FERMION MASSES AND MIXINGS IN MINIMAL SO(10) GUTS

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GRAND
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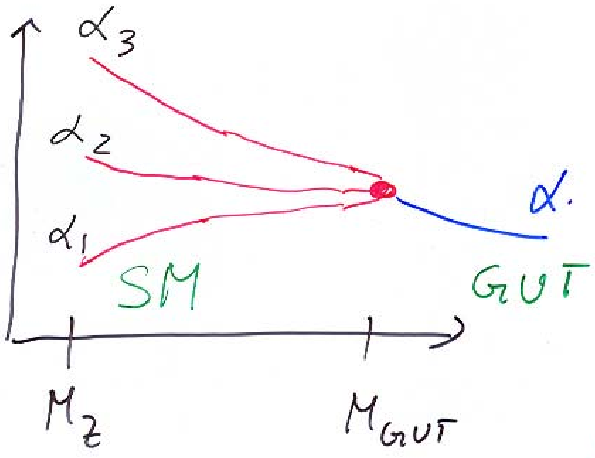
THEORIES



$M_{GUT} \simeq 10^{16}$ GeV

$$SU(3)_C \times SU(2)_L \times U(1)_Y \subset G$$

BIGGER SYMMETRY

$\rightarrow$ NEW CONSTRAINTS
& PREDICTIONS

① MAGNETIC MONOPOLES

MASS $\simeq M_{GUT} \gg M_Z$

$\Rightarrow$ COSMOLOGICAL ISSUE

② PROTON DECAY

SUPERKAMIOKANDE

NOT FOUND YET

③ FERMION MASSES AND MIXINGS

THIS TALK

IN SM OR MSSM

$$\mathcal{L}_Y = Q^T \phi_u Y_U u^c + Q^T \phi_d Y_D d^c$$

$$+ L^T \phi_d Y_E e^c + \text{NEUTRINO}$$

Q, L, u^c, d^c, e^c TRIPLETS IN GENERATION SPACE

Y_U, Y_D, Y_E 3X3 COMPLEX MATRICES

ϕ_u, ϕ_d HIGGS DOUBLETS

($\phi_d = i\tau_2 \phi_u^*$ IN SM)

! NO RELATIONS AMONG YUKAWAS !

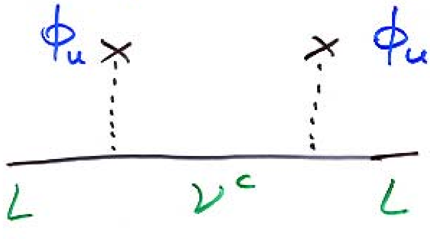
WHAT ABOUT NEUTRINO ?

$$\mathcal{L}_Y = \left(L^T \phi_u\right) \frac{Y_N}{M} \left(\phi_u^T L\right)$$

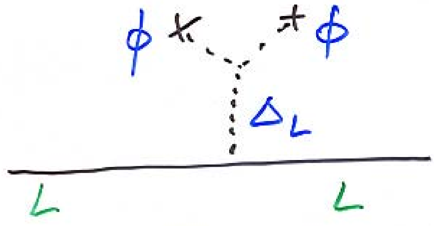
Y_N 3X3 SYMMETRIC COMPLEX MATRIX (AGAIN NO RELATION WITH OTHER YUKAWAS!)

M LARGE SCALE ($\gg M_W$)

THE FORM IMPLEMENTED BY THE SEE-SAW MECHANISM :



TYPE I



TYPE II

WHY SHOULD GUTS DO BETTER?

SO(10):

$$16_F^i = (Q, u^c, d^c, L, e^c, \nu^c)$$

$i = 1, 2, 3$

$\Rightarrow$ RIGHT-HANDED NEUTRINO ν^c AUTOMATIC

$\Rightarrow$ SEE-SAW

$\Rightarrow$ RELATIONS AMONG

$$Y_U, Y_D, Y_E, Y_N$$

AT $M_{GUT} \simeq 10^{16}$ GeV

WE WILL ASSUME :

