,\} \in H^{(2)}(Y)$

on IIA side : $C_3 = u^k \chi_k + A^a \wedge \omega_a + cc \quad , \quad B + iJ = z^a \omega_a$

gravi-photon : C_1

hyper multiplet : $\{ u^k , v^k \} \quad , \quad v^k \sim$ internal metric

univ. hyper multiplet : $\{ \phi , B^{ext} , C_3^{(3,0)} , C_3^{(0,3)} \}$

vector multiplet : $\{ A^a , z^a \}$

on IIB side : $C_4 = A^k \wedge \chi_k + cc \quad , \quad B + iJ = u^a \omega_a \quad , \quad {}^*C_4 + iC_2 = v^a \omega_a$

gravi-photon : A^0

hyper multiplet : $\{ u^k , v^k \}$

univ. hyper multiplet : $\{ \phi , C_0 , C_2^{ext} , B^{ext} \}$

vector multiplet : $\{ A^a , z^a \} \quad , \quad z^a \sim$ internal metric

on CY: complex structure and Kahler class deformation independent :

$$M = M_V \otimes M_H \qquad \text{special Kahler} \otimes \text{quaternionic Kahler}$$

KK – approach : $M_{10} = M_4 \otimes M_6$, $x^M = \{ x^\mu , y^m \}$

and integrate over y^m

in general : Fourier expansion in set of harmonic eigenfunctions

$\longrightarrow$ 4-d massive mode

simplest case : fields do not depend on y^m

= truncation on the massless sector

truncation possible : if massless field $\neq$ sources for massive

ie. no linear couplings $\longrightarrow$ consistent truncation

BUT : requires explicit metric (only few examples known Nicolai, deWit)

weaker version : if source terms contain derivatives
then integrating out massive modes
does not change Lagr. on 2-derivative level

Duff, Ferrara, Pope, Stelle

(only this form expected for KK-reductions related to gauged supergravity)