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Classical and Quantum Reduction of Couplings

SM very successful

BUT with >20 free parameters

ad hoc Higgs sector

ad hoc Yukawa couplings

Best candidate for Physics Beyond SM

MSSM with >100! free parameters mostly in its SSB sector:

- cures problem of quadratic divergencies of the SM (hierarchy problem)
- restricts the Higgs sector

- SM with two - Higgs doublets

$$V = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + \left(m_3^2 H_1 H_2 + h.c. \right)$$

$$+ \frac{1}{2} \lambda_1 \left(H_1^+ H_1 \right)^2 + \frac{1}{2} \lambda_2 \left(H_2^+ H_2 \right)^2$$

$$+ \lambda_3 \left(H_1^+ H_1 \right) \left(H_2^+ H_2 \right) + \lambda_4 \left(H_1 H_2 \right) \left(H_1^+ H_2^+ \right)$$

$$+ \left\{ \frac{1}{2} \lambda_5 \left(H_1 H_2 \right)^2 + \left[\lambda_6 \left(H_1^+ H_1 \right) + \lambda_7 \left(H_1^+ H_2^+ \right) \right] \left(H_1 H_2 \right) + h.c. \right]$$

Suresymmetry provides tree level relations among couplings

$$\lambda_1 = \lambda_2 = \frac{1}{4} \left(g^2 + g'^2 \right)$$

$$\lambda_3 = \frac{1}{4} \left(g^2 - g'^2 \right) , \quad \lambda_4 = - \frac{1}{4} g^2$$

$$\lambda_5 = \lambda_6 = \lambda_7 = 0$$

With $v_1 = \langle Re\, H_1^0 \rangle$, $v_2 = \langle Re\, H_2^0 \rangle$

and $v_1^2 + v_2^2 = (246 \text{ GeV})^2$, $\frac{v_2}{v_1} \equiv \tan\beta$

$\Longrightarrow h^0, H^0, H^\pm, A^0$

At tree level

$$M^2_{h^0,H^0} = \frac{1}{2}\left\{M_A^2 + M_Z^2 \mp \left[(M_A^2+M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta\right]^{1/2}\right\}$$

$$M^2_{H^\pm} = M_W^2 + M_A^2$$

$$\Rightarrow \begin{cases} M_{h^0} < M_Z \, |\cos 2\beta| \\ M_{H^0} > M_Z \\ M_{H^\pm} > M_W \end{cases}$$

Radiative corrections

$$M^2_{h^0} \simeq M_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{16\pi^2 M_W^2} \log \frac{\tilde{m}^2_{t_1} \tilde{m}^2_{t_2}}{m_t^4}$$

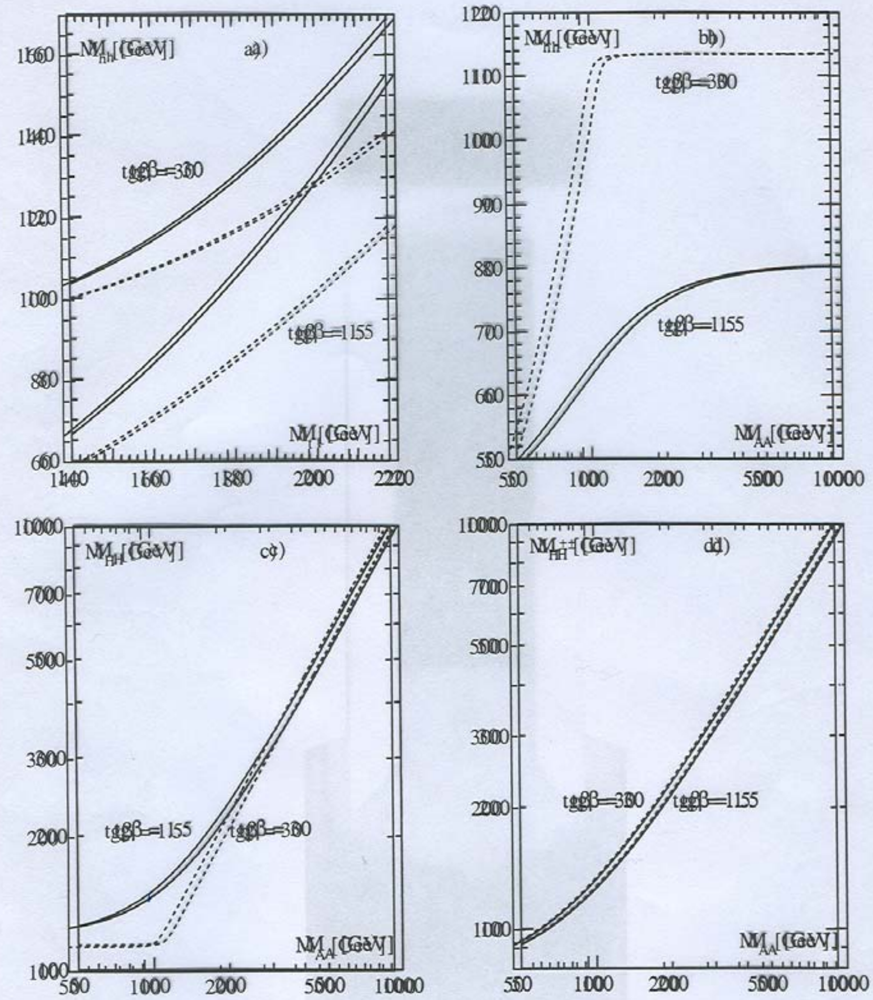


Figure 18: *(a) The upper limit on the light scalar Higgs pole mass in the MSSM as a function of the top quark mass for two values of tgβ = 1.5, 30; the common squark mass has been chosen as $M_S = 1$ TeV. The full lines correspond to the case of maximal mixing [$A_t = \sqrt{6} M_S, A_b = \mu = 0$] and the dashed lines to vanishing mixing. The pole masses of the other Higgs bosons, H, A, $H^\pm$, are shown as a function of the pseudoscalar mass in (b-d) for two values of tgβ = 1.5, 30, vanishing mixing and $M_t = 175$ GeV.*

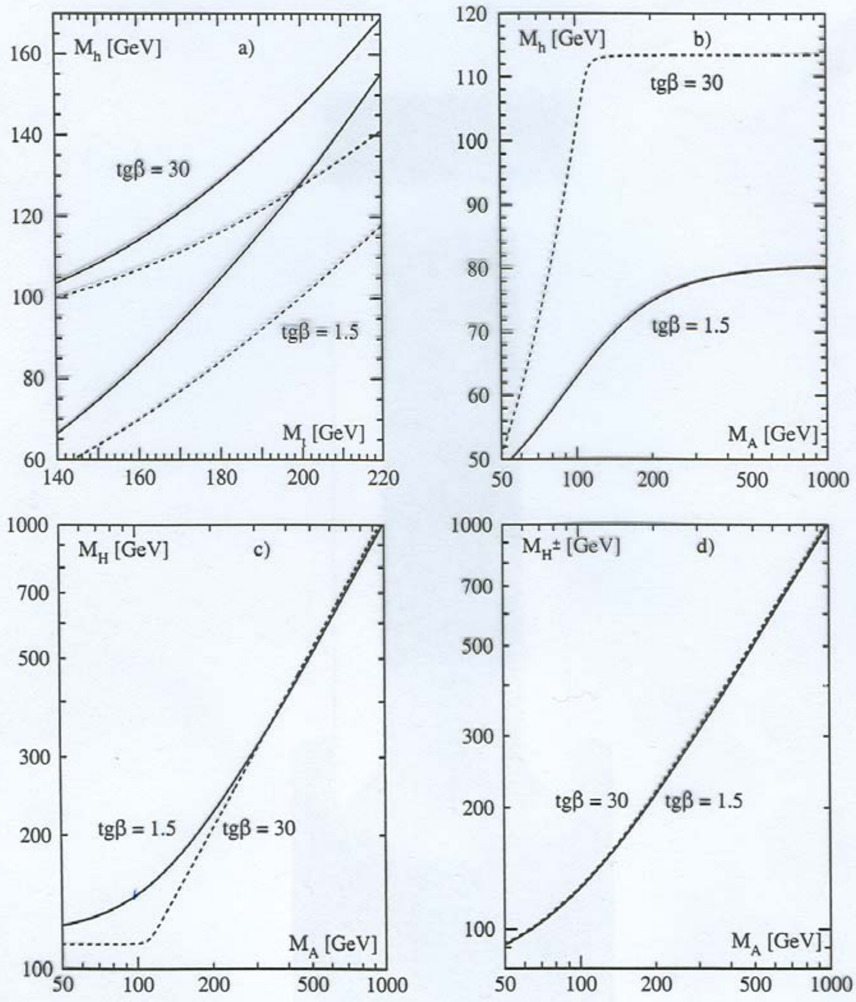
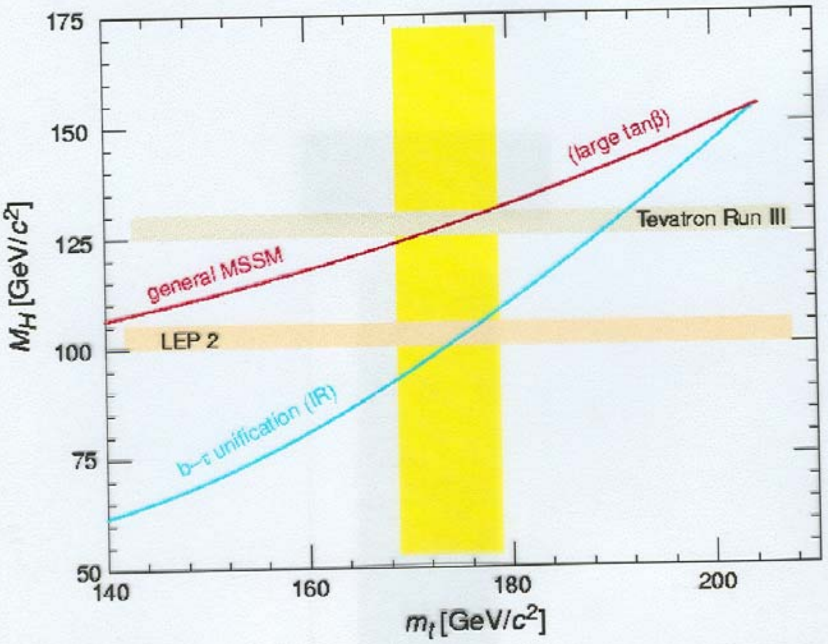


Figure 18: *(a) The upper limit on the light scalar Higgs pole mass in the MSSM as a function of the top quark mass for two values of* $tg\beta = 1.5, 30$*; the common squark mass has been chosen as* $M_S = 1$ *TeV. The full lines correspond to the case of maximal mixing* $[A_t = \sqrt{6}M_S,\ A_b = \mu = 0]$ *and the dashed lines to vanishing mixing. The pole masses of the other Higgs bosons,* $H, A, H^{\pm}$*, are shown as a function of the pseudoscalar mass in (b–d) for two values of* $tg\beta = 1.5, 30$*, vanishing mixing and* $M_t = 175$ *GeV.*



Carena
Espinosa
Quiros
Wagner

Fig Upper bounds on the mass of the lightest Higgs boson, as a function of the top-quark mass, in two variants of the minimal supersymmetric standard model. The upper curve refers to a general MSSM, in the large-tan β limit; the lower curve corresponds to an infrared-fixed-point scenario with b-τ unification, from Ref. [77].