1. ONLY SO(10)

NO FLAVOUR SYMMETRY

2. RENORMALIZABILITY

- NEED IT FOR PREDICTIONS
- NOT SO CRAZY :

$$W = \frac{c}{M_{Pl}} QQQL$$

$c \leq 10^{-7}$ PHENOMENOLOGICAL CONSTRAINT

IN SM

$$2 \times 1 = 2$$

Q, L $\qquad$ u^c, d^c, e^c $\qquad$ ϕ

IN SO(10) GUT

$$16 \times 16 = 10 + 120 + 126$$

1 INDEX

3 INDICES ANTISYMMETRIC

5 INDICES ANTISYMMETRIC SELF-DUAL

MATTER

HIGGS

THE MOST GENERAL YUKAWA

$$\mathcal{L}_Y = 16_F^T \left(10_H Y_{10} + 120_H Y_{120} + 126_H Y_{126} \right) 16_F$$

3 X 3 MATRICES

$$Y_{10,126}^T = + Y_{10,126}$$

$$Y_{120}^T = - Y_{120}$$

FROM SO(10)

THE MOST GENERAL CASE
NOT RESTRICTIVE :
TRY TO MODEL SIMPLE,
MINIMAL SUBCASES

ONLY ONE 10_H

$$\mathcal{L}_Y = 16_F^T \, 10_H \, Y_{10} \, 16_F$$

IN $PS = SU(2)_L \times SU(2)_R \times SU(4)_C$

$$10_H = \underbrace{(2,2,1)}_{\text{VEV} \begin{pmatrix} v_u & 0 \\ 0 & v_d \end{pmatrix}} + (1,1,6)$$

DOES NOT BREAK $SU(4)_C$

$$\Rightarrow \begin{pmatrix} u^1 & u^2 & u^3 & \nu \\ d^1 & d^2 & d^3 & e \end{pmatrix}_L = (2,1,4) \subset 16_F$$

$$16_F = (2,1,4) + (1,2,\bar{4})$$

HAVE THE SAME MASS

$$M_D = M_E$$

$$M_U = M_{\nu_D} \longrightarrow L Y_{\nu_D} \phi_u \nu^c$$

$$M_U \to Q Y_U \phi_u u^c$$

PREDICTIONS :

(A) $m_b = m_\tau$ GOOD!

$m_s = m_\mu$ BAD! AT M_{GUT}

$m_d = m_e$ BAD!

(B) 1 MATRIX (Y_{10}) ONLY :

$$16_F \longrightarrow U\, 16_F$$

SUCH THAT

$$U^T Y_{10} U = Y_{10}^{diag}$$

NO MIXING

(C) NEUTRINO HAVE LARGE DIRAC MASS $Y_{\nu_D} = Y_U$

ONLY ONE 126_H

$$\mathcal{L}_Y = 16_F \, 126_H \, Y_{126} \, 16_F$$

$$126_H = (3,1,\overline{10}) + (1,3,10) + (2,2,15) + (1,1,6)$$

GIVES TINY MAJORANA MASS $\nu_L \nu_L$

$\downarrow$

M_{ν_L}

GIVES HUGE MAJORANA MASS $\nu_R^{\bullet} \nu_R$

$\downarrow$

M_{ν_R}

$B\text{-}L \propto \begin{pmatrix} 1 & \\ & 1 \\ & & -3 \end{pmatrix}$

DIRAC MASSES

SEE-SAW:

$$M_N = -M_{\nu_D}^T M_{\nu_R}^{-1} M_{\nu_D} + M_{\nu_L}$$

PREDICTIONS :

(A) DUE TO $\langle 15 \rangle \propto \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix}$

$m_\tau = -3\, m_b$ BAD!

$m_\mu = -3\, m_s$ GOOD! AT M_{GUT}

$m_e = -3\, m_d$ BAD!

(B) NO MIXING!

(C) NEUTRINO MAJORANA,
SEE-SAW NATURAL

ONLY ONE 120_H

$$\mathcal{L}_Y = 16_F \, 120_H \, Y_{120} \, 16_F$$

$$120_H = (2,2,1) + (2,2,15) + (3,1,6) + (1,3,6) + $$
$$+ (1,1,10) + (1,1,\overline{10})$$

DIRAC MASSES

$$Y_{120}^T = - Y_{120}$$

LESS PARAMETERS !
THAN FOR Y_{10}, Y_{126}
(SYMMETRIC)

$$\langle (2,2,1) \rangle = \begin{pmatrix} v^u & 0 \\ 0 & v^d \end{pmatrix}$$

$$\langle (2,2,15) \rangle = \begin{pmatrix} w^u & 0 \\ 0 & w^d \end{pmatrix}$$

PREDICTIONS :

(A) Y_{120} TOO SIMPLE

$\Rightarrow$ EIGENVALUES = $+a, -a, 0$

BAD !

(B) NO RELATIONS AMONG DIFFERENT YUKAWAS $\rightarrow$

$$M_U = (v^u + w^u)\, Y_{120}$$
$$M_{\nu_D} = (v^u - 3w^u)\, Y_{120}$$
$$M_D = (v^d + w^d)\, Y_{120}$$
$$M_E = (v^d - 3w^d)\, Y_{120}$$

(C) NO MIXING

(D) NEUTRINO DIRAC TYPE

SUMMARY

$10_H \longrightarrow m_b = m_\tau$

126_H → BREAKS RANK

126_H → $m_\mu = -3 m_s$

126_H → SEE-SAW (M_{ν_R})

120_H NOT VERY PROMISING ??

TWO HIGGSES: $10_H + 126_H$

$$\mathcal{L}_Y = 16_F \left(10_H Y_{10} + 126_H Y_{126} \right) 16_F$$

$$16_F \rightarrow U\, 16_F$$

- LAZARIDES, SHAFI
- BABU, MOHAPATRA

Y_{10} DIAGONAL (3 REAL PARAMETERS)

Y_{126} SYMMETRIC (12 REAL PARAMETERS)

- MODEL COMPATIBLE WITH DATA

MOHAPATRA ET AL, 03, 04
BERTOLINI, MALINSKY, 05

- $U_{e3} \simeq 0.1 - 0.15$
- IN TYPE II SEESAW:

$b-\tau$ UNIFICATION $\Leftrightarrow$ LARGE θ_{ATM}

BAJC, SENJANOVIĆ, VISSANI, 02

POSSIBLE TO UNDERSTAND

WHY $\theta_{atm} \approx$ LARGE IN

TYPE II SEE-SAW :

$$M_{\nu_L} \propto Y_{126}$$

$$\begin{cases} M_d = v_{10}^d Y_{10} + v_{126}^d Y_{126} \\ M_e = v_{10}^d Y_{10} - 3 v_{126}^d Y_{126} \end{cases}$$

$$\Rightarrow Y_{126} \propto M_d - M_e$$

$$\Rightarrow M_{\nu_L} \propto M_d - M_e$$

BRAHMACHARI, MOHAPATRA, 98

APPROX.:

- 2-3 GENERATION CASE
- NEGLECT $m_{s,\mu} \ll m_{b,\tau}$
- M_d AND M_e HAVE SMALL MIXINGS

$$M_{\nu_L} \propto \begin{pmatrix} 0 & 0 \\ 0 & m_b - m_\tau \end{pmatrix}$$

BAJC, SENJANOVIĆ, VISSANI, 02

$$\boxed{\text{LARGE } \theta_{atm} \Longleftrightarrow b-\tau \text{ UNIFICATION}}$$

IS IT 126_H REALLY NECESSARY ?

(1) RANK:

$$126_H \longrightarrow 16_H$$

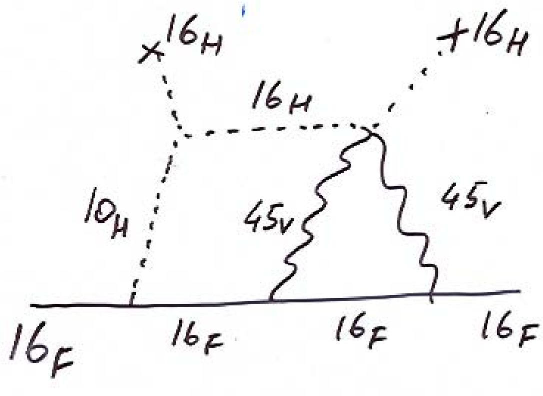
$$\langle 16_H \rangle = M_R$$

IN THE $(1,2,\bar{5})$ DIRECTION

(2) M_{ν_R}:

NO 126_H, AT TREE ORDER $M_{\nu_R} = 0$

WITTEN'S IDEA (80):



$$M_{\nu_R} \simeq \left(\frac{\alpha}{\pi}\right)^2 \frac{M_R^2}{M_{GUT}} Y_{10}$$

WHEN IS IT VALID?