Quantum Reduction of Couplings

Consider a GUT with

g – gauge coupling

g_i – other couplings (Yukawas, self-couplings)

Any relation among the couplings

$$\Phi(g, g_1, \cdots) = \text{const}$$

which is RGI should satisfy

$$\frac{d}{dt}\Phi = 0 \quad , \quad t = \ln\mu$$

$$\frac{d}{dt}\Phi = \frac{\partial \Phi}{\partial g} \overset{\beta_g}{\overset{|||}{\frac{dg}{dt}}} + \sum_i \frac{\partial \Phi}{\partial g_i} \overset{\beta_i}{\overset{|||}{\frac{dg_i}{dt}}} = 0$$

which is equivalent to

$$\frac{dg}{\beta_g} = \frac{dg_1}{\beta_1} = \frac{dg_2}{\beta_2} = \cdots \quad \text{characteristic system}$$

$$\Longrightarrow \quad \beta_g \frac{dg_i}{dg} = \beta_i$$

Reduction eqs
Oehme
Zimmermann

Demand power series solution to the REs

$$g_i = \sum_{n=0} \rho_i^{(n+1)} g^{2n+1}$$

Remarkably, uniqueness of these solutions can be decided already at 1-loop!

Assume

$$\beta_i = \frac{1}{16\pi^2}\left[\sum_{j,k,l} \beta_i^{(1)jkl} g_j g_k g_l + \sum_{j\neq g} \beta_i^{(1)j} g_i g^2\right] + \cdots$$

$$\beta_g = \frac{1}{16\pi^2} \beta_g^{(1)} g^3 + \cdots$$

Assume $\rho_i^{(n)}$, $n \leq r$ have been uniquely determined

To obtain $g_i^{(r+1)}$, insert g_i in REs and collect terms of $O\left(g^{2r+1}\right)$

$$\rightsquigarrow \sum_{\ell\neq g} M(r)_i^{\ell}\, \rho_\ell^{(r+1)} = \text{lower order quantities}$$

Known by assumption

where

$$M(r)_i^\ell = 3\sum_{j,k\neq g} b_i^{(1)jk\ell}\rho_j^{(1)}\rho_k^{(1)} + b_i^{(1)\ell} - (2r+1)\, b_g^{(1)}\delta_i^\ell$$

$$0 = \sum_{j,k,\ell\neq g} b_i^{(1)jk\ell}\rho_j^{(1)}\rho_k^{(1)}\rho_\ell^{(1)} + \sum_{\ell\neq g} b_i^{(1)\ell}\rho_\ell^{(1)} - b_g^{(1)}\rho_i^{(1)}$$

$\Rightarrow$ for a given set of $\rho_i^{(1)}$, the $\rho_i^{(n)}$ for all $n>1$ can be uniquely determined if

$$\det M(n)_i^\ell \neq 0$$

for all n

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$\Rightarrow$ for a given set of $\rho_i^{(1)}$, the $\rho_i^{(n)}$ for all $n > 1$ can be uniquely determined if

$$\det M(n)_i^\ell \neq 0$$

for all n

Kubo et al.

Consider an $SU(N)$ (non-susy) theory with

$\phi^i(N)$, $\hat{\phi}_i(\bar{N})$ - complex scalars:

$\psi^i(N)$, $\hat{\psi}_i(\bar{N})$ - Weyl spinors

$\lambda^a\,(a=1,\dots,N^2-1)$ - "

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\sqrt{2}\left[g_Y\bar{\psi}\lambda^a T^a\phi - \hat{g}_Y\bar{\hat{\psi}}\lambda^a T^a\hat{\phi} + h.c. - V(\phi,\hat{\phi}),\right.$$

$$V(\phi,\hat{\phi}) = \frac{1}{4}\lambda_1(\phi^i\phi^*_i)^2 + \frac{1}{4}\lambda_2(\hat{\phi}_i\hat{\phi}^{*i})^2 + \lambda_3(\phi^i\phi^*_i)(\hat{\phi}_j\hat{\phi}^{*j}) + \lambda_4(\phi^i\phi^*_j)(\hat{\phi}_i\hat{\phi}^{*j})$$

Searching for power series solution of the R.E.s we find

$$g_Y = \hat{g}_Y = g;\ \lambda_1 = \lambda_2 = \frac{N-1}{N}g^2;\ \lambda_3 = \frac{1}{2N}g^2;\ \lambda_4 = -\frac{1}{2}g^2$$

i.e. SUSY

Kubo et.al.

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$\lambda^a \, (a=1,\dots,N^2-1)$ - ″

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + i\sqrt{2} \left[g_Y \bar{\psi} \lambda^a T^a \phi - \hat{g}_Y \bar{\hat{\psi}} \lambda^a T^a \hat{\phi} + h.c. - V(\phi, \hat{\phi}), \right.$$

$$V(\phi, \hat{\phi}) = \frac{1}{4} \lambda_1 (\phi^i \phi_i^*)^2 + \frac{1}{4} \lambda_2 (\hat{\phi}_i \hat{\phi}^{*i})^2 + \lambda_3 (\phi^i \phi_i^*)(\hat{\phi}_j \hat{\phi}^{*j}) + \lambda_4 (\phi^i \phi_j^*)(\hat{\phi}_i \hat{\phi}^{*j})$$

Searching for power series solution of the R.E.s we find

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i.e. SUSY

$N=1$ gauge theories

Consider a chiral, anomaly free, $N=1$ globally supersymmetric gauge th. based on a group G with gauge coupling g.

Superpotential

$$W = \frac{1}{2} m_{ij} \phi^i \phi^j + \frac{1}{6} C_{ijk} \phi^i \phi^j \phi^k$$

m_{ij}, C_{ijk} - gauge invariant tensors

ϕ^i - matter fields transforming as an ir. rep. R_i of G.

Renormalization constants associated with W

$$\phi^{0i} = (Z^i_j)^{1/2} \phi^j \,, \quad m^0_{ij} = Z^{i'j'}_{ij} m_{i'j'} \,, \quad C^0_{ijk} = Z^{i'j'k'}_{ijk} C_{i'j'k'}$$

$N=1$ non-renormalization thm ensures absence of mass and cubic-int-term infinities

$$Z^{ijk}_{i'j'k'} Z^{1/2\, i'}_{i''} Z^{1/2\, j'}_{j''} Z^{1/2\, k'}_{k''} = \delta^i_{(i''} \delta^j_{j''} \delta^k_{k'')}$$

$$Z^{ij}_{i'j'} Z^{1/2\, i'}_{i''} Z^{1/2\, j'}_{j''} = \delta^i_{(i''} \delta^j_{j'')}$$

(In the background field method)

$$Z_g Z_V^{1/2} = 1$$

→ Only surviving infinities are Z^i_j (Z_V)
i.e. one infinity for each field.

Renormalization constants associated with W

$$\phi^{0i} = (Z^i_j)^{1/2} \phi^j \,, \quad m^0_{ij} = Z^{i'j'}_{ij} m_{i'j'} \,, \quad C^0_{ijk} = Z^{i'j'k'}_{ijk} C_{i'j'k'}$$

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$$Z^{ij}_{i'j'} Z^{1/2\, i'}_{i''} Z^{1/2\, j'}_{j''} = \delta^i_{(i''} \delta^j_{j'')}$$

(In the background field method)

$$Z_g Z_V^{1/2} = 1$$

$\rightarrow$ Only surviving infinities are $Z^i_j (Z_V)$ i.e. one infinity for each field.

The 1-loop β-function of the gauge coupling is

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2}\left[\sum_i \ell(R_i) - 3C_2(G)\right]$$

$\ell(R_i)$ – Dynkin index of R_i

$C_2(G)$ – quadratic Casimir of the adjoint rep.

β-functions of C_{ijk}, by virtue of the non-renormalization thm, are related with the anomalous dim. matrix γ_i^j of ϕ^i

$$\beta_{ijk}^{(1)} = \frac{dC_{ijk}}{dt} = C_{ijl}\gamma^l_k + C_{ikl}\gamma^l_j + C_{jkl}\gamma^l_i$$

$$\gamma^{(1)j}_i = Z^{-\frac{1}{2}}{}^k_i \frac{d}{dt} Z^{\frac{1}{2}}{}^j_k$$

$$= \frac{1}{32\pi^2}\left[C^{jkl}C_{ikl} - 2g^2 C_2(R_i)\delta_i^j\right]$$

$C_2(R_i)$ – quadratic Casimir of R_i

$C^{ijk} = C^*_{ijk}$

$$b_g^{(2)} = \frac{1}{(16\pi^2)^2}\, 2\, g^5 \left[\sum_i \ell(R_i) - 3C_2(G)\right]$$

$$- \frac{1}{(16\pi^2)^2}\, \frac{g^3}{r}\, C_2(R_i)\left[C^{jkl}\, C_{ikl} - 2g^2 C_2(R_i)\,\delta^j_i\right]$$

$r : \mathrm{tr}\,\delta^{ab}$

Parkes, West, Jones
Mezincescu, Yau
Machacek, Vaughn

$$\gamma^{(2)i}_{\ j} = \frac{1}{(16\pi^2)^2}\, 2\, g^4\, C_2(R_i)\left[\sum_i \ell(R_i) - 3C_2(G)\right]$$

$$- \frac{1}{(16\pi^2)^2}\, \frac{1}{2}\left[C^{ikl}\, C_{jkm} + 2g^2 (R^a)^i_{\ m} (R^a)^l_{\ j}\right]$$

$$\cdot \left[C^{mpq}\, C_{lpq} - 2\,\delta^m_l\, g^2\, C_2(R_i)\right]$$

$$b_g^{NSVZ} = \frac{g^3}{16\pi^2}\left[\frac{\sum_i \ell(R_i)\left(1 - 2\gamma_i\right) - 3\,C_2(G)}{1 - g^2 C_2(G)/8\pi^2}\right]$$

Novikov – Shifman – Vainstein – Zakharov

$$\ell(R_i)\,\delta_{ab} = \mathrm{Tr}\,(T_a T_b)$$

$$C_2(R_i)\,\delta_{ij} = \sum_a (T^a T^a)_{ij}$$

where $T_a\,[a = 1, \dots, \dim(adj)]$ is the a-th generator in the R_i rep.

$$C_2(R_i) = \frac{\dim(adj)}{\dim(R_i)}\,\ell(R_i) \qquad \parallel$$

e.g. in $SU(N)$

N has $\ell = 1 \rightsquigarrow C_2(N) = \frac{N^2-1}{N}$

N^2-1 " $\ell = 2N \rightsquigarrow C_2(N^2-1) = 2N$

Finite Unification

Old days...