NONSUSY : DIFFICULT,

WITHOUT FINE-TUNING $M_R \ll M_{GUT}$

FOR SUCCESSFUL GAUGE UNIFICATION

$\Rightarrow$ M_{ν_R} TOO SUPPRESSED

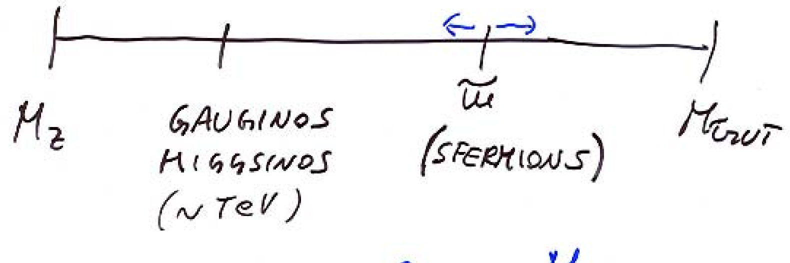
SUSY : NO, FOR LOW-ENERGY SUSY BREAKING

$M_{\nu_R} \propto m_{SUSY}$

SPLIT SUSY :

(ARKANI-HAMED, DIMOPOULOS, 04)

$\tilde{m} = ?$



YES, IF $\tilde{m} \simeq M_{GUT}$

(BAJC, SENJANOVIĆ, 04)

(3) WITTEN'S MODEL TOO SIMPLE:

ONLY ONE 10_H

$$M_{\nu_D} \sim M_{\nu_R} \sim Y_{10}$$

$$\Rightarrow \boxed{M_N \propto Y_{10}} \quad \checkmark \quad \boxed{m_b = m_\tau} \quad \checkmark\checkmark$$

- NO MIXING
- $m_\mu = m_s$ BAD
- $m_e = m_d$

$\Rightarrow$ ADD ANOTHER 120_H

Y_{10} 3 REAL PARAMETERS

Y_{120} 6 REAL -"-

CAN WE FIT THE DATA

WITH 10_H AND 120_H ?

$$(2,2,1)_{10} \longrightarrow v_{10}^u , v_{10}^d$$

$$(2,2,1)_{120} \longrightarrow v_{120}^u , v_{120}^d$$

$$(2,2,15)_{120} \longrightarrow w_{120}^u , w_{120}^d$$

$$M_U = v_{10}^u Y_{10} + \left(v_{120}^u + w_{120}^u\right) Y_{120}$$

$$M_D = v_{10}^d Y_{10} + \left(v_{120}^d + w_{120}^d\right) Y_{120}$$

$$M_E = v_{10}^d Y_{10} + \left(v_{120}^d - \underline{\underline{3}}\, w_{120}^d\right) Y_{120}$$

$$M_{\nu_D} = v_{10}^u Y_{10} + \left(v_{120}^u - \underline{\underline{3}}\, w_{120}^u\right) Y_{120}$$

$M_{\nu_R} \propto Y_{10}$ (WITTEN)

$M_N = -M_{\nu_D}^T M_{\nu_R}^{-1} M_{\nu_D}$ (TYPE I ONLY !?)

ONE CAN REWRITE

$$M_D = M_0 + M_2$$
$$M_U = C_0 M_0 + C_2 M_2$$
$$M_E = M_0 + C_3 M_2$$
$$M_{\nu_D} = C_0 M_0 + C_4 M_2$$

THE FULL 3X3 NUMERICAL ANALYSIS HARD, TIME CONSUMING.