... divergences are "hidden under the carpet" (Dirac, Lects on Q.F.T.,'64)

Recent years ...

... divergences reflect existence of a higher scale where new degrees of freedom are excited. Not just artifacts of pert. th.

However the presence of quadratic divergences means that physics at one scale are very sensitive to unkown physics at higher scales.

⤳ SUSY ths which are free of quadratic divergences in spite of any experimental evidence...

⤳ Natural to expect that beyond unification scale the theory should be completely finite.

- $N=4$ ⤳ finite to all orders in pert.
- $N=2$ ⤳ only 1-loop contributions to β-function. Possible to arrange the spectrum so that theory is finite.

Multiplicities for massless irreducible reps with maximal helicity 1

N Spin	1	1	2	2	4
1	–	1	–	1	1
1/2	1	1	2	2	4
0	2	–	4	2	6

$$N=2 \ : \ b(g) = \frac{2g^3}{(4\pi)^2}\left(\sum_i T(R_i) - C_2(G)\right)$$

e.g. SU(N) with 2N fundamental reps $\longrightarrow b(g) = 0$

$SU(5) : p(5+\bar{5})\,;\ q(10+\overline{10})\,;\ r(15+\overline{15})$

with $p+3q+7r=10$

$SO(10) : p(10+\overline{10})\,;\ q(16+\overline{16})$

with $p+2q=8$

$E_6 \ : 4(27+\overline{27})$

Consider a chiral, anomaly free, $N=1$ gauge theory with group G.

The superpotential is

$$W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j$$

$$\left.\begin{matrix} Y^{ijk} \\ \mu^{ij} \end{matrix}\right\} \text{gauge invariant}$$

Yukawa couplings

Φ_i - matter superfields in irreducible reps of G

Necessary and sufficient conditio
for $N=1$ 1-loop finiteness

- Vanishing of $b_g^{(1)}$ implies

$$\sum_i \ell(R_i) = 3\, C_2(G) \qquad \|$$

$\ell(R_i)$ - Dynkin index of R_i

$C_2(G)$ - Quadratic Casmir of G (adjoint)

$\Rightarrow$ Selection of the field content (representations) of the theory

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CHIRAL TWO-LOOP-FINITE SUPERSYMMETRIC THEORIES ☆

Shahram HAMIDI, J. PATERA [1] and John H. SCHWARZ
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Received 2 April 1984

Any globally supersymmetric theory in four dimensions that is one-loop finite is automatically (at least) two-loop finite. We classify all such theories that are chiral and have a simple gauge group.

One of the less satisfying aspects of GUTs ("grand-unified theories") and super GUTs is that they contain many arbitrary parameters. One principle that could serve to limit the number of parameters is the requirement of finiteness. This possibility has been raised with the discovery that there are large classes of supersymmetric gauge theories that are free from ultraviolet divergences at all orders of perturbation theory [‡1]. The theories for which this has been established are all $N = 4$ and some [2] $N = 2$ super Yang–Mills theories. Finiteness allows some parameters to be introduced via mass and soft supersymmetry-breaking terms, but it relates all the dimensionless couplings. Unfortunately, all $N = 2$ or $N = 4$ theories are nonchiral ("vector-like") and do not appear suited to the construction of a realistic model. Recently, the possibility has been raised that certain $N = 1$ theories could also be finite [3,4]. A theory containing Yang–Mills and chiral superfields is ultraviolet finite at loop provided that certain conditions (described below) restricting the representations and couplings of the chiral superfields are satisfied. It has been proved by direct calculation [3] and by considerations involving the chiral anomaly [4] that these conditions ensure two-loop finiteness as well, without any additional restrictions. It is an open question whether any of the $N = 1$ theories of this class are finite beyond two loops. The purpose of this letter is to list all chiral solutions of the one-loop conditions that are based on a simple gauge group.

Consider a globally supersymmetric $N = 1$ theory in four dimensions with a simple Yang–Mills group G. In addition to the gauge superfield, it can contain chiral superfields in an arbitrary representation R of G with irreducible components R_i:

$$\mathrm{R} = \bigoplus_i \mathrm{R}_i \,. \tag{1}$$

Our task is to find the possible choices of R and associated couplings that ensure one-loop finiteness. We only consider chiral theories ($\mathrm{R} \neq \overline{\mathrm{R}}$). This restricts G to those groups that have complex representations, namely SU(n) with $n \geqslant 3$, SO($4k + 2$) with $k \geqslant 2$, and E_6. Cancellation of the gauge-current anomaly

$$A(\mathrm{R}) = \sum_i A(\mathrm{R}_i) = 0 \tag{2}$$

is also imposed, since it is a necessary requirement for a consistent quantum theory. The anomaly condition is nontrivial only for SU(n).

There are two additional conditions required by one-loop finiteness [3,4]. The first is the one-loop finiteness of the gauge-field self energy. The condition is

$$l(\mathrm{R}) = \sum_i l(\mathrm{R}_i) = 3C_2(\mathrm{G}) \,, \tag{3}$$

where $l(\mathrm{R}_i)$ is the "index" of R_i [5] and $C_2(\mathrm{G})$ is

---

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[‡1] For a review see ref. [1].

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the eigenvalue of the second-order Casimir operator (which coincides with the index of the adjoint representation). Since indices are always positive (except for singlets, which are excluded), eq. (3) already limits R to a finite number of possibilities for a given group G.

The second condition is the one-loop finiteness of the chiral superfield self-energy. In terms of coefficients d describing the cubic self-coupling of the chiral superfields in the superpotential, the condition is

$$\sum_{\substack{b,c\\ j,k}} d^{ijk}_{abc}\bar{d}^{i'jk}_{a'bc} = 2g^2\delta_{aa'}\delta_{ii'}C_2(\mathrm{R}_i)\,. \qquad (4)$$

The subscripts a, b, c label components of the representations $\mathrm{R}_i, R_j, \mathrm{R}_k$.

The only irreducible representations that can occur in R are ones whose indices do not exceed $3C_2(\mathrm{G})$. Singlets (with nonzero couplings) are excluded by (4). All relevant representations of the groups with complex representations are listed in table 1. It also gives the indices and anomalies, normalized to be unity for the fundamental representation. Complex-conjugate representations, which have the same index and opposite anomaly, are not shown.

In seeking solutions to eqs. (2)–(4) it is convenient to consider first the trace of (4) given by summing over $a = a'$ and the m_α values of $i = i'$ for which $\mathrm{R}_i = \mathrm{R}_\alpha$. This results in conditions of the form

$$\sum_{\beta\gamma} |C_{\alpha\beta\gamma}|^2 = m_\alpha l(\mathrm{R}_\alpha)\,. \qquad (5)$$

Eq. (5) is weaker than (4), but it is useful for quickly eliminating many candidates from the list of admissible R's. Detailed examination of (4) then eliminates

Table 1
Properties of relevant irreducible representations

	SU(n)					
Representation						
						
Dimension	n	n(n-1)/2	n(n+1)/2	n^2-1	n(n-1)(n-2)/6	n(n-1)(n-2)(n-3)/24
Index	1	n-2	n+2	2n	(n-2)(n-3)/2	(n-2)(n-3)(n-4)/6
Anomaly	1	n-4	n+4	0	(n-3)(n-6)/2	(n-3)(n-4)(n-8)/6

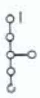
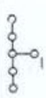
	O(4k+2)				E(6)	
Representation						
				↑		
Dimension	4k + 2	2(k+1)(4k+1)	(4k+2)(4k+1)/2	2^{2k}	27	78
Index	1	4k + 4	4k	2^{2k-3}	1	4

Table 2
Multiplicities m_α of the irreducible components R_α of R for all the solutions.

Irrep	27	$\overline{27}$	Comments
E(6)	n	$12 - n$	$7 \leqslant n \leqslant 12$

Irrep	10	54	45	16	$\overline{16}$	Comments
SO(10)	8	0	0	n	$8 - n$	$5 \leqslant n \leqslant 8$
	2	1	1	1	0	
	$12 - 2m - 2n$	1	0	n	m	$n + m \leqslant 4$, $n > m$

Irrep	□	$\overline{□}$	⊟	$\overline{⊟}$	□□	$\overline{□□}$	Adj
SU(n)	$2n - 4$	$2n + 4$	0	1	1	0	0
$n \geqslant 7$	$n - 4$	$n + 4$	0	1	1	0	1

Irrep	8	$\overline{8}$	28	70	63
SU(8)	1	5	1	1	1

Irrep	3	$\overline{3}$	6	8
SU(3)	3	10	1	0
	0	7	1	1

Irrep	4	$\overline{4}$	15	6	10
SU(4)	0	8	1	1	1
	4	12	0	1	1

Irrep	5	$\overline{5}$	10	$\overline{10}$	15	$\overline{15}$	24
SU(5)	3	14	2	0	1	0	0
	6	14	0	1	1	0	0
	5	7	4	2	0	0	0
	5	10	5	0	0	0	0
	I → 6	9	4	1	0	0	0
	7	8	3	2	0	0	0
	8	10	3	1	0	0	0
	1	2	1	0	1	1	1
	1	9	0	1	1	0	1
	2	3	3	2	0	0	1
	3	5	3	1	0	0	1
	II → 4	7	3	0	0	0	1
	5	6	2	1	0	0	1
	6	8	2	0	0	0	1
	8	9	1	0	0	0	1
	3	4	1	0	0	0	2

Irrep	6	$\overline{6}$	15	$\overline{15}$	21	$\overline{21}$	20	35
SU(6)	0	16	3	0	1	0	0	0
Sibold et. al. →	8	16	0	1	1	0	0	0
	0	4	5	3	0	0	0	0
	3	5	4	3	0	0	0	0
	0	12	6	0	0	0	0	0
	2	10	5	1	0	0	0	0
	4	8	4	2	0	0	0	0
	8	12	3	1	0	0	0	0
	0	2	1	0	0	0	3	1
	0	4	2	0	0	0	2	1
	3	5	1	0	0	0	2	1
	0	6	3	0	0	0	1	1
	2	4	2	1	0	0	1	1
	3	7	2	0	0	0	1	1
	6	8	1	0	0	0	1	1
	1	3	1	0	1	1	0	1
	2	10	0	1	1	0	0	1
	1	3	3	2	0	0	0	1
	0	8	4	0	0	0	0	1
	2	6	3	1	0	0	0	1
	3	9	3	0	0	0	0	1
	5	7	2	1	0	0	0	1
	6	10	2	0	0	0	0	1
	9	11	1	0	0	0	0	1
	0	4	2	0	0	0	0	2
	3	5	1	0	0	0	0	2
	0	2	1	0	0	0	1	2

some [: for SU(5) and 11 for SU(6)] of the class allowed by (2), (3), and (5). The complete list of complex representations satisfying (2), (3) and (4) is given in table 2. The conjugate representations, which are also solutions, are not tabulated. In most cases the couplings are uniquely determined (up to a change of basis), but in a few cases there are free parameters or discrete alternatives. Note that there are no solutions for SO($4k + 2$) with $k > 2$.

Scanning the tables for potentially realistic schemes,

•• Vanishing of $\gamma^{(1)i}_{\ \ j}$ implies

$$\underset{\text{Yukawa}}{\underset{\uparrow}{Y^{ikl}}}\, Y_{jkl} = 2\,\delta^i_j\, \underset{\text{gauge}}{\underset{\uparrow}{g}}{}^2 C_2(R_i) \quad \|$$

$C_2(R_i)$ – quadratic Casimir of R_i

$Y_{ijk} = (Y_{ijk})^*$

⟹ Yukawa and gauge couplings are related.

Note • μ^{ij} are not restricted

•• Appearence of $U(1)$ is incopatible with • 1st condt.

••• 2nd condt forbids the presence of singlets with nonvanishing couplings.

∴ ⟹ ~~Susy~~ by G-invariant soft terms

* 1-loop finiteness condts necessary and sufficient to guarantee 2-loop finiteness

* 1-loop finiteness condts ensure that $b_g^{(3)}$ in 3-loops vanishes but in general $\gamma^{(3)}$ does not.

Grisaru - Milewski - Zanon

Parkes - West

What happens in higher loops?

So far 1-loop finiteness condts (on γs) are telling us

$$Y^{ijk} = Y^{ijk}(g)$$

$$b_Y^{(1)ijk} = 0$$

* * Necessary and sufficient condts for vanishing β_g and β_{ijk} to all orders

Lucchesi
Piguet
Sibold

1. $\beta_g^{(1)} = 0$

2. $\gamma^{(1)i}_{\ \ \ \ j} = 0$

3. $\beta_Y^{ijk} = \beta_g \dfrac{dY^{ijk}}{dg}$

admit power series solution which in lowest order is a solution of condt 2.

3. ↗ 3'. There exist solutions to $\gamma^{(1)i}_{\ \ \ \ j}=0$ of the form
$Y^{ijk} = \rho^{ijk} g$, ρ^{ijk} - complex

↘ 4. These solutions are isolated and non-degenerate considered as solutions of $\beta_Y^{(1)ijk} = 0$

Recall

R-invariance, axial anomaly

In massless N=1 ths

U(1) chiral transformation R:

$$A_\mu \to A_\mu \quad , \quad \lambda \to e^{-i\alpha}\lambda \, ,$$

$$\phi \to e^{-i\frac{2}{3}\alpha}\phi \, , \quad \psi \to e^{i\frac{1}{3}\alpha}\psi \, , \quad \cdots$$

$$\psi_D = \begin{pmatrix} \psi \\ \bar{\chi} \end{pmatrix} \longrightarrow e^{i\alpha\gamma_5}\psi_D$$

Noether current $J^\mu_R = \bar{\lambda}_D \gamma^\mu \gamma^5 \lambda_D + \cdots$

$$\rightsquigarrow \partial_\mu J^\mu_R = r\left(\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \cdots\right)$$

$r = b_g^{(1)}$!

Only 1-loop contributions due to Adler-Bardeen non-renormalization thm

Supercurrent

$$\mathcal{J} \equiv \{ J'^{\mu}_{R} , \; Q^{\mu}_{\alpha} , \; T^{\mu}_{\nu} \} , \cdots$$

vector super multiplet

associated to R-invariance

associated to susy

associated to translation inv.

Ferrara + Zumino

(supercurrent is represented as vector superfield

$$V_{\mu}(x,\theta,\bar{\theta}) = R_{\mu}(x) - i\theta^{\alpha} Q_{\mu\alpha}(x) + i\bar{\theta}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}}_{\mu}(x) - 2(\theta\sigma^{\nu}\bar{\theta}) T_{\mu\nu}(x) + \cdots$$
)

- $J'^{\mu}_{R} \neq J^{\mu}_{R}$

- $J'^{\mu}_{R} = J^{\mu}_{R} + O(\hbar)$

In addition

Clark
Piquet
Sibold

$$\mathcal{S} = \{ b_g F^{\mu\nu} F_{\mu\nu} + \cdots , \; b_g \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \cdots , \; b_g \lambda^{\beta} \sigma^{\mu\nu}_{\alpha\beta} F_{\mu\nu} + \cdots , \; \cdots \}$$

Super trace anomaly

trace anomaly of T^{μ}_{ν}

anomaly of R-current

trace anomaly of susy current

chiral super multiplet

There is a relation, whose structure is independent from the renormalization scheme, although individual coefficients (except the 1-loop values of β-functions) may be scheme dependent

$$r = \beta_g \left(1 + x_g\right) + \beta_{ijk}\, x^{ijk} - \gamma_A\, r^A$$

radiative corrections

unrenormalized coefficients of anomalies associated to chiral inv. of superpotential

Linear combinations of anomalous dims

Thm: If (i) no gauge anomaly
(ii) $\beta^{(1)}(g) = 0$ i.e. no R-current anomaly
(iii) $\gamma^{(1)}{}^i_j = 0$ implies also $r^A = 0$
(iv) exist solutions to $\gamma^{(1)} = 0$ of the form $C_{ijk} = \rho_{ijk}\, g$, ρ_{ijk} - complex
(v) these solutions are isolated + non-degenerate

when considered as solutions of $b^{(1)}_{ijk} = 0$.

- Then each of the solutions can be uniquely extended to a formal power series in g, and the $N = 1$ Y-M models depend on the single coupling constant g with a b-function vanishing to all orders.

Proof: Inserting $b_{ijk} = b_g \frac{d f_{ijk}}{dg}$ in the identity and taking into account the vanishing of r, r^A

$$\rightsquigarrow \quad 0 = b_g \left(1 + O(\hbar) \right)$$

Its solution (as formal power series in $\hbar$) is: $b_g = 0$ and $b_{ijk} = 0$ too. //

2-loop RGEs for SSB parameters

Martin-Vaughn-Yamada-Jack-Jones '94

Consider N=1 gauge thy with

$$W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j$$

and SSB terms

$$-\mathcal{L}_{SB} = \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j$$

$$+ \frac{1}{2} (m^2)^i_j \phi^{*i} \phi_j + \frac{1}{2} M \lambda\lambda + \text{h.c.}$$

- 1-loop finiteness conditions

$$h^{ijk} = - M Y^{ijk}$$

universality

$$(m^2)^i_j = \frac{1}{3} M M^* \delta^i_j$$

in addition to $\beta^{(1)}_g = \gamma^{(1)i}_j = 0$

- - 1-loop finiteness

$\rightarrow$ 2-loop finiteness

Assuming

- $b_g^{(1)} = \gamma^{(1)}{}^j_i = 0$
- the reduction eq

$$b_Y^{ijk} = b_g \, dY^{ijk}/dg$$

admits power series solution

$$Y^{ijk} = g \sum_{n=0} \rho_{(n)}^{ijk} g^{2n}$$

- $(m^2)^i_j = m_j^2 \delta^i_j$

$$\rightsquigarrow (m_i^2 + m_j^2 + m_k^2)/MM^* = 1 + \frac{g^2}{16\pi^2} \Delta^{(1)}$$

for i,j,k with $\rho_{(0)}^{ijk} \neq 0$

where $\Delta^{(1)} = -2 \sum_l [(m_l^2/MM^*) - \frac{1}{3}] C(R_l)$

- $\Delta^{(1)} = 0$ for $N=4$ with STr cond
- $\Delta^{(1)} = 0$ for the $N=1$, $SU(5)$ FUTs !

The SU(5) finite model

Kapetanakis, Mondragon, Z

Kobayashi, Kubo, Mondragon, Z

Content

$$3(\bar{5}+10)+4(\overset{H_\alpha}{5}+\overset{\bar{H}_\alpha}{\bar{5}})+24$$

fermion supermultiplets

scalar supermultiplets

Hamidi-Schwarz

Jones-Raby

Quiros et al.

Kazakov

Babu-Enkhbat

Gogoladze

Imposing a discrete symmetry

$$\to W=\sum_{i=1}^{3}\sum_{\alpha=1}^{4}\left[\frac{1}{2}g^{u}_{i\alpha}\,10_i\,10_i\,H_\alpha + g^{d}_{i\alpha}\,10_i\,\bar{5}_i\,\bar{H}_\alpha\right]+\sum_{\alpha=1}^{4}g^{f}_{\alpha}H_\alpha\,24\,\bar{H}_\alpha + \frac{g^{\lambda}}{3}(24)^3$$

with $g^{u,d}_{i\alpha}=0$ for $i\neq\alpha$

We find

$$b_g^{(1)} = 0$$

$$b_{i\alpha}^{u(1)} = \frac{1}{16\pi^2}\Big[-\frac{96}{5}g^2 + \sum_{b=1}^{4}\left(g^u_{ib}\right)^2 + 3\sum_{j=1}^{3}\left(g^u_{ja}\right)^2 + \frac{24}{5}\left(g^f_\alpha\right)^2 + 4\sum_{b=1}^{4}\left(g^d_{ib}\right)^2\Big]g^u_{i\alpha}$$

$$b_{i\alpha}^{d(1)} = \frac{1}{16\pi^2}\Big[-\frac{84}{5}g^2 + 3\sum_{b=1}^{4}\left(g^u_{ib}\right)^2 + \frac{24}{5}\left(g^f_\alpha\right)^2 + 4\sum_{j=1}^{3}\left(g^d_{j\alpha}\right)^2 + 6\sum_{b=1}^{4}\left(g^d_{ib}\right)^2\Big]g^d_{i\alpha}$$

$$b^{\lambda(1)} = \frac{1}{16\pi^2}\Big[-30g^2 + \frac{63}{5}\left(g^\lambda\right)^2 + 3\sum_{\alpha=1}^{4}\left(g^f_\alpha\right)^2\Big]g^\lambda$$

$$b_\alpha^{f(1)} = \frac{1}{16\pi^2}\Big[-\frac{98}{5}g^2 + 3\sum_{i=1}^{3}\left(g^u_{i\alpha}\right)^2 + 4\sum_{i=1}^{3}\left(g^d_{i\alpha}\right)^2 + \frac{48}{5}\left(g^f_\alpha\right)^2 + \sum_{b=1}^{4}\left(g^f_b\right)^2 + \frac{21}{5}\left(g^\lambda\right)^2\Big]g^f_\alpha$$

Considering g as the primary coupling, we solve the Red. Eqs.

$$\beta_g = \beta_a \frac{dg}{dg_a}$$

requiring power series ansatz.