BETTER FIRST USE AN APPROXIMATE SCHEME :

- IS THE SYSTEM WORTH STUDYING IN DETAIL ?
- FIND SOME ANALYTICAL RESULTS

STUDY FIRST THE 2ND AND 3RD GENERATIONS ONLY:

$$M_0 = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 0 & i\alpha \\ -i\alpha & 0 \end{pmatrix}$$

$c_0, a, b \in \mathbb{R}$ $(a, b > 0)$

$\alpha, c_2, c_3, c_4 \in \mathbb{C}$

ONLY 4 REAL PARAMETERS FROM YUKAWA SECTOR (a, b, α)

BUT 7 REAL PARAMETERS FROM HIGGS SECTOR (c_0, c_2, c_3, c_4)

IN THE LIMIT

$$\frac{m_s}{m_b},\ \frac{m_\mu}{m_\tau},\ \frac{m_c}{m_t} \longrightarrow 0$$

THE ANALYSES SIMPLIFY ENORMOUSLY:

$$\Downarrow$$

① $b-\tau$ UNIFICATION $\quad m_b = m_\tau$

② IF $\underset{\theta_{ATM}}{\theta_\ell} \simeq \frac{\pi}{4} \Longrightarrow \underset{\theta_{bc}}{\theta_q} \simeq 0$

③ LIGHT NEUTRINO DEGENERATE

BAJC, SENJANOVIĆ,
TO APPEAR SOON

PROOF VERY SIMPLE:

$$M_D = \begin{pmatrix} a & i\alpha \\ -i\alpha & b \end{pmatrix}$$

$$M_D M_D^+ = \begin{pmatrix} a^2 + |\alpha|^2, & i(a\alpha^* + b\alpha) \\ -i(a\alpha + b\alpha^*), & b^2 + |\alpha|^2 \end{pmatrix}$$

$$\det(M_D M_D^+) = |ab - \alpha|^2 = m_b^2 m_s^2 \to 0$$

$$\Rightarrow \quad ab = \alpha^2$$

$$\tan(2\theta_D) = \frac{2\alpha}{a-b}$$

$$M_U = \begin{pmatrix} c_0 a & i c_2 \alpha \\ -i c_2 \alpha & c_0 b \end{pmatrix}$$

$$M_U M_U^+ \longrightarrow c_0^2 = c_2^2$$

$$\tan(2\theta_U) = \frac{\pm 2\alpha}{a-b}$$

$$\theta_2 = \theta_U - \theta_D = $$
$$= 0$$
OR
$$\pm 2\theta_D$$

$$M_E = \begin{pmatrix} a & , & i c_3 \alpha \\ -i c_3 \alpha & , & b \end{pmatrix}$$

$$M_E M_E^+ \longrightarrow c_3^2 = 1 \rightarrow \theta_E = \pm \theta_D$$

$$\boxed{m_\tau = m_b}$$ 1ST PREDICTION

$$M_{\nu_R} \propto M_0$$

$$M_{\nu_D} = c_0 M_0 + c_4 M_2$$

$$\Rightarrow \quad M_N \propto \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = M_0$$

DIAGONAL

$$\theta_q = \theta_D - \theta_U = 0 \quad \text{OR} \quad \pm 2\theta_E$$

$$\theta_\ell = \theta_E - \theta_N = \theta_E$$

$$\boxed{\text{IF } \theta_\ell \approx \frac{\pi}{4} \Rightarrow \theta_q \approx 0}$$ 2ND PREDICTION

FROM $\theta_E \simeq \frac{\pi}{4}$

AND $\tan(2\theta_E) \simeq \frac{2\alpha}{a-b} \Rightarrow a \simeq b$

$$\Rightarrow M_\nu \propto M_0 \sim I$$

$\boxed{\text{NEUTRINO DEGENERATE}}$

3^{RD} PREDICTION

WITTEN + 10_H + 120_H

1. COUNTEREXAMPLE TO THE COMMON BELIEF THAT IN GUTS ν MASSES HIERARCHICAL

2. θ_q SMALL BECAUSE EQUAL TO THE DIFFERENCE OF TWO LARGE ANGLES

3. θ_ℓ LARGE BECAUSE IT FAILS TO CANCEL (M_N DIAGONAL)

4. $B-\tau$ UNIFICATION AUTOMATIC (IN SPITE OF 120_H)