$\Rightarrow (g^u_{ii})^2 = \frac{8}{5} g^2 + \cdots , (g^d_{ii})^2 = \frac{6}{5} g^2 + \cdots$

$(g^\lambda)^2 = \frac{15}{7} g^2 + \cdots , (g^f_4)^2 = g^2 , (g^f_\alpha)^2 = 0 + \cdots (\alpha = 1,2,3)$

Higher order terms can be uniquely determined.

$\Rightarrow$ All 1-loop β-functions vanish

Moreover

All 1-loop anomalous dimensions of chiral fields vanish.

$$\gamma^{(1)}_{10_i} = \frac{1}{16\pi^2}\left[-\frac{36}{5}g^2 + 3\sum_{b=1}^{4}\left(g^u_{ib}\right)^2 + 2\sum_{b=1}^{4}\left(g^d_{ib}\right)^2\right]$$

$$\gamma^{(1)}_{\bar{5}_i} = \frac{1}{16\pi^2}\left[-\frac{24}{5}g^2 + 4\sum_{b=1}^{4}\left(g^d_{ib}\right)^2\right]$$

$$\gamma^{(1)}_{H_\alpha} = \frac{1}{16\pi^2}\left[-\frac{24}{5}g^2 + 3\sum_{i=1}^{3}\left(g^u_{i\alpha}\right)^2 + \frac{24}{5}\left(g^f_\alpha\right)^2\right]$$

$$\gamma^{(1)}_{\bar{H}_\alpha} = \frac{1}{16\pi^2}\left[-\frac{24}{5}g^2 + 4\sum_{i=1}^{3}\left(g^d_{i\alpha}\right)^2 + \frac{24}{5}\left(g^f_\alpha\right)^2\right]$$

$$\gamma^{(1)}_{24} = \frac{1}{16\pi^2}\left[-\frac{10}{5}g^2 + \sum_{\alpha=1}^{4}\left(g^f_\alpha\right)^2 + \frac{21}{5}\left(g^\lambda\right)^2\right]$$

$\Rightarrow$ Necessary and sufficient conds for finiteness to all orders are satisfied

- SU(5) breaks down to the standard model due to <24>

- Use the freedom in mass parameters to obtain only a pair of Higgs fields light, acquiring v.e.v.

- Get rid of unwanted triplets rotating the Higgs sector (after a fine tuning)

  see Quiros et. al., Kazakov et. al
  Yioshioka

- Adding soft terms we can achieve SUSY breaking.

1) Gauge Couplings Unification

$\sin^2\theta_W , \alpha_{em} \rightsquigarrow \alpha_3(M_Z)$

Marciano+Senjanović
⋮
Amaldi et. al.

2) Bottom-Tau Yukawa Unif.

SU(5)-type

$\rightsquigarrow m_t \sim 100-200$ GeV

Barger et. al.
Carena et. al.

*3) Top-Bottom-Tau Yuk Unif.

$h_t^2 = \frac{4}{3} h_{b,\tau}^2$ in SU(5)-FUT

Similar to SO(10)

$\rightsquigarrow m_t \sim 160-200$ GeV

Ananthanarayan et. al.
Barger et. al.
Carena et. al.

*4) Gauge-Top-Bottom-Tau Unif.

e.g. FUT-SU(5): $h_t^2 = \frac{8}{5} g_U^2 ; h_{b,\tau}^2 = \frac{6}{5} g_U^2$

M_s [GeV]	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	$M_{\rm GUT}$ [GeV]	M_b [GeV]	M_t [GeV]
300	0.123	54.1	2.2×10^{16}	5.3	183
500	0.122	54.2	1.9×10^{16}	5.3	183
10^3	0.120	54.3	1.5×10^{16}	5.2	184

FUTA

M_s [GeV]	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	$M_{\rm GUT}$ [GeV]	M_b [GeV]	M_t [GeV]
800	0.120	48.2	1.5×10^{16}	5.4	174
10^3	0.119	48.2	1.4×10^{16}	5.4	174
1.2×10^3	0.118	48.2	1.3×10^{16}	5.4	174

FUTB

M_s [GeV]	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	$M_{\rm GUT}$ [GeV]	M_b [GeV]	M_t [GeV]
300	0.123	47.9	2.2×10^{16}	5.5	178
500	0.122	47.8	1.8×10^{16}	5.4	178
1000	0.119	47.7	1.5×10^{16}	5.4	178

MIN SU(5)

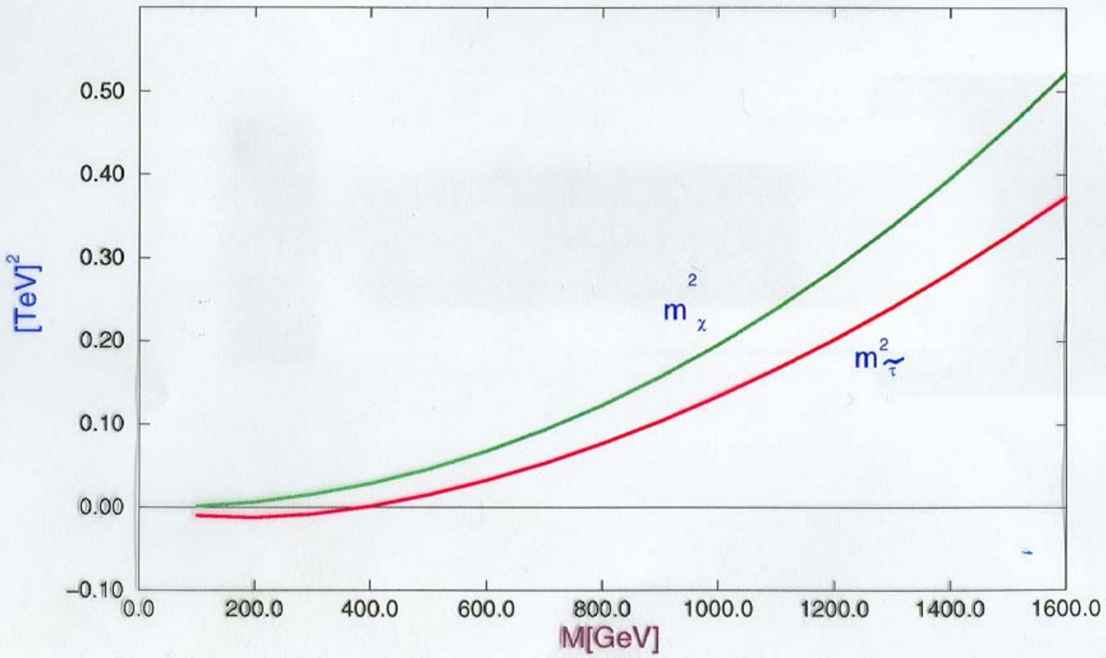
The predictions for the three models for different M_s

With theoretical corrections and uncertainties
~ 4%

$M_t = 173.8 \pm 5$ GeV; 178.0 ± 4.3 GeV
CDF + D0

Model A

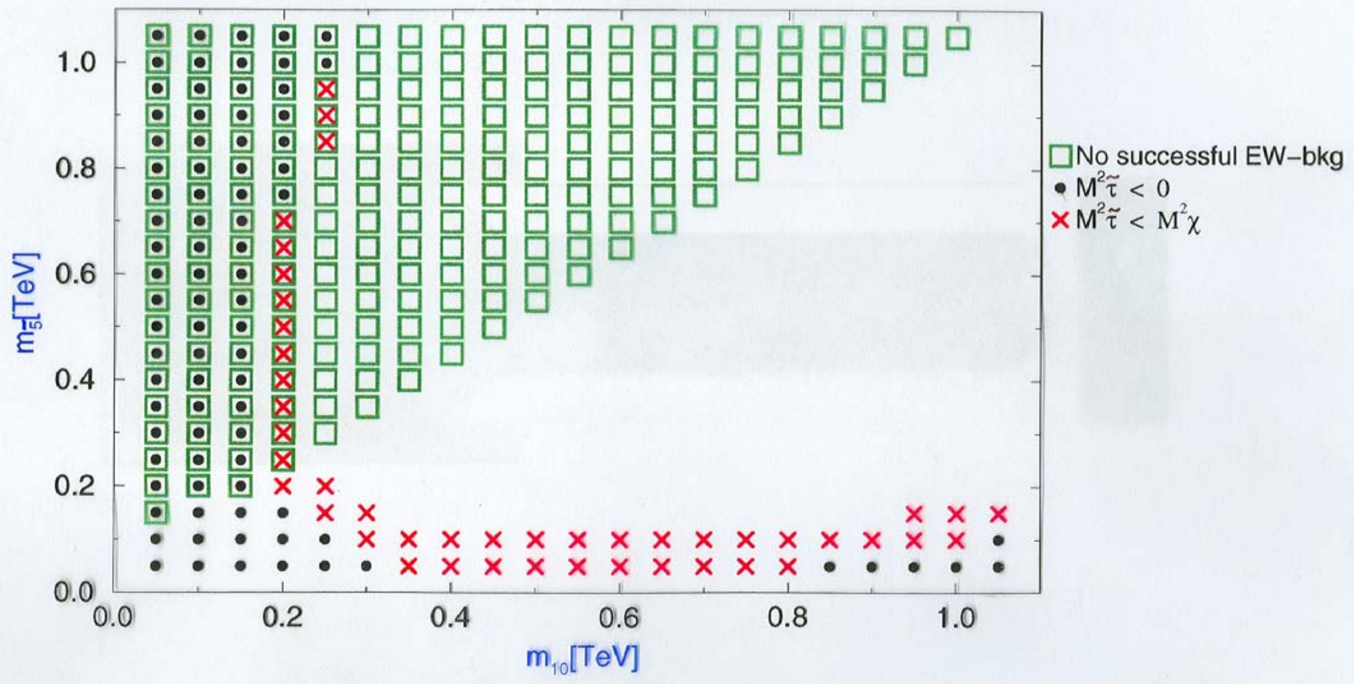
Similar behaviour holds for Model B too



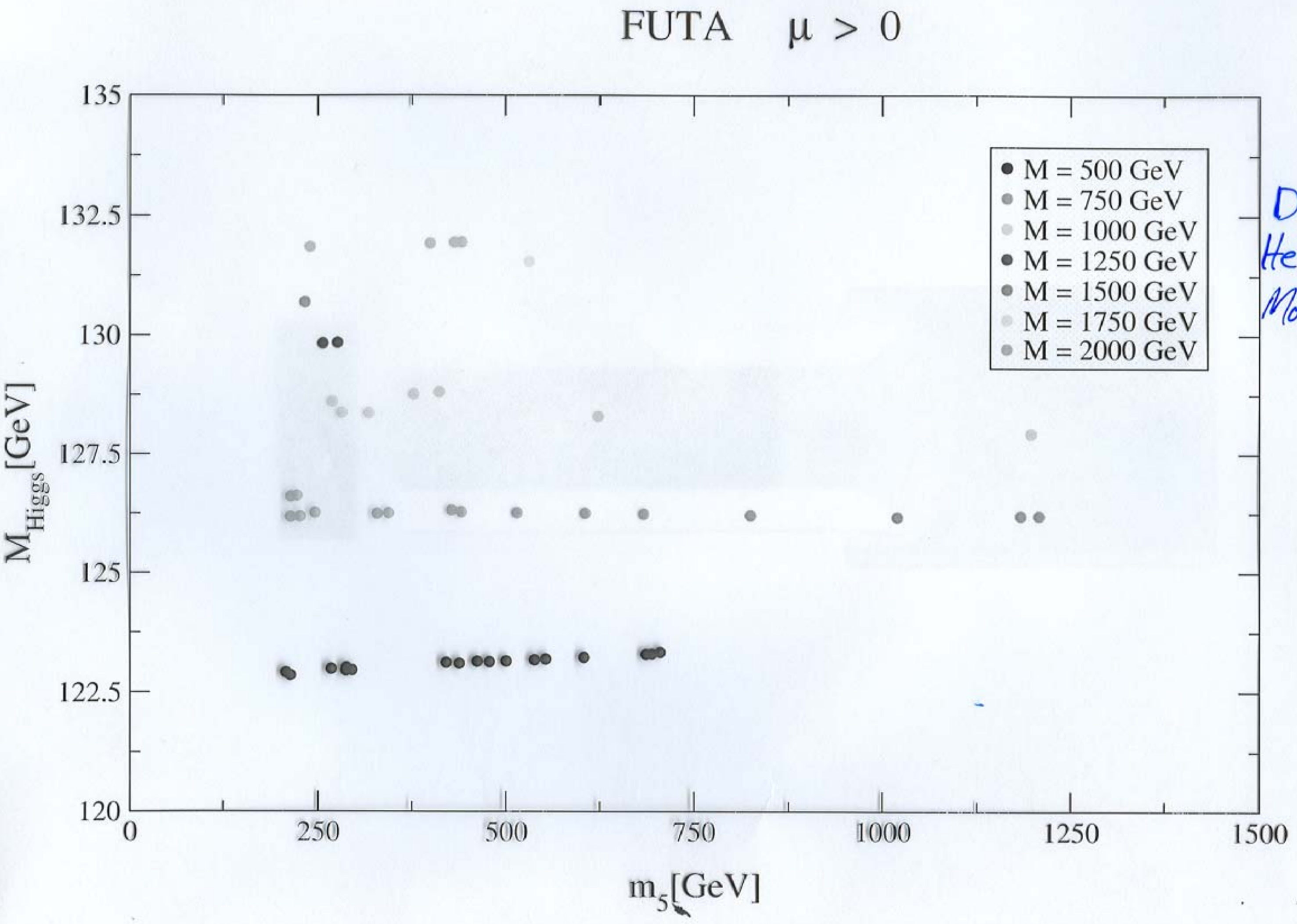
$m^2_{\tilde{\tau}}$ and m^2_χ for the universal choice of soft scalar masses

Model A

$M_{susy} = 0.3$ TeV



The empty region yields a neutralino as LSP



Djouadi
Heinemeyer
Mondragon
Z

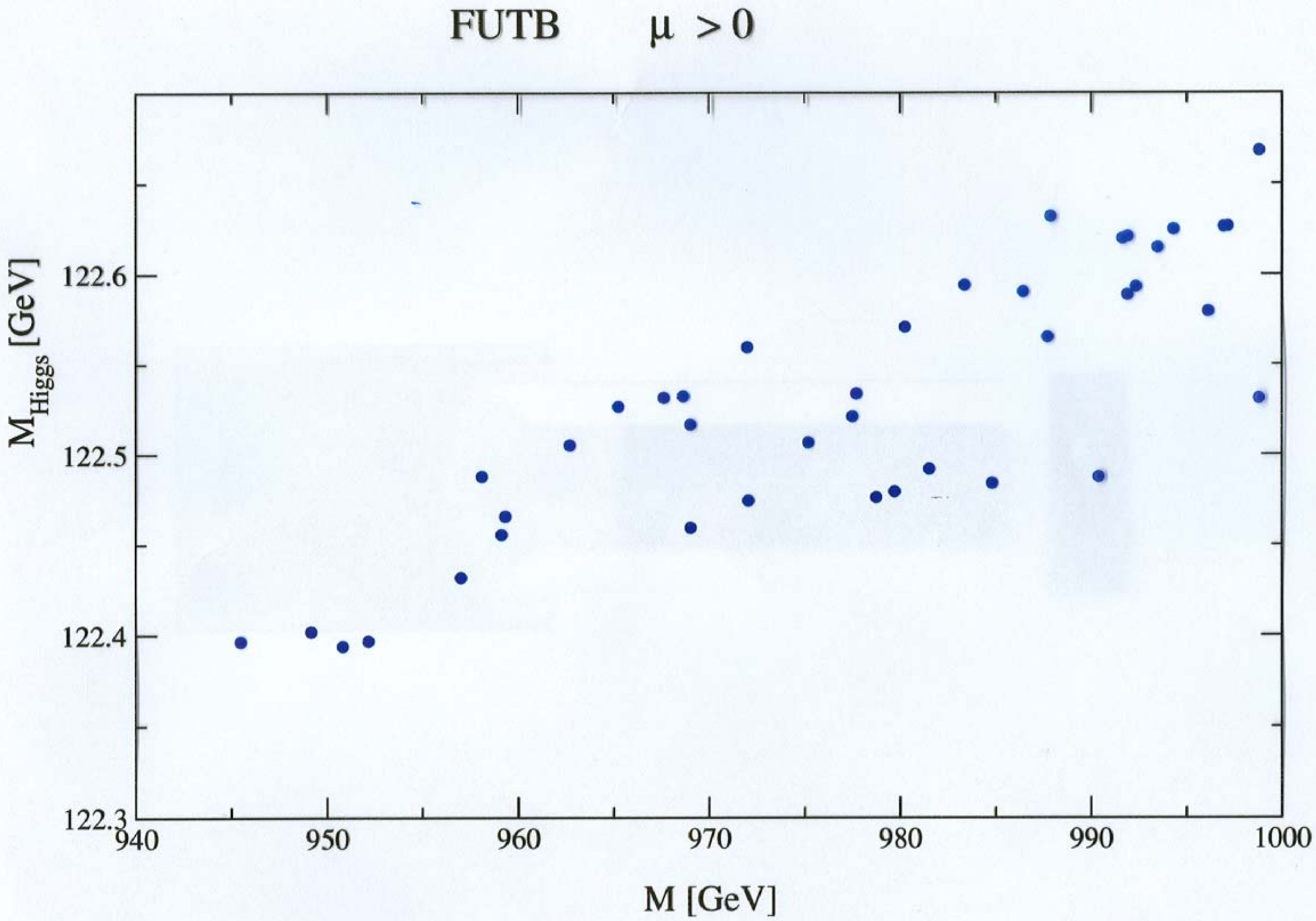

FUTB
μ > 0
M_Higgs [GeV]
122.3
122.4
122.5
122.6
940
950
960
970
980
990
1000
M [GeV]

Table 3: A representative example of the predictions for the s-spectrum for the model **A**.

$m_\chi = m_{\chi_1}$ (TeV)	0.22	$m_{\tilde{b}_2}$ (TeV)	1.06
m_{χ_2} (TeV)	0.41	$m_{\tilde{\tau}} = m_{\tilde{\tau}_1}$ (TeV)	0.33
m_{χ_3} (TeV)	0.93	$m_{\tilde{\tau}_2}$ (TeV)	0.54
m_{χ_4} (TeV)	0.94	$m_{\tilde{\nu}_1}$ (TeV)	0.41
$m_{\chi_1^\pm}$ (TeV)	0.41	m_A (TeV)	0.44
$m_{\chi_2^\pm}$ (TeV)	0.94	$m_{H^\pm}$ (TeV)	0.45
$m_{\tilde{t}_1}$ (TeV)	0.94	m_H (TeV)	0.44
$m_{\tilde{t}_2}$ (TeV)	1.09	m_h (TeV)	0.12
$m_{\tilde{b}_1}$ (TeV)	0.86		

Table 4: A representative example of the predictions of the s-spectrum for the model **B**.

$m_\chi = m_{\chi_1}$ (TeV)	0.44	$m_{\tilde{b}_2}$ (TeV)	1.79
m_{χ_2} (TeV)	0.84	$m_{\tilde{\tau}} = m_{\tilde{\tau}_1}$ (TeV)	0.47
m_{χ_3} (TeV)	1.38	$m_{\tilde{\tau}_2}$ (TeV)	0.69
m_{χ_4} (TeV)	1.39	$m_{\tilde{\nu}_1}$ (TeV)	0.62
$m_{\chi_1^\pm}$ (TeV)	0.84	m_A (TeV)	0.74
$m_{\chi_2^\pm}$ (TeV)	1.39	$m_{H^\pm}$ (TeV	0.75
$m_{\tilde{t}_1}$ (TeV)	1.60	m_H (TeV	0.74
$m_{\tilde{t}_2}$ (TeV)	1.82	m_h (TeV)	0.12
$m_{\tilde{b}_1}$ (TeV)	1.56		

Coset Space Dimensional Reduction
(CSDR)

Original motivation

Use higher dimensions

- to unify the gauge and Higgs sectors
- to unify the fermion interactions with gauge and Higgs fields

Supersymmetry provides further unification (fermions in adj reps)

Forgacs + Manton ; Manton

Chapline + Slansky

Kubyshin + Mourao + Rudolph + Volobujev - book

Kapetanakis + G. Z. - Phys. Rept

Manousselis + G. Z., Phys. Lett. B504, 122 (01); PLB518, 171 (01); JHEP03, 002 (02); JHEP11, 025 (04)

Further successes

(a) chiral fermions in 4 dims from vector-like reps in the higher dim thy.

(b) the metric can be deformed (in certain non-symmetric coset sp) and more than one scales can be introduced

(c) Wilson flux breaking can be used

ADD

- Softly broken susy chiral ths in 4 dims can result from a higher dimensional susy theory

Theory in D dims → Thy in 4 dims

1. Compactification $\underset{x^M}{M^D} \to \underset{x^\mu}{M^4} \times \underset{y^a}{B}$

B – a compact space

$\dim B = D - 4 = d$

2. Dimensional Reduction

Demand that $\mathcal{L}$ is independent of the extra y^a coordinates

- One way: Discard the field dependence on y^a coordinates

•• An elegant way: Allow field dependence on y^a and employ a symmetry of the Lagrangian to compensate.

Obvious choice: Gauge Symmetry

Allow a non-trivial dependence on y^a, but impose the condition that a symmetry transformation by an element of the isometry group S of B is compensated by a gauge transformation.

$\Longrightarrow$ $\mathcal{L}$ independent of y^a just because is gauge invariant.

Integrate out extra coordinates

CSDR: $B = S/R$

$$S: \; Q_A = \{ \underset{\substack{| \\ R}}{Q_i} , \underset{\substack{| \\ S/R}}{Q_a} \}$$

$$[Q_i, Q_j] = f_{ij}^{\ \ k} Q_k \quad , \quad [Q_i, Q_a] = f_{ia}^{\ \ b} Q_b$$

$$[Q_a, Q_b] = f_{ab}^{\ \ i} Q_i + \underset{\uparrow}{f_{ab}^{\ \ c}} Q_c$$

vanishes in symmetric S/R

Consider a Yang-Mills-Dirac thy in D dims based on group G defined on $M^D \longrightarrow M^4 \times S/R$, $D=4+d$

$$g^{MN} = \begin{pmatrix} \eta^{\mu\nu} & 0 \\ 0 & -g^{ab} \end{pmatrix}, \quad \eta^{\mu\nu} = \mathrm{dig}(1,-1,-1,-1)$$

g^{ab} - coset space metric

$d = \dim S - \dim R$

$$A = \int d^4x\, d^dy \sqrt{-g} \Big[-\frac{1}{4} Tr(F_{MN} F_{K\Lambda})\, g^{MK} g^{N\Lambda} + \frac{i}{2} \Psi \Gamma^M D_M \Psi \Big]$$

$$D_M = \partial_M - \theta_M - A_M \;, \quad \underset{\text{spin connection of } M^D}{\theta_M} = \frac{1}{2} \theta_{MN\Lambda} \Sigma^{N\Lambda}$$

Ψ in rep F of G

We require that any transformation by an element of S acting on S/R is compensated by gauge transformations.

$$A_\mu(x,y) = g(s)\, A_\mu(x, s^{-1}y)\, g^{-1}(s)$$

$$A_\alpha(x,y) = g(s)\, J_\alpha^{\,b} A_b(x, s^{-1}y)\, g^{-1}(s) + g(s)\, \partial_\alpha\, g^{-1}(s)$$

$$\psi(x,y) = f(s)\, \Omega\, \psi(x, s^{-1}y)\, f^{-1}(s)$$

g, f - gauge transformations in the ad_J, F of G corresponding to the s transf. of S acting on S/R

$J_\alpha^{\,b}$ - Jacobian for s

Ω - Jacobian + local Lorentz rotation in tangent space

Above conditions imply constraints that D-dims fields should obey.

Solution of constraints $\left\{ \right.$

- 4-dim fields
- Potential
- Remaining gauge invariance

Taking into account all the constraints and integrating out the extra coordinates, we obtain in 4 dims

$$A = C \int d^4x \Big(-\frac{1}{4} \operatorname{Tr} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \sum_{\alpha} \operatorname{Tr}(D_\mu \phi_a D^\mu \phi_a)$$
$$+ V(\phi) + \frac{i}{2} \bar{\psi} \underset{\text{Kinetic terms}}{\Gamma^\mu D_\mu} \psi - \frac{i}{2} \bar{\psi} \underset{\text{mass terms}}{\Gamma^a D_a} \psi \Big)$$

$$D_\mu = \partial_\mu - A_\mu , \; D_a = \partial_a - \theta_a - \phi_a , \; \underset{\substack{\text{spin connection} \\ \text{of coset space}}}{\theta_a} = \frac{1}{2} \theta_{abc} \Sigma^{bc}$$

C - volume of coset space

$$V(\phi) = -\frac{1}{4} g^{ac} g^{bd} \operatorname{Tr}\Big\{ \big(f_{ab}^{\;\;C} \phi_C - [\phi_a, \phi_b] \big) \big(f_{cd}^{\;\;D} \phi_D - [\phi_c, \phi_d] \big) \Big\}$$

$A = 1, \ldots, \dim S$, f - structure const. of S

Still $V(\phi)$ only formal since ϕ_a must satisfy $f_{ai}^{\;\;D} \phi_D - [\phi_a, \phi_i] = 0$

- The 4-dim gauge group

$$H = C_G(R_G)$$

i.e. $$\underset{\substack{\uparrow\\ \text{higher}\\ \text{dim group}}}{G} \supset R_G \times \underset{\substack{\uparrow\\ \text{4-dim}\\ \text{group}}}{H}$$

- • Scalar fields

$$S \supset R$$

$$\text{adj}\, S = \text{adj}\, R + v$$

$$G \supset R_G \times H$$

$$\text{adj}\, G = (\text{adj}\, R, 1) + (1, \text{adj}\, H) + \sum (r_i, h_i)$$

If $v = \sum s_i$

when $s_i = r_i \Longrightarrow h_i$ survives in 4 dims

• • • Fermions

$G \supset R_G \times H$

$F = \sum (t_i, h_i)$

spinor of $SO(d)$ under R

$\sigma_d = \sum \sigma_j$

for every $t_i = \sigma_i \Rightarrow h_i$ survives in 4 dims

Possible to obtain a chiral theory in 4 dims even starting with Weyl (+ Majorana) fermions in vector-like reps of G in $D = 4n+2$ dims.

$\Gamma^{D+1}\psi = \pm\psi$ Weyl cond

In $D = 4n+2$ dims Dirac spinors ($2^{[D/2]}$ comp) ψ are defined as direct sum of Weyl spinors

$$\psi = \psi_+ \oplus \psi_- = 6_D + \bar{6}_D$$

non-self conjugate spinors

The $SO(4) \times SO(N)$ branching rule is

$$6_D = (2, 1; 6_N) + (1, 2; \bar{6}_N)$$

$$\bar{6}_D = (2, 1; \bar{6}_N) + (1, 2; 6_N)$$

The Weyl condition picks up 6_D or $\bar{6}_D$ and the Majorana cond (reverses the sign of all int. qu. nos) identifies the two irreducible parts of the decomposition

An easy case in calculating the potential and its minimization

If $G \supset S \Rightarrow H$ breaks to $K = C_G(S)$

$$\begin{array}{ccccl} G & \supset & S & \times & K \leftarrow \text{gauge group after spontaneous sym. breaking} \\ & & \cup & & \cap \\ G & \supset & R & \times & H \\ & & & & \uparrow \\ & & & & \text{gauge group in 4 dims} \end{array}$$

But

fermion masses

$$M^2 \psi = D_a D^a \psi - \frac{1}{4} R \psi - \frac{1}{2} \Sigma^{ab} \underset{\underset{0}{\shortparallel}}{F_{ab}} \psi > 0$$

if $\overset{0}{S} \subset G$

comparable to the compactification scale

Supersymmetry breaking by dim reduction over Symmetric CS. $\left(\text{e.g. } SO_7/SO_6 \right)$

Consider $G = E_8$ in 10 dims with Weyl-Majorana fermions in the adjoint of E_8, i.e. a susy E_8

Embedding of $R = SO(6)$ in E_8 is suggested by the decomposition

$$E_8 > SO(6) \times SO(10)$$

$$248 = (15,1) + (1,45) + (6,10) + (4,16) + (\bar{4},\overline{16})$$

$$\text{adj } S = \text{adj } R + v$$

$$21 = 15 + 6 \leftarrow \text{vector}$$

Spinor of $SO(6)$: 4

In 4 dims we obtain a gauge thy based on

$$H = C_{E_8}(SO(6)) = SO(10)$$

with scalars in 10

and fermions in 16

- Theorem: When S/R symmetric the potential necessarily leads to spont. breakdown of H.

- - Moreover in this case we have

$$\begin{array}{c} E_8 \supset SO(7) \times SO(9) \\ \quad\quad\; \cup \quad\quad\;\; \cap \\ E_8 \supset SO(6) \times SO(10) \end{array}$$

⇒ Final gauge group after breaking

$$K = C_{E_8}(SO(7)) = SO(9)$$

CSDR over symmetric coset spaces breaks completely original supersymmetry

Soft Supersymmetry Breaking
by CSDR over non-symmetric CS.

We have examined the dim. red of a supersymmetric E_8 over the 3 existing 6-dim CS: G_2/SU_3, $SP(4)/(SU(2)\times U(1))_{non-max}$, $SU(3)/U(1)\times U(1)$

$\Rightarrow$ Softly Broken Supersymmetric Theories in 4 dims without any further assumption

Non-symmetric CS admit torsion and the two latter more than one radii

Consider supersymmetric E_8 in 10 dims and $S/R = G_2/SU(3)$

We use the decomposition

$$E_8 \supset SU(3) \times E_6$$
$$248 = (8,1) + (1,78) + (3,27) + (\bar{3},\overline{27})$$

and choose $R = SU(3)$

$$\text{adj } S = \text{adj } R + v$$
$$14 = 8 + \underbrace{3+\bar{3}}_{\text{vector}}$$

Spinor : $1 + 3$ under $R = SU(3)$

$\Rightarrow$ 4 dim thy : $H = C_{E_8}(SU(3)) = E_6$

with scalars in $27 = 6$

and fermions in $27, 78$

i.e. spectrum of a supersymmetric E_6 thy in 4 dims

The Higgs potential of the genuine Higgs β

$$V(\beta) = 8 - \frac{40}{3}\beta^2 - \left[4 d_{ijk}\,\beta^i\beta^j\beta^k + h.c.\right]$$
$$+ \beta^i\beta^j d_{ijk}\, d^{klm}\,\beta_l\beta_m$$
$$+ \frac{11}{4}\sum_\alpha \beta^i (G^\alpha)_i^{\ j}\beta_j\,\beta^k (G^\alpha)_k^{\ l}\beta_l$$

which obtains F-terms contributions from the superpotential

$$W(B) = \frac{1}{3} d_{ijk} B^i B^j B^k$$

D-term contributions

$$\frac{1}{2} D^\alpha D^\alpha \quad , \quad D^\alpha = \sqrt{\frac{11}{2}}\;\beta^i (G^\alpha)_i^{\ j}\beta_j$$

The rest terms belong to the SSB part of the Lagrangian

$$\mathcal{L}_{scalar\,SSB} = -\frac{1}{R^2}\frac{40}{3}\beta^2 - \left[4 d_{ijk}\,\beta^i\beta^j\beta^k + h.c\right]\frac{9}{R}$$

$$M_{gaugino} = (1 + 3\tau)\frac{6}{\sqrt{3}}\;\frac{1}{R}$$

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Z

Fuzzy CSDR

$$M^D = M^4 \times (S/R)_F$$

matrix manifold
e.g. fuzzy sphere S^2_F

Instead of algebra of functions

$$\mathrm{Fun}(M^D) \sim \mathrm{Fun}(M^4) \times \mathrm{fun}(S/R)$$

we consider the algebra $\mathrm{fun}(M^4) \times M_n$

M_n - finite dim NC (non-com) algebra of matrices that approximates S/R

e.g. $S^2 \longrightarrow S^2_F$

Algebra of functions on S^2 can be generated by the coordinates of R^3 modulo the relation $\sum_{\hat{a}=1}^{3} x_{\hat{a}} x_{\hat{a}} = r^2$

S^2_F at fuzziness level N is the NC manifold with coordinate functions

$\hat{X}_{\hat{a}} = \kappa J^{\hat{a}}$ – $(N+1)$-dim rep of $SU(2)$

$(N+1)\times(N+1)$ herm matrices

$$\sum_{\hat{a}=1}^{3} \hat{X}_{\hat{a}} \hat{X}_{\hat{b}} = r^2, \qquad [\hat{X}_{\hat{a}}, \hat{X}_{\hat{b}}] = i\kappa\, C_{\hat{a}\hat{b}\hat{c}} \hat{X}_{\hat{c}}$$

$$\left\{ C_{\hat{a}\hat{b}\hat{c}} = \epsilon_{\hat{a}\hat{b}\hat{c}}/r, \quad \kappa = \lambda_N r, \quad \lambda_N = r/\sqrt{N/2\,(N/2+1)} \right.$$

or, if $X_{\hat{a}} = 1/i\kappa\, \hat{X}_{\hat{a}}$

- $$\sum_{\hat{a}=1}^{3} X_{\hat{a}} X_{\hat{a}} = -r^2/\kappa^2 = -\lambda_N^2, \quad [X_{\hat{a}}, X_{\hat{b}}] = C_{\hat{a}\hat{b}\hat{c}} X_{\hat{c}}$$

A function on S^2_F is a symmetric polynomial in $X^{\hat{a}}$. A convenient basis

$$1, \quad \hat{Y}_{lm} = r^{-l} \sum_{\hat{a}} f^{(lm)}_{\hat{a}_1, \hat{a}_2, \dots, \hat{a}_l} X^{\hat{a}_1} \dots X^{\hat{a}_l}, \quad l \leq N$$

NC spherical harmonic up to level N

$$f = \sum_{l=0}^{N} \sum_{m=-l}^{l} f_{lm} \hat{Y}_{lm}$$

generic function on S^2_F

On S_F^2 there is a natural SU(2) differential 3-dimensional calculus.

Derivations $e_{\hat{a}}(f) = [X_{\hat{a}}, f]$ of a function f along $X_{\hat{a}}$.

Lie derivatives on functions

$$\mathcal{L}_{\hat{a}} f = [X_{\hat{a}}, f]$$

Lie derivatives on 1-forms $\theta^{\hat{a}}$ (dual to $e_{\hat{a}}$)

$$\mathcal{L}_{\hat{a}}(\theta^{\hat{b}}) = C_{\hat{a}\hat{b}\hat{c}}\,\theta^{\hat{c}}$$

1-form A on $M^4 \times S_F^2$

$$A = \underset{A_\mu(X^\mu, X_{\hat{a}})}{A_\mu}\, dx^\mu + \underset{A_{\hat{a}} = A_{\hat{a}}(X^\mu, X_{\hat{a}})}{A_{\hat{a}}}\, \theta^{\hat{a}}$$

An infinitesimal gauge transformation

$$\delta\phi(X) = \underbrace{\lambda(X)}_{\text{gauge transf parameter}} \underbrace{\phi(X)}_{\text{field on fuzzy space described by NC coord. } X^{\hat{a}}}$$

$U(1)$ if $\lambda(X)$ function of $X^{\hat a}$

$U(P)$ if $\lambda(X)$ valued in Lie algebra of $P \times P$ matrices

- $\delta X_{\hat a} = 0 \rightsquigarrow \delta(X_{\hat a}\phi) = X_{\hat a}\lambda(X)\phi \neq \lambda(X)X_{a}\phi$
- $\delta(\underset{\text{covariant coordinates}}{\varphi_{\hat a}}\phi) = \lambda\varphi_{\hat a}\phi$

if $\delta(\varphi_{\hat a}) = [\lambda, \varphi_{\hat a}]$

- $\underset{\text{NC analogue of covariant der.}}{\varphi_{\hat a}} = X_{\hat a} + \underset{\text{interpreted as gauge fields}}{A_{\hat a}}$

$$\delta A_{\hat a} = -[X_{\hat a}, \lambda] + [\lambda, A_{\hat a}]$$

- $F_{\hat a\hat b} = [X_{\hat a}, A_{\hat b}] - [X_{\hat b}, A_{\hat a}] + [A_{\hat a}, A_{\hat b}] - C^{\hat c}_{\hat a\hat b}A_{\hat c}$

  $= [\varphi_{\hat a}, \varphi_{\hat b}] - C^{\hat c}_{\hat a\hat b}\varphi_{\hat c}$ analogue of field strength

  $\delta F_{\hat a\hat b} = [\lambda, F_{\hat a\hat b}]$
- $\delta\psi = [\lambda, \psi]$, spinor ψ in the adjoint

CSDR over $(S/R)_F$

$$G = U(P) \quad \text{on} \quad M^4 \times (S/R)_F$$

$$A_{YM} = \frac{1}{4} \int d^4x \, \underset{\substack{\text{integration}\\ \text{over } (S/R)_F}}{Tr} \, tr_G \, f_{MN} f^{MN}$$

$$f_{MN} \longrightarrow \left(f_{\mu\nu}, f_{\mu\hat{b}}, f_{\hat{a}\hat{b}} \right)$$

$$f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$

$$f_{\mu\hat{a}} = \partial_\mu A_{\hat{a}} - [X_{\hat{a}}, A_\mu] + [A_\mu, A_{\hat{a}}]$$

$$= \partial_\mu \varphi_{\hat{a}} + [A_\mu, \varphi_{\hat{a}}] = D_\mu \varphi_{\hat{a}}$$

$$f_{\hat{a}\hat{b}} = [\varphi_{\hat{a}}, \varphi_{\hat{b}}] - C^{\hat{c}}_{\hat{a}\hat{b}} \varphi_{\hat{c}}$$

$$\rightsquigarrow A_{YM}' = \int d^4x \, Tr \, tr_G \left(\frac{1}{4} f_{\mu\nu}^2 + \frac{1}{2} (D_\mu \varphi_{\hat{a}})^2 - V(\varphi) \right)$$

$$V(\varphi) = -\frac{1}{4} \, Tr \, tr_G \sum_{\hat{a}\hat{b}} f_{\hat{a}\hat{b}} f_{\hat{a}\hat{b}}$$

The G gauge transformation with parameter $\lambda(x^\mu, X^{\hat{a}})$ is now interpreted as M^4 gauge trans.

$$\lambda(x^\mu, X^{\hat{a}}) = \lambda^\alpha(x^\mu, X^{\hat{a}})\, T^\alpha$$
$$= \lambda^{h,\alpha}(x_\mu)\, T^h\, T^\alpha$$

T^α - generators of $U(P)$

$\lambda^\alpha(x^\mu, X^{\hat{a}})$ - $(N+1)(N+1)$ matrices expressible as $\lambda(x^\mu)^{\alpha,h} T^h$

T^h - generators of $U(N+1)$

$$\rightsquigarrow U((N+1)P)$$

Then we reduce the number of of gauge fields and scalars in A_{YM} applying the CSDR constraints e.g. $G = U(1)$, $(S/R)_F = S_F^2$

CSDR constraints are satisfied by embedding $SU(2)$ in $U(N+1)$

We find in four dims

- No H group (due to the fact that the diff. calculus is based on dim S derivations instead of dim S - dim R in ordinary case.)
- $K = C_{U(N+1)}(SU(2)) = U(N-1) \times U(1)$ as the final gauge group
- a harmless surviving Higgs

Similar results are obtained for $G = U(P)$

CSDR for more general $(S/R)_F$

(e.g. CP^M described by $(N+1)\times(N+1)$ matr.)

CSDR constraints are satisfied by embedding S in $U((N+1)P)$ and the 4-dim gauge theory is

$$K = C_{U((N+1)P)}(S)$$

Concerning fermions to solve the corresponding constraints we embed

$$S \subset SO(\dim S)$$

$$U((N+1)P) \supset S_{U((N+1)\cdot P)} \times K$$

$$\mathrm{adj}\, U((N+1)P) = (\mathrm{adj}\, S, 1) + (1, \mathrm{adj}\, K) + \sum_i (s_i, k_i)$$

$$SO(\dim S) \supset S$$

$$\text{spinor } \sigma = \sum_i \sigma_i$$

for $s_i = \sigma_i \rightarrow k_i$ surviving

Major difference among ordinary and fuzzy-CSDR

- 4-dim gauge theory appears already spontaneously broken

⤳ in 4 dims appears only the physical Higgs that survives SSB

⤳ Yukawa sector

(i) massive fermions

(ii) interactions among fermions and physical Higgs fields.

⟹ if we obtain in fuzzy-CSDR the SM ⤳ large extra dims

Fundamental differences among ordinary and fuzzy-CSDR:

• A non-abelian gauge group is not necessary in high dims.
The presence of a $U(1)$ in the higher-dim theory is enough to obtain non-abelian gauge theories in 4 dims.

•• The theory is renormalizable in the sense that divergencies can be removed by a finite number of counterterms.